$$y_n = \sum_m c_{nm} f_m$$

$$(f_l, y_n) = \sum_m c_{nm} (f_l, f_m)$$

$$c_{nl} = (f_l, y_n)$$

$$H_0 f_n(x) = \Omega \omega_{0_n}^2 f_n(x)$$

$$E \frac{d^2}{dx^2} \left(W T^3 \frac{d^2 y_n}{dx^2} \right) = -\rho W T \frac{d^2 y_n}{dt^2}$$

Tension (T) is constant, so it can be divided out:

$$E \frac{d^2}{dx^2} \left(W \frac{d^2 y}{dx^2} \right) = -\frac{\rho W}{T^2} \frac{d^2 y}{dt^2}$$

The derivatives need to be distributed, because Width (W) is dependent on x:

$$E \frac{d}{dx} \left(\frac{dW}{dx} \frac{d^2 y}{dx^2} + W \frac{d^3 y}{dx^3} \right) = \frac{\rho W}{T^2} \omega^2 y$$

$$\frac{d^2 W}{dx^2} \frac{d^2 y}{dx^2} + 2 \frac{dW}{dx} \frac{d^3 y}{dx^3} + W \frac{d^4 y}{dx^4} = \frac{\rho W}{E T^2} \omega^2 y$$

$$\Omega = \frac{\rho}{E T^2}$$