Reconciling Landau's and my Normalizations

In my scheme I write the cross section from first and then derive a likelihood function from it.

\[ \frac{A_{\text{eff}}}{d\Omega} = \sigma_1 \hat{B}(m, \Omega)[c^\star W(\Omega)c] + \sigma_2 \hat{P}(m) U(\Omega) \]

where \( \sigma_1 \) = total \( \phi \) cross section
\( \sigma_2 \) = total non-resonant cross section
\( B(m) \) = \( \phi \) mass plot density normalized to 1 over the peak (\( \Lambert \) over whole mass plot)
\( P(m) \) = \( \phi \) phase space mass plot density normalized to 3 over the whole mass plot.

\[ W(\Omega) \] = \( \phi \) angular distribution in \( d\Omega^2 \) that has been normalized to 1 over all angles
\( U(\Omega) \) = phase space angular distribution in \( d\Omega^2 \) normalized to 1 over all angles = \( \frac{4\pi}{3} \)

Measured counts are given by

\[ \frac{dN}{d\sigma d\Omega} = \frac{d\Omega}{d\Omega} \cdot L \cdot A(m, \Omega) \]

where \( L \) = integrated luminosity
\( A(m, \Omega) \) = detector reconstruction efficiency as a function of kinematics.

Eq. (2) tells the normalization in terms of event counts. For example, the total \( \phi \) count is

\[ N_{\phi} = \int d\Omega d\Omega \cdot L \cdot \int \frac{d\Omega}{d\Omega} B(m) A(m, \Omega) [c^\star W(\Omega)c] \equiv \sigma_1 [c^\star \hat{W}_1 c] \]

where for convenience I introduce a new matrix \( \hat{W}_1 \) defined as

\[ \hat{W}_1 \equiv \int \frac{d\Omega}{d\Omega} B(m) A(m, \Omega) W(\Omega) \]

which plays the role of (luminosity \times acceptance) when sandwiched between \( c^\star \) and \( c \).
From this I can define my likelihood function as

$$L_i = \left[ \sum_{m} P(m) \frac{W_i(m)}{\tau_i (\Omega_c)} + \sigma_0 P(m) \Omega_c \right] A(m_i, \Omega_c)$$

where by analogy to \( \hat{\Omega} \), \( \Omega_c = L \int d^3p \rho(p) A(m, \Omega_c) \). \( \Omega_c \) is just the ordinary phase-space acceptance.

I impose the normalization of the \( \Omega \) for fitting with \( C_j = 1 \) and then after the fit re-educating \( \Omega \) and forcing the condition \( \int d^2W \Omega_c = 1 \) to make \( \Omega_c \) into the total fiducial cross section.

When this is done, the partial cross section for event \( j \) is just \( C_j \Omega_c \Sigma \left( \frac{d^2 \sigma}{d^2W} \right) \). The later factor comes from the normalization we used for the \( W_i(\Omega) \) function.

In Christy's picture, we write the likelihood fact and then derive the cross section later.

$$L_i = \frac{w_i [\hat{\Omega}_\Sigma W_i(m_i) \Omega_c \Sigma \hat{I}_i] + (1-w_i)}{\hat{I}_i}$$

where \( w_i \) is the weight \( 0 \leq w_i \leq 1 \) for event \( i \) from channel likelihood fit

$$\hat{I}_i = \text{normalization integral defined on Monte Carlo reconstructed data set as}$$

$$\hat{I}_i = \frac{1}{M_{gg}} \sum_v W_i(\Omega_c)$$

This fit has fewer degrees of freedom because it takes the channel likelihood results as given. The normalization of \( \{C_j\} \) in the fit as taken as \( C_j = 1 \). The results then have to be interpreted to give a cross section.

So get this result, I need an expression for \( w_i \).
\[(A) \quad w_i = \frac{N_{sp} B(m_i)}{N_{sp} B(m_i) + N_{o} P(m_i)} - \text{local 1st fraction on the numerator}\]

So if I rewrite the likelihood factoring out \((1-w_i)\)

\[L_i \rightarrow \left(\frac{w_i}{1-w_i}\right) \left[\frac{C^* W(m_i) c}{C^* I c}\right] + 1\]

then \[\frac{w_i}{1-w_i} = \left(\frac{N_{sp}}{N_{o}}\right) \left(\frac{B(m_i)}{P(m_i)}\right)\]

this allows me to recast the likelihood as equivalent to

\[L_i \rightarrow \left(\frac{N_{sp}}{N_{o}} B(m_i) \frac{C^* W(m_i) c}{C^* I c} + N_{o} P(m_i)\right)\]

which is equivalent to (1)

where \(N_{sp}\) and \(N_{o}\) are the results from channel likelihood up to an overall constant, \(L_i\) has to be the measured density, that is, the cross section times (acceptance \(c\)).

We need to find the overall constant \(K\) so that

\[(A) \quad \frac{dN}{d^3 \Omega} = K \cdot B(m, \Omega) \cdot C(m, \Omega) L\]

To find \(K\) we look for consistency with channel likelihood:

\[\int_{d^3 \Omega} \frac{dN}{d^3 \Omega} \ d^3 \Omega = N_{sp}\]

\[= K \cdot \int_{d^3 \Omega} B(m, \Omega) \frac{C^* W(\Omega) c}{C^* I c} \ d^3 \Omega\]

\[= K \cdot N_{sp} \frac{[C^* \Omega, c]}{C^* I c}\]
In terms of integrals,

\[ \langle e^2 I_0 \rangle = \frac{1}{N_0} \sum \left\{ e^2 W((\alpha_\psi c) I_0) \right\} = \frac{\int d^3 m \, d^3 \Omega \, B(m) \, A(m, \Omega) \, e^{i \hat{W}(\Omega) c}}{\int d^3 m \, d^3 \Omega \, B(m) \, A(m, \Omega)} \]

where \( \bar{A}_4 = \frac{1}{(4\pi)^2} \int d^3 m \, B(m) \int d^A_2 \, A(m, \Omega) \)

Therefore

\[
\left( \frac{\langle e^2 I_0 \rangle}{\langle e^2 \bar{I}_0 \rangle} \right) = \left( \frac{4\pi^2}{L} \right) \left( \frac{\bar{A}_4}{L} \right) = \Rightarrow K = \left( \frac{4\pi^2}{L} \right) \left( \frac{\bar{A}_4}{L} \right)
\]

So get cross section from \( \frac{d^2 \sigma}{d^3 m \, d^3 \Omega} \) just drop \( L \) and \( A(m, \Omega) \)

\[
(4A) \quad \frac{d^2 \sigma}{d^3 m \, d^3 \Omega} = \left( \frac{4\pi^2}{L} \right)^2 \left( \frac{N_0}{L} \right) B(m) \left[ \frac{e^{i \hat{W}(\Omega) c}}{e^{i \hat{I} c}} \right] \left[ \frac{1}{L \bar{A}_4} \right] (r \left( \frac{N_0}{L \bar{A}_4} \right) P(m))
\]

She gives us the answer. Integrating gives the total cross section for \( \psi \psi \) as a sum over idempotents.

\[
(5A) \quad \sigma_{\psi \psi} = \left( \frac{4\pi^2}{L} \right)^2 \left( \frac{N_0}{L} \right) \sum_{n=0} \left| \lambda_n \right|^2 \left( \frac{2J_n + 1}{4\pi} \right)
\]

\[ \text{full acceptance corr. } \quad \text{cross section} \]

so all never get renormalized by the same acceptance correction.

\[ (4) \quad \text{The consistency condition is not quite fulfilled for the } V_\psi \text{ part, however, for that part we would have arrived the same way and obtained } K = \left( \frac{4\pi^2}{L} \right) \left( \frac{\bar{A}_4}{L} \right). \]

\[ \text{The inconsistency shows that our starting equation (1A) is not fully consistent with the channel selection condition (2A). It would have been correct if we had started with (1A) instead.} \]

\[ \text{(1A)} \quad \mathcal{L}_0 = W_i \left[ \frac{e^{i \hat{W}(\Omega) c}}{e^{i \hat{I} c}} \right] + (1 - W_i) \left( \frac{\bar{A}_4}{L} \right)
\]

\[ \text{This is a potentially important correction as this ratio is usually quite different from 1, perhaps 2 or 3.} \]