Preliminary Exam : Classical Physics

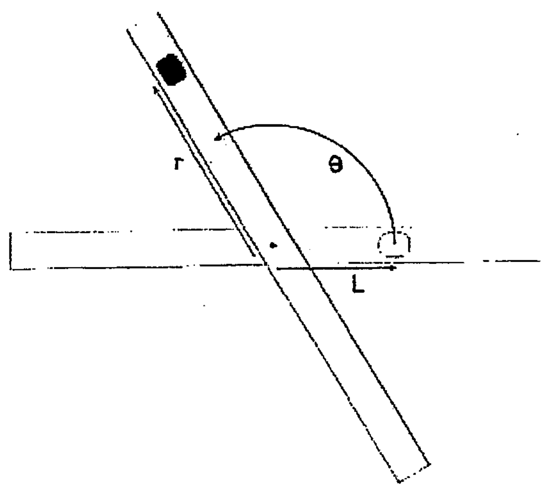
January 17, 2003, 9:00 a.m. - 1:00 p.m.

Please answer 3 QUESTIONS from each of the two sections.

Please use a separate book FOR EACH QUESTION.

Section I: Classical Mechanics

1. A tube spins in a vertical plane around an axis perpendicular to its length, making an angle $\theta(t)$ with respect to the horizontal. An object of mass M slides without friction inside the tube and can pass freely though the center. The axis of rotation is in a horizontal plane and gravity, g, acts in the vertical direction. The distance of the mass from the axis of rotation is $r(t)$.



   (a) Write down the Lagrangian, L, for this system, assuming r and θ are independent variables. Also, write down the function giving the constraint on θ, for the case of constant rotation, $\theta = \omega t$.

   (b) Obtain the equations of motion using the method of undetermined multipliers. In this situation, what is the physical meaning of the undetermined multiplier?

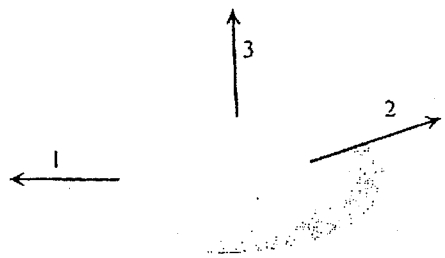
   (c) Solve the equation of motion for $r(t)$ for constant rotation using the initial conditions $r(0) = L$ and $dr/dt(0) = 0$.

   (d) Under what conditions can one obtain a periodic solution to $r(t)$, for large t? Is this motion stable?

2. Consider an object with moment of inertia around its principal axes equal to I_1, I_2 and I_3 such that

$$I_3 > I_2 > I_1.$$

We want to study the stability of the motion for rotations about each axis separately.



Euler's equations for the case of force-free motion are:

$$(I_i - I_j)\omega_i\omega_j - \sum_k I_k\dot{\omega}_k\epsilon_{ijk} = 0$$

where the rotation vector is $\vec{\omega} = \omega_1\hat{e}_1 + \omega_2\hat{e}_2 + \omega_3\hat{e}_3$, and the vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ are the unit vectors along the 3 principal axes.

(a) To determine the stability around axis 1, assume that the rotation vector is given by $\vec{\omega} = \omega_1\hat{e}_1 + \lambda\hat{e}_2 + \mu\hat{e}_3$, where $\omega_1 \gg \lambda, \mu$. Determine whether the perturbations λ and μ grow in time or not.

(b) Using similar reasoning, determine whether rotations about axis 2 and axis 3 are stable or not.

(c) Finally, consider the case where $I_1 = I_2$. Show whether or not rotations around axis 1 and axis 3 are stable.

3. (a) Write the Lagrangian for a point particle of mass m moving in a plane under the influence of a central force $V(r)$.

(b) Find a cyclic (ignorable) coordinate, show that the corresponding canonical momenta is a constant of motion, and identify that constant of motion as a well-known physical quantity.

(c) Write the Hamiltonian for this case and find an equation of motion that involves only one coordinate and its first time derivative, as well as constants of motion that you identify as common physical quantities.

(d) One of Kepler's laws of planetary motion states that each planet sweeps out equal areas δA in equal times δt, regardless of its orbit's eccentricity. Derive this law and find an expression for dA/dt in terms of constants of motion.

4. Consider a massive point particle moving in one dimension in the potential of the uniform gravitational field given by

$$V(z) = mgz$$

(a) For this system construct and solve the Hamilton-Jacobi equation for S

$$H\left(q_1, ..., q_s; \frac{\partial S}{\partial q_1}, ..., \frac{\partial S}{\partial q_s}; t\right) + \frac{\partial S}{\partial t} = 0$$

(b) Using $\beta_i = \frac{\partial S}{\partial \alpha_i}$ = constant, with separation constants α_i from part(a.), determine the motion $q_i(t)$ of the system.

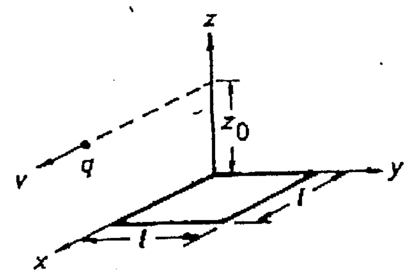
Section II: Electromagnetism

5. A neutral, insulated conducting sphere of radius a is situated in free space. A point charge q is at a distance R from the center of the sphere, where $R > a$.

   (a) Find the force exerted on the conducting sphere by the point charge.

   (b) Find the potential of the conducting sphere and the potential everywhere in its interior.

6. Consider a square loop of wire, of side length l, lying in the x, y plane as shown in the figure below. Suppose a particle of charge q is moving with a constant velocity v where $v \ll c$, in the xz -plane at a constant distance z_0 from the xy -plane. Suppose the particle crosses the z -axis at $t = 0$ .. The electromagnetic field due to the uniformly moving charge is $\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\vec{r}}{r^3}$, and $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$ with $\vec{r} = (x - vt)\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Give the induced emf in the loop due to the moving charge as a function of time.



7. An electromagnetic plane wave of frequency $\omega/2\pi$ is incident on a very large rectangular dielectric block that has a dielectric constant ϵ and the magnetic permeability of free space. The wave is linearly polarized and its direction of propagation is perpendicular to the face of the dielectric on which it is incident.

   (a) Use Maxwell's equations to derive expressions for the relation between the electric and magnetic field vectors in free space as well as in the dielectric medium.

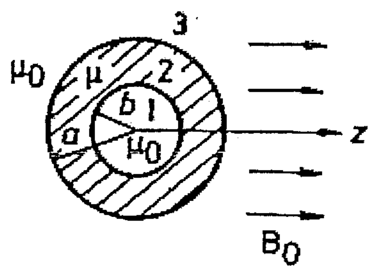
   (b) Derive an expression for the velocity of the electromagnetic wave in the dielectric medium.

   (c) Specify the boundary conditions for the electric and magnetic fields at the interface between free space and the dielectric medium.

   (d) Find the fraction of the incident electromagnetic energy that is reflected from the surface of the dielectric.

8. A long hollow right circular cylinder made of iron of permeability μ is placed with its axis perpendicular to an initially uniform magnetic flux density $\vec{B}_0$ (see figure). Assume that $\vec{B}_0$ is small enough so that it does not saturate the iron, and that the permeability μ is a constant in the field range of this problem. Let the inner and outer radii of the cylinder be b and a respectively.

   Derive an expression for the magnetostatic potential ϕ_1 inside the cylinder (region 1 of the figure). What is ϕ_1 in the limit $\mu \gg \mu_0$ with $\mu_0 = 1$? What does this result say about magnetic shielding?



Note: in polar coordinates $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$