Preliminary Exam : Classical Physics

August 21, 2002, 9:00 a.m. - 1:00 p.m.

Please answer **3 QUESTIONS** from each of the two sections.
Please use a separate book **FOR EACH QUESTION**.

Section I: Classical Mechanics

1.) In a rotating frame of reference, Newton's equation of motion is modified by including "fictitious" forces:

$$\vec{F}_{\text{eff}} = m\,\vec{a}_r = m\,\vec{a}_f + m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v}$$

where the subscript r refers to the rotating frame and f to the fixed frame.

(a) If a projectile is fired due east from a point on the surface of the Earth at a northern latitude λ , with a speed V_0, and at an angle with respect to the horizontal of α, find the lateral deflection when the projectile strikes the Earth.

(b) If $V_0 = 300$ m/s, $\alpha = 30^o$, and $\lambda = 60^o$, what is the total lateral deflection?

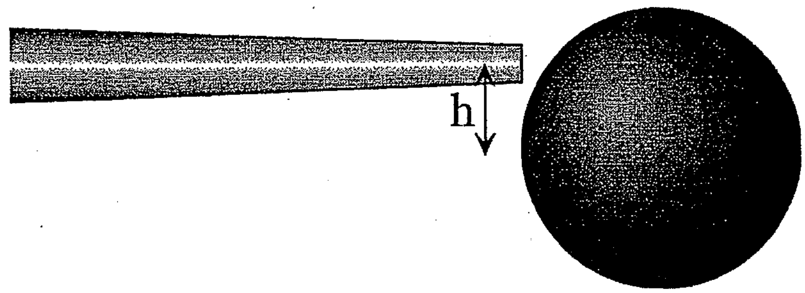
2.) Consider a central force given by:

$$F(r) = -\frac{k}{r^2} - \frac{k'}{r^4}$$

(a) What is the minimum value of L, the angular momentum, such that circular orbits exist and what is the radius of this orbit?

(b) If L is greater than this minimum value how many solutions for circular orbits exist? Find a simple condition between the radius of the orbit, r, and k, and k', which determines the stability of the orbit. Which orbits are stable and which are unstable?

3.) Consider hitting a billiard ball (of radius R, mass M, and uniform density) horizontally with a cue stick at a height, h, above (or below) the center of the ball. The tip of the cue stick does not slip on contact with the ball. The moment of inertia of a sphere about an axis through its center is $I = \frac{2}{5}MR^2$.

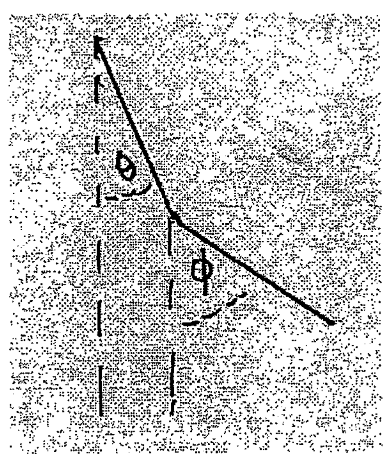


(a) If the ball is given an initial velocity of V_o, what is the final velocity of the ball, V, after it starts rolling without slipping due to friction?

(b) At what height should you hit the ball so that $V = V_o$?

(c) Is there a height, h, such that $V > V_o$?

4.) Two identical uniform thin rods of mass M and length l are suspended, as shown, from a smooth pivot and connected smoothly so that they can swing freely in a vertical plane (as shown in diagram).



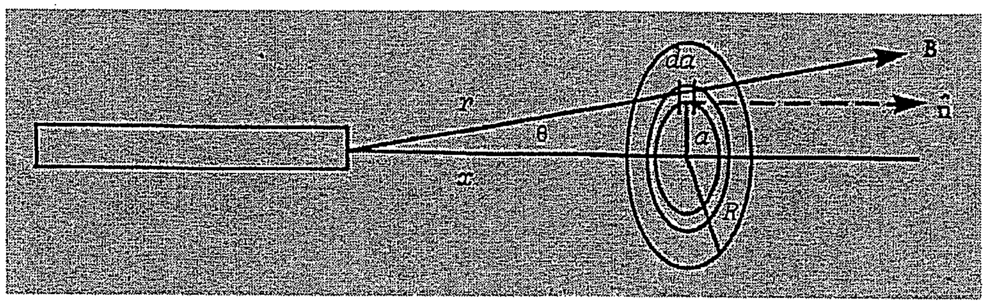
(a) Find the Lagrangian for this system, in the limit of small oscillations.

(b) From the Lagrangian in part (a), derive the equations of motion for small oscillations.

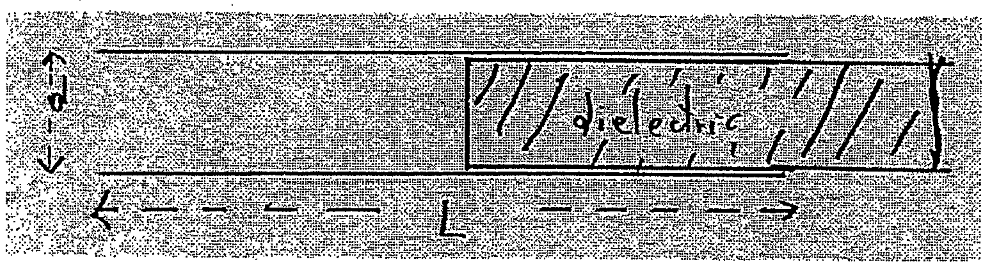
(c) In this limit of small oscillations, determine whether the angles θ and ϕ are, or are not, normal mode coordinates.

Section II: Electromagnetism

5.) A pole of a bar magnet produces a magnetic induction field that is approximately radial and spherically symmetrical with magnitude $B = k/(4\pi r^2)$, where $k = 4 \times 10^{-5} W$. This pole is moved along the axis of a coil of 200 turns of high resistance wire bound together in the form of a circular loop of radius $R = 5\,cm$. When the pole is just passing through the center of the coil with a velocity of 20 cm/s, what is the induced emf in the coil?



6.) Two identical square conducting plates, with side length L, are placed one directly above the other so that they are parallel and a distance d apart, with $d \ll L$. A dielectric slab with dielectric constant κ has dimensions that allow it to exactly fill the space between the two plates completely, allowing it to slide smoothly and freely between the plates. An experimenter exerts a force that slides the dielectric out from between the plates, while the plates carry equal and opposite total charges Q. You may ignore the edge effects at the edges of the conducting plates.



(a) when the dielectric slab is positioned so that two of its edges are lined up with the plate and it projects from the space between the plates with one set of edges a distance x displaced from their initial positions (as shown in diagram), find the force (magnitude and direction) that must be exerted on the slab to maintain it in that position for an arbitrary value of x and fixed values of Q.

(b) If the plates are attached to a potential source that maintains a constant potential difference V_0 between them, how much work must be done to move the dielectric slab to a position in which $x = L/2$?

7.) A perfectly conducting planar sheet of infinite extent has a circular section of radius a separated from the remainder of the sheet by a very thin perfectly insulating ring. The circular section is raised to a potential V_0 above ground, and the remainder of the sheet is grounded. Derive an expression for the potential everywhere on a line perpendicular to the sheet and passing through the center of the circle.

8.) (a) Using Maxwell's equations and Ohm's law with a zero charge density and a nonzero current density, derive the wave equation for the E-field.

(b) Solve the wave equation, in a good conductor at low frequencies ($4\pi\sigma/(\omega\epsilon) \gg 1$), for the E-field traveling along the z-axis and polarized along the x-axis. Discuss the physical meaning of your results.