LGD shower resolution (NIM article section)

Mihajlo Kornicer
Richard T. Jones
University of Connecticut, Storrs, CT

Dec 8, 2003

1 The LGD resolution

The energy and spatial resolution in the LGD are measured using 2-shower invariant mass distributions from $\eta \to 2\gamma$ and $\pi^0 \to 2\gamma$ decays, exploring the formula for the square of the invariant mass

$$M^2 = 4E_1E_2\sin^2\frac{\theta_{12}}{2} = 2E_1E_2(1-\cos\theta_{12}), \tag{1}$$

where E_1 and E_2 represent photon energies, while θ_{12} is their angular separation. The variance of the squared invariant mass in terms of the variances of energy and shower position in the LGD is given by

$$V(M^2) = \sum_{i=1}^{2}\left[\left(\frac{\partial M^2}{\partial E_i}\right)^2 V_E + \left(\frac{\partial M^2}{\partial x_i}\right)^2 V_X + \left(\frac{\partial M^2}{\partial y_i}\right)^2 V_Y + 2\left(\frac{\partial M^2}{\partial x_i}\right)\left(\frac{\partial M^2}{\partial y_i}\right)V_{XY}\right], \tag{2}$$

where index i counts photons. The expression for the energy resolution $\sigma_E = \sqrt{V_E}$ is given by Eq. 2 (NIM:$AE + B\sqrt{E}$) where parameters A and B have to be determined. Spatial variances are taken to be

$$\begin{aligned} V_X = \sigma_x^2 &= \frac{C_E^2}{E} + (X_0\sin\theta\cos\phi)^2, \\ V_Y = \sigma_y^2 &= \frac{C_E^2}{E} + (X_0\sin\theta\sin\phi)^2, \\ V_{XY} &= X_0^2\sin^2\theta\cos\phi\sin\phi, \end{aligned} \tag{3}$$

where C_E depends on the size of the LGD block, and X_0 is radiation length [1]. For this LGD $C_E = 7.1$ mm$\cdot$GeV$^{\frac{1}{2}}$, and $X_0 = 31.6$ mm [2]. Finding energy derivatives from Eq. 2 is straight forward while all spatial derivatives can be expressed in terms of measured photon momenta [3]. Taking all this into account, Eq. (2) can be simplified as

$$\frac{V(M^2)}{M^4} = \frac{V_E(E_1)}{E_1^2} + \frac{V_E(E_2)}{E_2^2} + \frac{V_S(M^2)}{M^4}, \tag{4}$$

where spatial derivatives and variances are grouped into the term $V_S(M^2)$.

From events with two reconstructed showers the π^0 sample was selected by limiting invariant mass to $M < 0.5\ GeV$ and the η sample was obtained by selecting pairs with shower separation $D_{\gamma\gamma} \geq 20\ cm$. The mass resolution was measured by forming the invariant mass distributions for a set of well-defined shower energies $(E_1, E_2)_k$, with $k = 1\ldots 18$ for the η and $k = 1\ldots 16$ for the π^0 sample. The π^0 and η peaks from these distributions were fitted with a Gaussian over a polynomial background. From the Gaussian fit the value $V(M^2)/M^4$ has been calculated. The errors are predominantly systematic, governed by the choice of background function, and estimated to be 5% and 10% for the η and π^0 data respectively.

Examining the $V_S(M^2)$ distributions from η and π^0 decays it was seen that spatial part in the η mass resolution is at 10% level, while it gives significant 50% contribution to the π^0 mass width [3]. Consequently, in the first approximation the spatial contribution to the η mass resolution has been neglected. Following Eq. (4) the set of $k = 18$ equations

$$\left(\frac{V(M^2)}{M^4}\right)_k = F_i + F_j, \tag{5}$$

was formed, where indexes i, j map different photon energies being used. The least-square solution for the F_i values, $i = 1, \ldots, 11$, was found without assuming the functional form of $F(E)$. This model-independent solution is free of any restrictions on the F values except that $F_i = F_j$ for $i = j$. The corresponding $\sigma_E(E_i) = E_i\sqrt{F_i}$ from the free solution is plotted in Fig. 1. These points agree very well with the standard expression for the energy resolution (Eq. 2), with $A = 0.037$ and $B = 0.080$ GeV$^{\frac{1}{2}}$.

In the next step, the spatial corrections to the mass resolution has been included by recording the mean of the $V_s(M^2)/M^4$ distributions for each (E_1, E_2) choice, for both the η and π^0 samples. The expression for the variance function (Eq. (4)) was modified, using the standard expression for the energy resolution,

$$\frac{V(M^2)}{M^4} = \left(A + \frac{B}{\sqrt{E_1}}\right)^2 + \left(A + \frac{B}{\sqrt{E_2}}\right)^2 + C\frac{V_s(M^2)}{M^4}. \tag{6}$$

In addition to the parameters A and B, parameter C that scales the spatial resolution contribution has been introduced. Within this simple model the η data fit with C set to zero is shown as solid line in Fig. 1, while

Table 1: The parameter values from different fits to the η and π^0 mass resolutions, obtained using Eq. (6). The corresponding fits are shown in Fig. 1

Fit	A	B $\left[GeV^{-1/2}\right]$	C	χ^2
η ($C=0$)	0.0371 ± 0.0006	0.0797 ± 0.0008	0	1.05
η (full)	0.036 ± 0.008	0.080 ± 0.003	0.26 ± 1.76	1.04
$\eta + \pi^0$ (full)	0.035 ± 0.004	0.073 ± 0.006	0.90 ± 0.02	1.50

the fit parameters are shown in the first row of Table 1. The η data fit when C was let to vary is represented by the dashed line in Fig. 1 and the second row in Table 1. The fit confirms that the η mass resolution is not very sensitive to spatial corrections. In the next step all measured points from the η and π^0 samples were fitted together. The fit is shown as the dotted line in Fig. 1, and the resulting parameters are shown in the last row of Table 1. Both the π^0 and η mass distributions are described well with this single set of parameters.

The scale parameter C can be viewed as a correction to the nominal value of $C_E = 7.1$ mm$\cdot$GeV$^{\frac{1}{2}}$ for the LGD, since the energy-dependent term in the spatial errors is dominant at small and moderate angles $\theta < 15\,\text{deg}$, while the angular part in Eq. (3) becomes important only at large angles $\theta > 15\,\text{deg}$. According to the fit results the energy resolution function is

$$\tfrac{\sigma_E}{E} = 3.6\% + \tfrac{7.3\%}{\sqrt{E}},$$

while the shower position uncertainty can be expressed as

$$\sigma_\rho = \sqrt{\left(\tfrac{0.64}{\sqrt{E}}\right)^2 + \left(X_0 \sin\theta\right)^2}, \quad \rho\sigma_\phi = \tfrac{0.64}{\sqrt{E}},$$

where (ρ, ϕ) are the polar coordinates of the shower in the face of the LGD. All lengths in these relations are in mm and all energies in GeV.

References

[1] "Calorimetry: Energy Measurements in Particle Physics", R. Wingman, International Series of Monographs on Physics 107, Oxford University Press (2000).

[2] C. P. Steffen, Ph.D. thesis, Indiana University (October 2001).

[3] "LGD shower resolution ", M.Kornicer, R.T Jones, Radphi TechNote *2003-701* (July 2003).

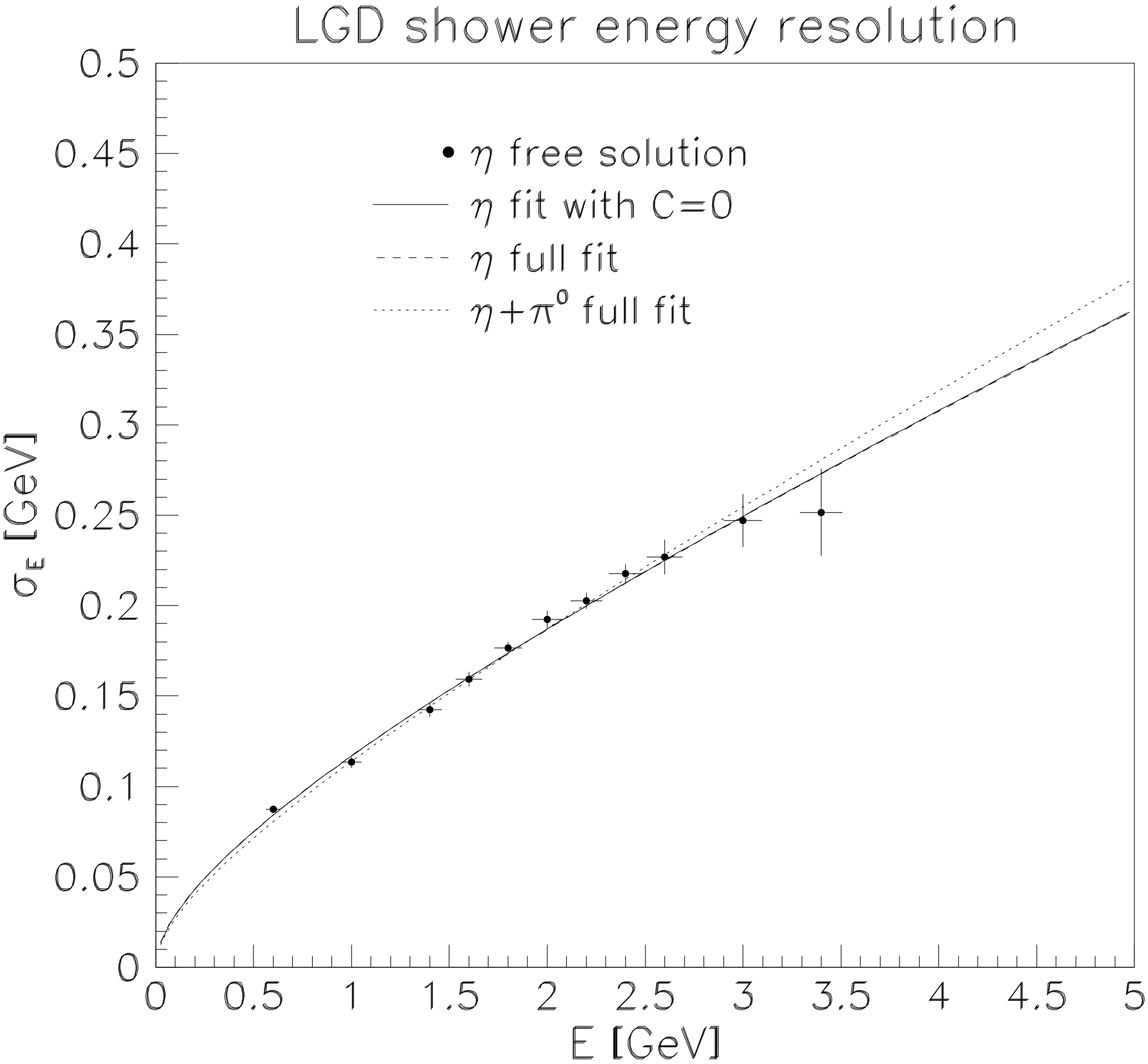

Figure 1: Energy resolution of showers in the LGD obtained from analysis of the 2γ sample. Points represent the free solution to the η mass resolution measurements when the contribution from the spatial resolution has been neglected. The solid line represents the fit to the η data with the standard energy resolution model (Eq. (6)with $C = 0$). The dashed line represents the fit to the η data when the spatial contribution is taken into account by Eq. (6). The dotted line corresponds to the simultaneous fit to the η and π^0 data with the same function (Eq. (6)). Corresponding fit parameters and χ^2 are shown in Table 1.