CLASSICAL MECHANICS

Preliminary Examination

Monday August 20, 2012
09:00 - 12:00, P-121

Answer a total of THREE questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented separately in an answer book or on individual sheets of paper. Make sure you clearly indicate who you are and the problem you are solving. Double-check that you include everything you want graded, and nothing else.

1. A particle of mass m in an isotropic three-dimensional harmonic oscillator potential has a natural angular frequency ω_0. The particle has a charge e and is simultaneously acted on by uniform magnetic and electric fields $\vec{B} = B\hat{z}$ and $\vec{E} = E\hat{x}$ with a Lagrangian given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2}m\omega_o^2(x^2 + y^2 + z^2)$$

$$+ eEx + \frac{1}{2}eB(-\dot{x}y + x\dot{y})$$

(a) Using Lagrange's equations, find the equations of motion for the particle.

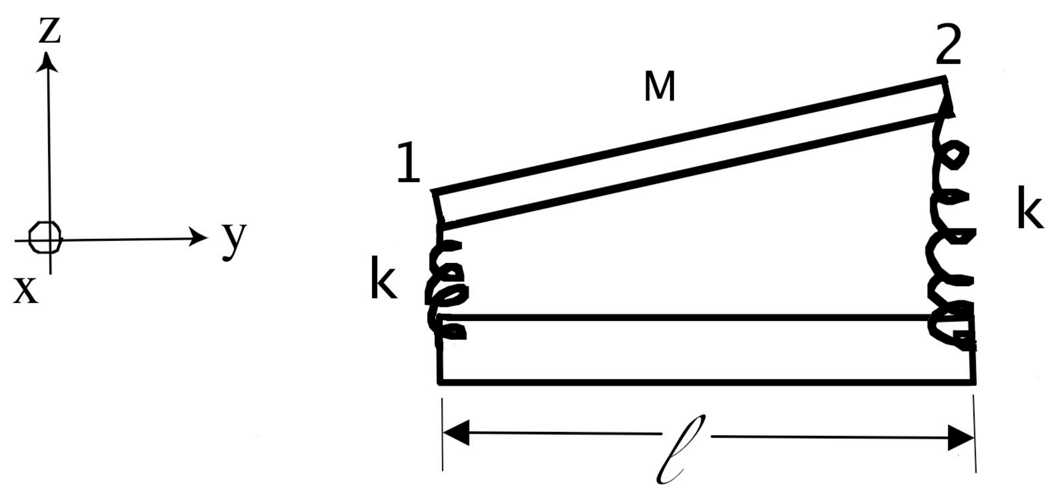
(b) Letting $x = x' + eE/m\omega_o^2$ rewrite the equations of motion for x and y of part (a) and find the vibrational frequencies of the particle. Hint: try solutions of the type $x' = A'e^{-i\omega t}$ and $y = y' = B'e^{-i\omega t}$.

2. Consider a uniform rod of length l and mass M, which is supported at each end by a massless spring with spring constant k. In equilibrium, the weight of the rod is balanced by the springs and, as a result, the rod rests horizontally above the table. Take this equilibrium position to be z=0. The springs are attached to the table and are constrained to move principally in the z direction. Consider only small displacements from equilibrium and ignore any translational motion in the x or y directions.

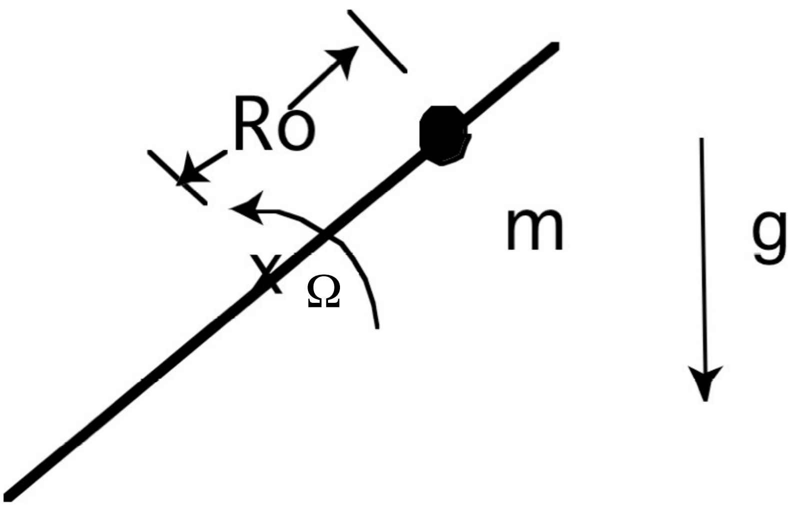
(a) Write down the equations describing the z motions of points 1 and 2.

(b) Find the normal mode frequencies of the system.

(c) Describe qualitatively the motion associated with each normal mode.



3. A particle of mass m is constrained to move along a straight frictionless wire. The wire is rotating in a vertical plane at a <u>constant</u> angular velocity Ω as shown in the figure below. Assume a right-handed Cartesian coordinate system (x,y,z) with x measured along the rotating wire. The system is in a gravitational field with the gravitational acceleration g pointing downwards. At t = 0 the wire is horizontal (perpendicular to g), and the mass is stationary with respect to the wire and lies at (x, y, z) = $(R_0, 0, 0)$.



a) Write the Lagrangian using the coordinate system (x, y, z) that rotates with the wire.

b) Write the equations of motion.

c) Solve the equation of motion for x for the case in which g = 0.

d) Write expressions for the forces of constraint when $g \neq 0$, and give a physical interpretation of the constraint forces.

4. A particle moves in a central force field given by the potential $\dfrac{-ke^{-ar}}{r}$. Find the conditions for a circular orbit. Find the period of small radial oscillations about this circular orbit.