Reconciling Antimo's and my Normalizations

Nov 14

In my scheme I write the cross section down first and then derive a likelihood function from it.

$$(1) \qquad \frac{d^5\sigma}{d^2m\, d^3\Omega} = \sigma_1 B(m) \left[c^+ \hat{W}(\Omega) c\right] + \sigma_0 P(m)\, U(\Omega)$$

where σ_1 = total $\phi\phi$ cross section
σ_0 = total non-resonant cross section (4K)

$B(m)$ = $\phi\phi$ mass plot density normalized to 1 over the peak (3 over whole mass plot)
$P(m)$ = 4K phase-space mass plot density normalized to 3 over the whole mass plot.

$[c^+\hat{W}(\Omega)c]$ = $\phi\phi$ angular distribution on $d^3\Omega$ that has been normalized to 1 over all angles
$U(\Omega)$ = phase-space angular distribution on $d^3\Omega$ normalized to 1 over all angles $= (4\pi)^{-3}$

Measured counts are given by

$$(2) \qquad \frac{d^5N}{d^2m\, d^3\Omega} = \frac{d\sigma}{d^2m\, d^3\Omega} \cdot L \cdot \mathcal{A}(m, \Omega)$$

where L = integrated luminosity

$\mathcal{A}(m,\Omega)$ = detector/reconstruction efficiency as a function of kinematics.

Eq. (2) tells the normalization in terms of event counts. For example, the total $\phi\phi$ count is

$$N_{\phi\phi} = \int \frac{d^5N_{\phi\phi}}{d^2m\, d^3\Omega} d^2m\, d^3\Omega = L\sigma_1 \int d^2m\, d^3\Omega\, B(m)\, \mathcal{A}(m,\Omega) \left[c^+ \hat{W}(\Omega) c\right]$$
$$\equiv \sigma_1 \left[c^+ \hat{\Omega}\, c\right]$$

where for convenience I introduce a new matrix $\hat{\Omega}$ defined as

$$\hat{\Omega}_{ij} \equiv L \int d^2m\, d^3\Omega\, B(m)\, \mathcal{A}(m,\Omega)\, \hat{W}_{ij}(\Omega)$$

which plays the role of (luminosity $\times$ acceptance) when sandwiched between c^+ and c.

From this I can define my likelihood function for event i

$$(3) \qquad l_i = \left[\frac{\sigma_1 B(m_i) [c^+ \hat{W}(\Omega_i) c] + \sigma_0 P(m_i) \mathcal{U}}{\sigma_1 [c^+ \hat{\Omega} c] + \sigma_0 \Omega_0} \right] a(m_i, \Omega_i)$$

where by analogy to $\hat{\Omega}$, $\Omega_0 \equiv L \int d^2 m \, d^3\Omega \, P(m) \, a(m, \Omega) \, \mathcal{U}$.
Ω_0 is just the ordinary phase-space acceptance.

I impose the normalization of the $\{c_i\}$ by fitting with one of the $c_i = 1$ and then after the fit rescaling σ and forcing the condition $\int d^3\Omega \, [c^+ W c] = 1$ to make σ_1 into the total $\phi\phi$ cross section.

When this is done then the partial cross section for wave j is just $\sigma_1 |c_j|^2 \left(\frac{2J_j+1}{4\pi}\right)$. The later factor comes from the normalization we used for the $W_{ij}(\Omega)$ functions.

- 14 In Ardimo's scheme, we write the likelihood first and then derive the cross section later.

$$(1A) \qquad l_i = \frac{w_i \, [c^+ \hat{W}(\Omega_i) c]}{[c^+ \hat{I} c]} + (1 - w_i)$$

where w_i = $\phi\phi$ weight $(0 \leq w_i \leq 1)$ for event i from channel likelihood fit

$\hat{I}$ = normalization integral defined on Monte Carlo reconstructed data set as

$$\hat{I}_{ij} = \frac{1}{M_{\phi\phi}} \sum_{\nu}^{M_{\phi\phi}} \hat{W}_{ij}(\Omega_\nu)$$

This fit has fewer degrees of freedom because it takes the channel likelihood results as given. The normalization of $\{c_i\}$ in the fit is taken as $c_1 = 1$. The results then have to be interpreted to give a cross section.

To get this result, I need an expression for w_i

$$(2A) \qquad w_i = \frac{N_{\phi\phi} B(m_i)}{N_{\phi\phi} B(m_i) + N_0 P(m_i)} = \text{local } \phi\phi \text{ fraction on the mass plot}$$

So if I rewrite the likelihood factoring out $(1-w_i)$

$$l_i \to \left(\frac{w_i}{1-w_i} \frac{[c^+ \hat{W}(\Omega_i) c]}{[c^+ \hat{I} c]} + 1 \right)$$

then
$$\frac{w_i}{1-w_i} = \left(\frac{N_{\phi\phi}}{N_0}\right)\left(\frac{B(m_i)}{P(m_i)}\right)$$

This allows me to recast the likelihood as equivalent to

$$l_i \to \left(N_{\phi\phi} \; B(m_i) \frac{[c^+ \hat{W}(\Omega_i) c]}{[c^+ \hat{I} c]} + N_0 P(m_i) \right)$$

$$\cancel{= \sigma_1 B(m_i) [c^+ \hat{W}(\Omega_i) c] \left\{ \frac{[c^+ \hat{\Omega} c]}{[c^+ \hat{I} c]} \right\} + \sigma_0 P(m_i) \mathcal{U} \left\{ \frac{\Omega_0}{\mathcal{U}} \right\}}$$

~~which is equivalent to (1)~~

where $N_{\phi\phi}$ and N_0 are the results from channel likelhd. Up to an overall constant, l_i has to be the measured density, that is the cross section times (acceptance $\times L$). We need to find the overall constant K so that

$$(3A) \qquad \frac{d^5 N}{d^2 m \, d^3 \Omega} = K \, l(m,\Omega) \, a(m,\Omega) \, L$$
$$= K \left(N_{\phi\phi} B(m) \frac{[c^+ \hat{W}(\Omega) c]}{[c^+ \hat{I} c]} + N_0 P(m) \right) a(m,\Omega) L$$

To find K I look for consistency with channel likelhd.

$$\int \frac{d^5 N}{d^2 m \, d^3 \Omega} \, d^2 m \, d^3 \Omega = N_{\phi\phi}$$

$$= \frac{K L N_{\phi\phi}}{[c^+ \hat{I} c]} \int d^2 m \, d^3 \Omega \, B(m) [c^+ \hat{W}(\Omega) c] \, a(m,\Omega)$$

$$= K L N_{\phi\phi} \frac{[c^+ \hat{\Omega} c]}{[c^+ \hat{I} c]}$$

In terms of integrals

$$[c^\dagger \hat{I}_j c] \equiv \frac{1}{M_{\phi\phi}} \sum_{\nu}^{M_{\phi\phi}} [c^\dagger \hat{W}(\Omega_\nu) c] = \frac{\int d^2m\, d^3\Omega\, B(m)\, a(m,\Omega)\, [c^\dagger \hat{W}(\Omega) c]}{\int d^2m\, d^3\Omega\, B(m)\, a(m,\Omega)}$$

$$= \frac{[c^\dagger \hat{\Omega} c]}{L\,(4\pi)^3\, \bar{a}_1} \quad \text{where} \quad \bar{a}_1 \equiv \frac{1}{(4\pi)^3} \int d^2m\, B(m) \int d^3\Omega\, a(m,\Omega)$$

Therefore $\dfrac{[c^\dagger \hat{\Omega} c]}{[c^\dagger \hat{I} c]} = (4\pi)^3 L \bar{a}_1$, $\Rightarrow$ $K = \dfrac{1}{(4\pi)^3 L \bar{a}_1}$ (*)

So get cross section from $\dfrac{d^5N}{d^2m\, d^3\Omega}$ just drop L and $a(m,\Omega)$

$$(4A) \qquad \frac{d^5\sigma}{d^2m\, d^3\Omega} = \left(\frac{1}{4\pi}\right)^3 \left\{ \left(\frac{N_{\phi\phi}}{L\bar{a}_1}\right) B(m) \frac{[c^\dagger \hat{W}(\Omega) c]}{[c^\dagger \hat{I} c]} + \left(\frac{N_0}{L\bar{a}_1}\right) P(m) \right\}$$

This gives us the answer. Integrating gives the total cross section for $\phi\phi$ as a sum over waves

$$(5A) \qquad \sigma_{\phi\phi} = \frac{1}{(4\pi)^3 [c^\dagger \hat{I} c]} \left(\frac{N_{\phi\phi}}{L\,\bar{a}_1}\right) \sum_{\mu=0} |C_\mu|^2 \left(\frac{2J_\mu + 1}{4\pi}\right)$$

<u>full acceptance corr.</u>   <u>naive cross section</u>

So all waves get renormalized by <u>the</u> <u>same</u> acceptance correction

---

(*) This consistency condition is not quite fulfilled for the 4K part, however. For that part we would have solved the same way and obtained $K = \dfrac{1}{(4\pi)^3 L \bar{a}_0}$. This inconsistency shows that our starting equation (1A) is not fully consistent with the channel likelihood condition (2A). This would have been correct if we had started with (1A)$'$ instead

$$(1A)' \qquad l_i = w_i \frac{[c^\dagger \hat{W}(\Omega_i) c]}{[c^\dagger \hat{I} c]} + (1 - w_i)\left(\frac{\bar{a}_1}{\bar{a}_0}\right)$$

This is a potentially important correction, as this ratio is usually quite different from 1, perhaps 2 or 3.