

# The Mini-Torus Option

## Outline

1. Fixing  $Q^2$  : systematic  $\frac{\delta A}{A}$
2. Fixing  $E_0$  : figure of merit
  - statistical  $\frac{\delta A}{A}$
  - spectrometer resolution
  - backgrounds
3. Straw-man mini-torus
  - what do we gain?  $(\frac{\Delta \theta}{\theta}) \cdot (\frac{\Delta \phi}{2\pi})$
  - design issues: optics, coil geometry
  - favorable case study

1. Fixing  $Q^2$ : Systematic errors

$$A_{LR} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = a_0 \tau \left[ \xi_v^P + \Delta_n + \Delta_s - \right]$$

proton part:  $\xi_v^P$  ( $Q_{\text{weak}}$ )

neutron part:  $\Delta_n = \xi_v^n \frac{\varepsilon G_E^P G_E^n + \tau G_M^P G_M^n}{\varepsilon (G_E^P)^2 + \tau (G_M^P)^2}$

strange part:  $\Delta_s = \xi_v^{(0)} \frac{\varepsilon G_E^P G_E^S + \tau G_M^P G_M^S}{\varepsilon (G_E^P)^2 + \tau (G_M^P)^2}$

axial part:  $\Delta_A = - \frac{\sqrt{1-\varepsilon^2} \sqrt{\tau(1+\tau)} (1-4\sin^2\theta_w) G_M^P G_A}{\varepsilon (G_E^P)^2 + \tau (G_M^P)^2}$



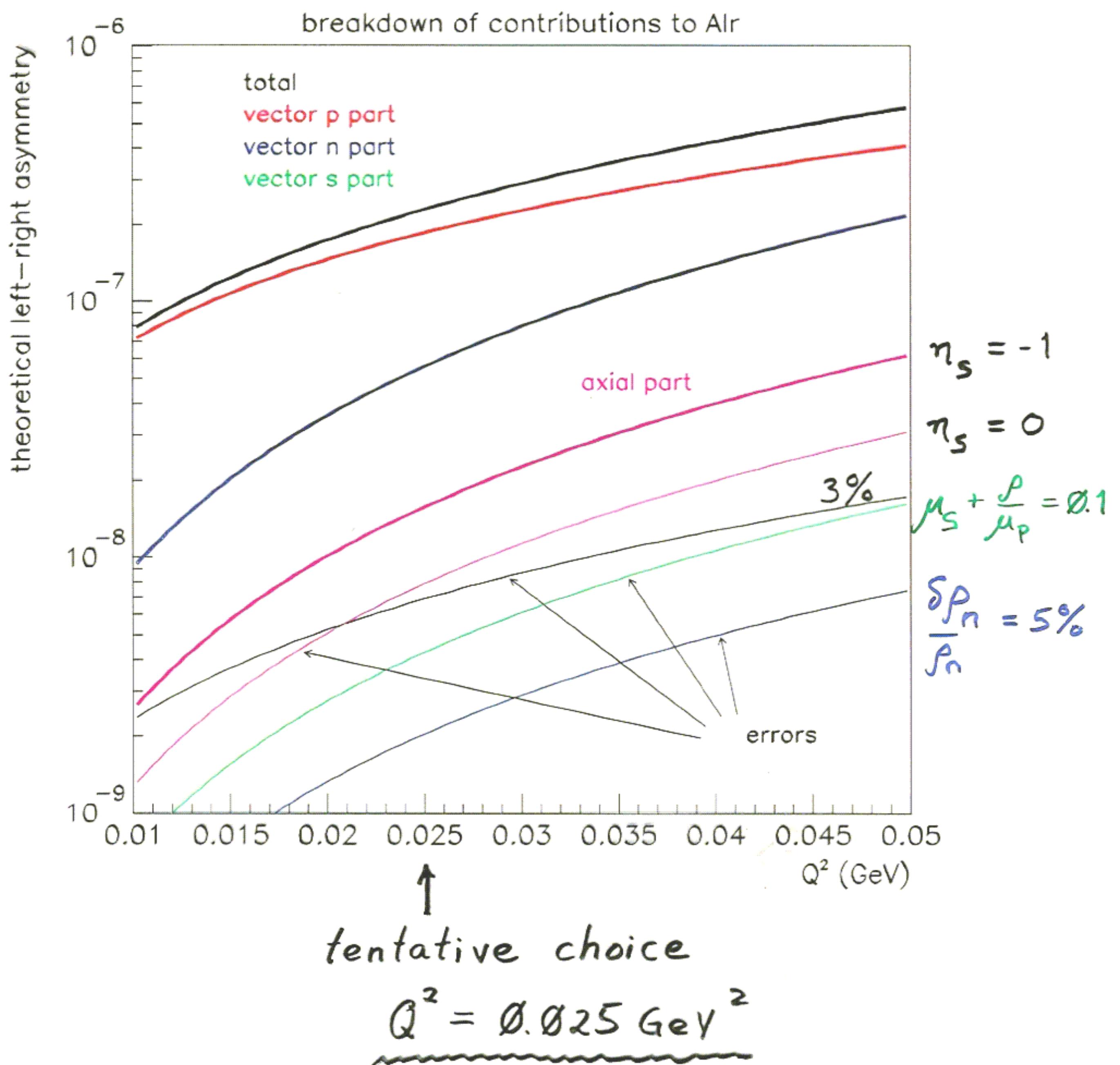
At forward angles  $\sim \theta \sqrt{\tau} \mu_p G_A^P \times \xi_v^P$

$\Rightarrow$  not negligible for this measurement

Key question:

How well do we think we will know

$$\eta_s \equiv \frac{G_A^S(0)}{g_A} ?$$



2. Fixing  $E_0$ : figure of merit

$$\left(\frac{\delta A}{A}\right)_{\text{meas.}} = \left[ A_{LR} P \left(\frac{d\sigma}{d\Omega}\right)^{1/2} \sqrt{L \Delta\Omega T f_{el}} \right]^{-1}$$

$A_{LR}$  = theoretical asymmetry

$P$  = beam polarization

$\left(\frac{d\sigma}{d\Omega}\right)$  = ep elastic cross section

$L$  = instantaneous luminosity

$\Delta\Omega$  = solid angle

$T$  = duration of measurement

$f_{el}$  = fraction of scattered electrons  
in elastic window



internal, external bremsstrahlung correc...

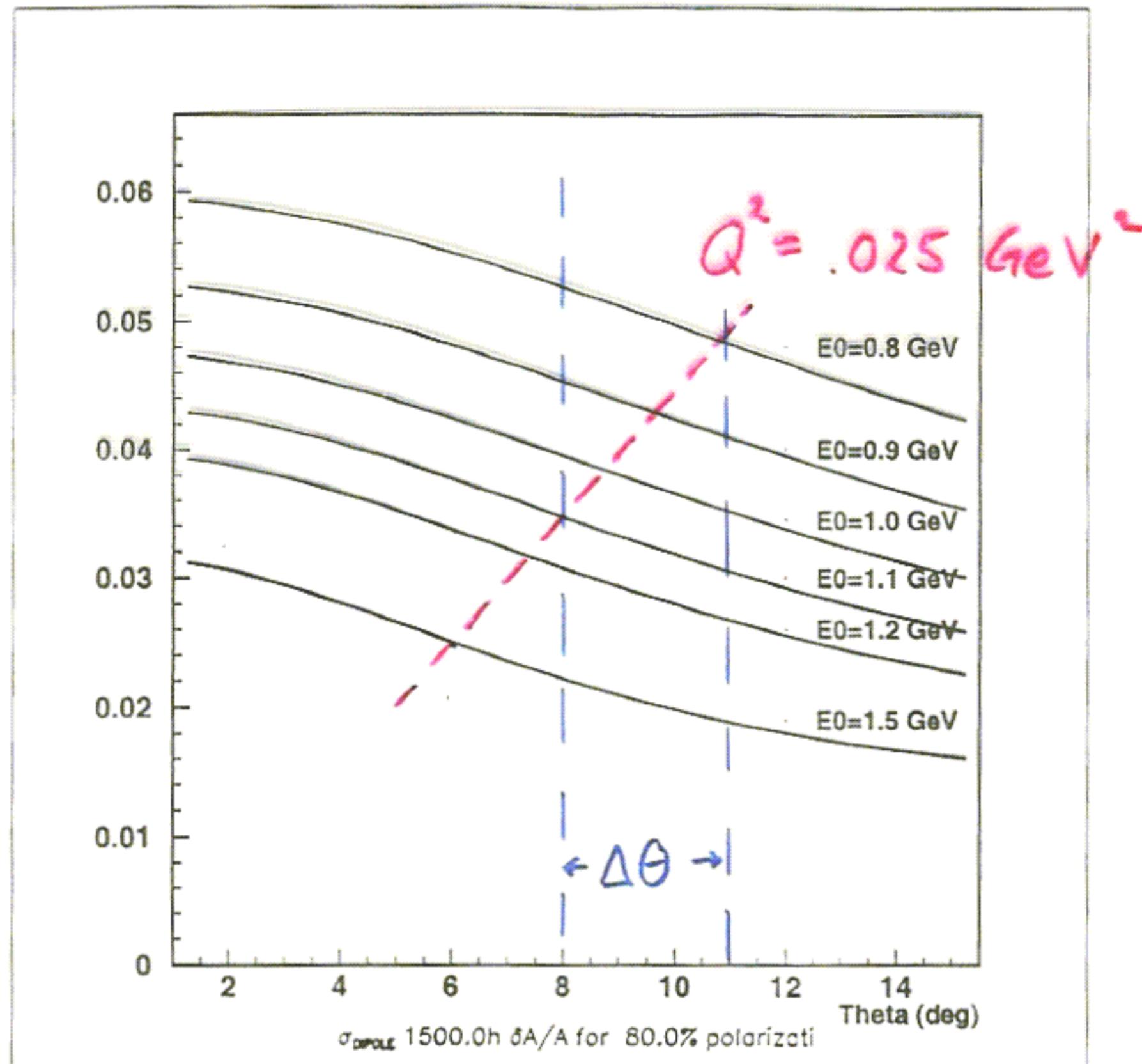


Figure 6: Relative error of the asymmetry for the elastically scattered longitudinally polarized electrons on protons for the forward angle measurement. The beam energy ranges from 0.8 GeV up to 1.5 GeV.

From Neven 7/2000

$$100 \mu\text{A} \rightarrow 150 \mu\text{A}$$

40cm target

1500 h

80% polarization

So 3% measurement appears feasible

but going to smaller θ would help...

F.O.M. : depends on  $\theta, Q^2$ ,

or  $E_0, Q^2$  equivalently,

or  $D, Q^2$  (given  $G\emptyset$  geometry)

where  $D$  is target-spectrometer distance

1) Fixed  $Q^2$ :  $\theta \propto \frac{1}{D} \propto \frac{1}{E_0}$

$\Rightarrow$  choice of 1 implies choice of all 3.

There is just one curve in Fig. 6

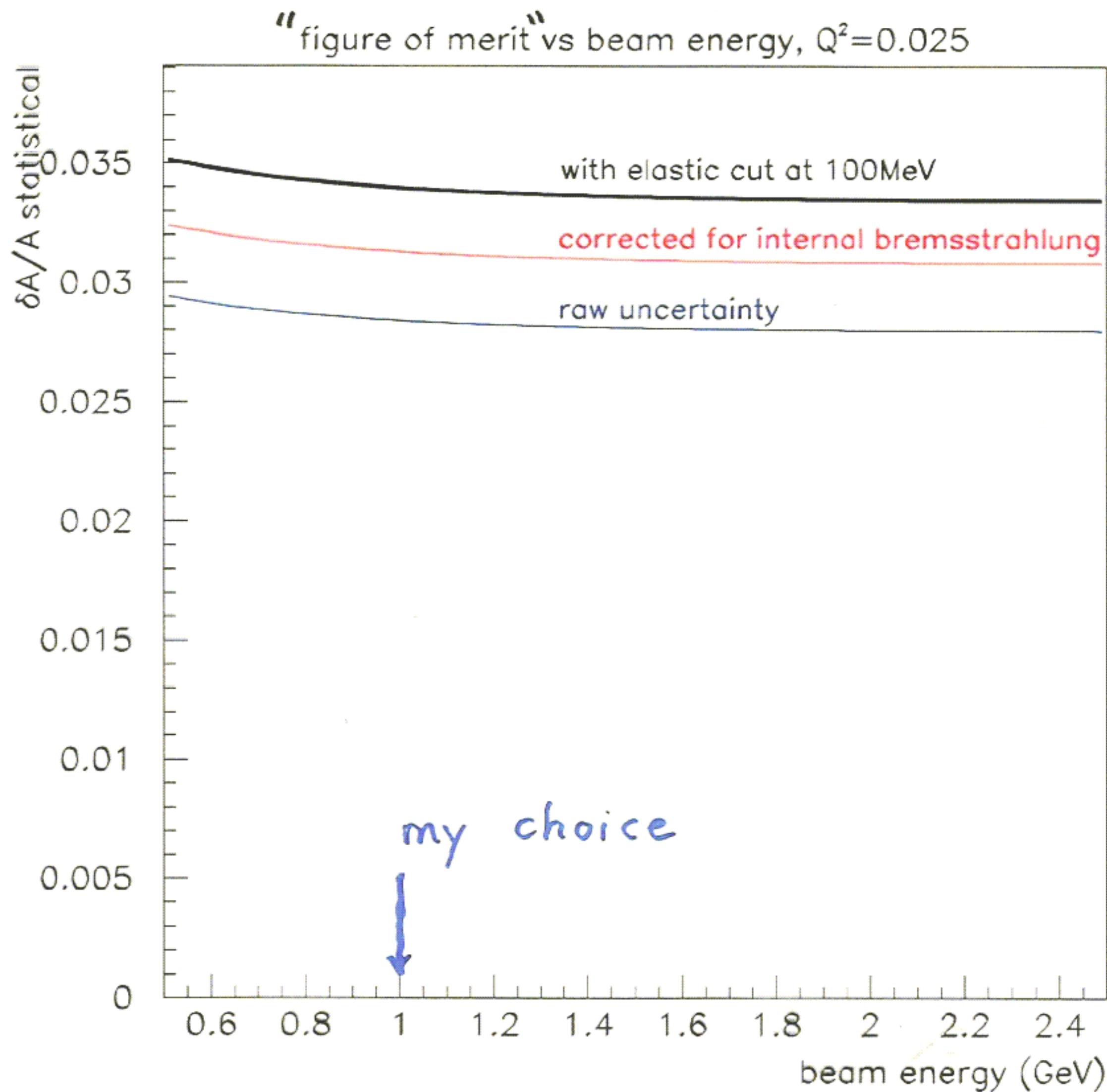
(once we lock  $\theta, D, E_0$  to one d.o.f.)

To optimize  $E_0$ , chose the minimum of FOM.

... but the curve is flat

why?

$$\frac{d\sigma}{d\Omega} \propto E_0^2 \iff \Delta\Omega \propto \frac{1}{D^2}$$



internal brems fel ~ external brems fel ~ 85%

Other considerations in choosing  $E_0$

- spectrometer resolution
- backgrounds

> Spectrometer resolution:

- ✓ solution demonstrated at  $E \sim 1 \text{ GeV}$
- ✓ probably scales to  $E_0 > 1 \text{ GeV}$  but elastic separation may decrease

> Backgrounds

- ✓ lower  $E_0$  is better

Choose

$$\boxed{\begin{aligned} E_0 &= 1 \text{ GeV} \\ \theta_0 &= 10.5^\circ \\ D &= 500 \text{ cm} \end{aligned}}$$

### 3. Straw-man mini-torus

★  $\frac{\delta A}{A}$  is flat in  $\theta, D, E_0$  @ fixed  $Q^2$

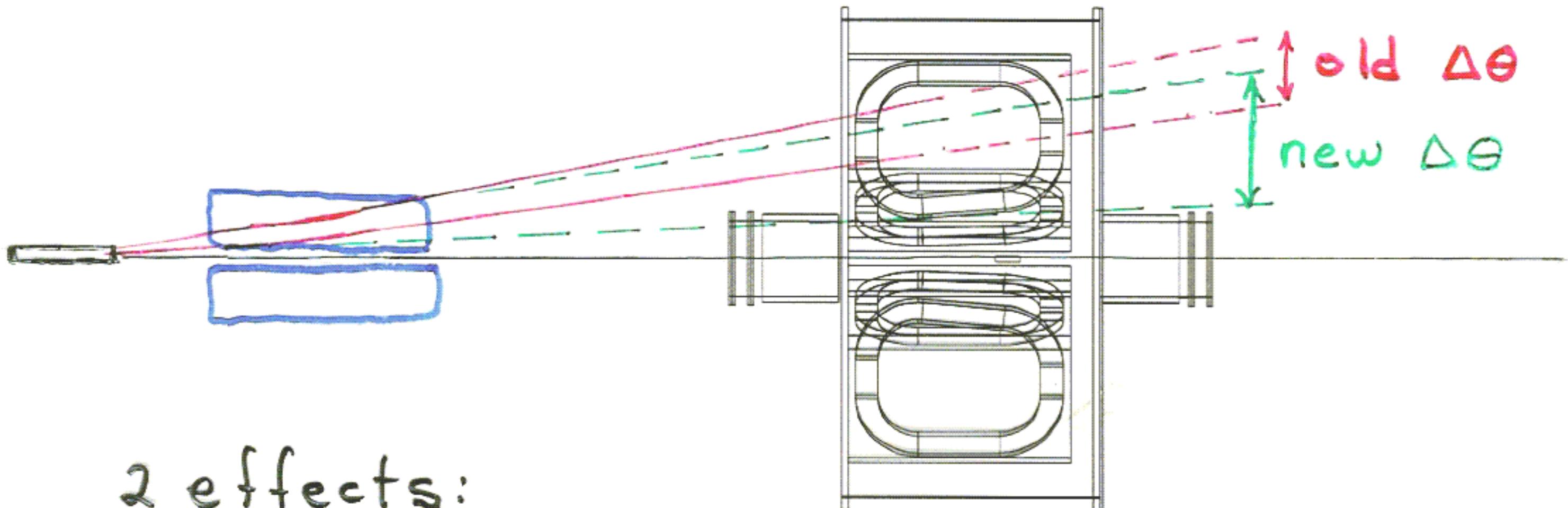
Where can we gain?

→ By changing the aperture

$$\left( \frac{\Delta\theta}{\theta} \right) \left( \frac{\Delta\phi}{2\pi} \right)$$

33%      67%

Consider:



2 effects:

- 1) angular shift (bend)      denominator ↓
- 2) angular compression (focus)      numerator ↑

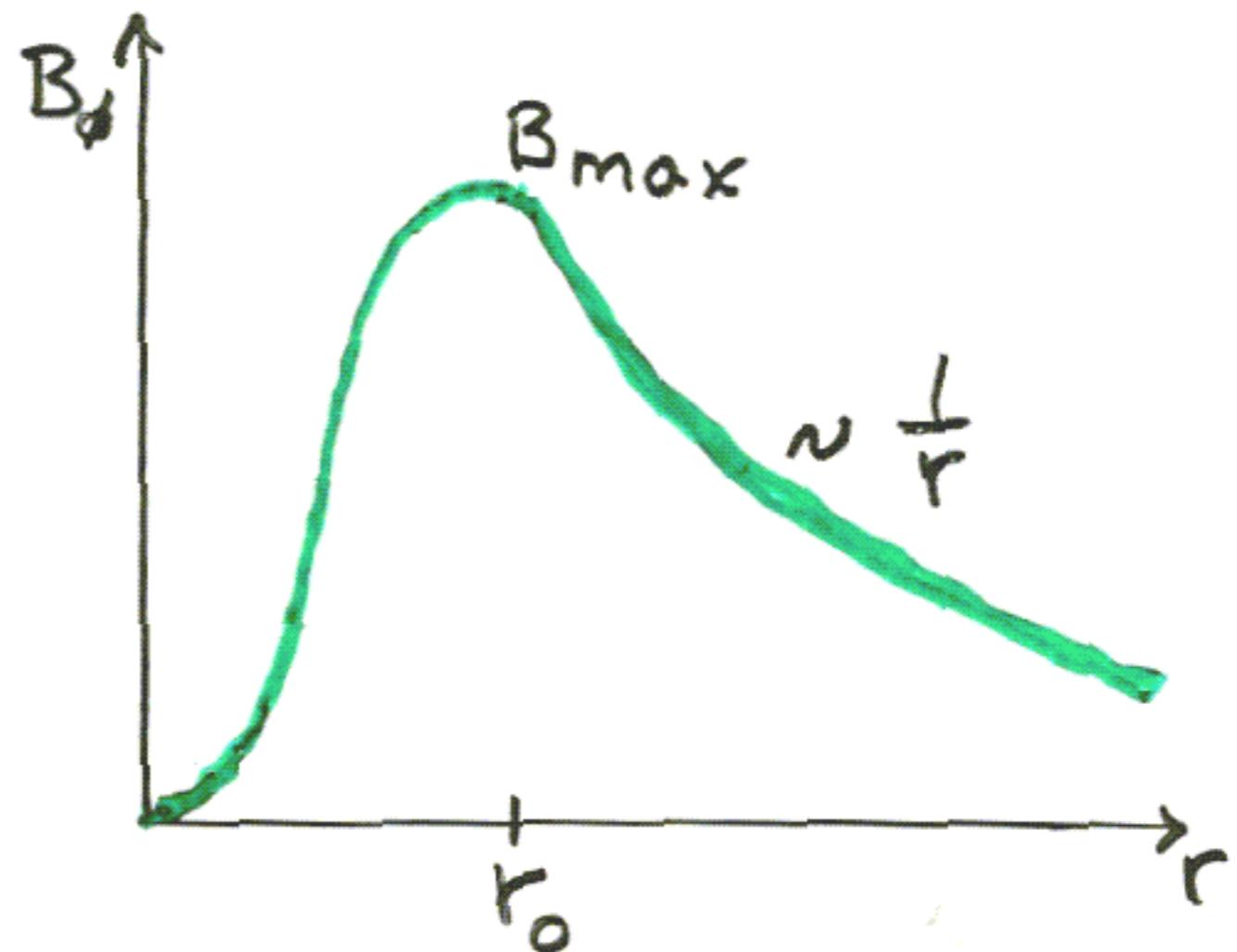
- Optics

... more quantitatively,

Consider an idealized toroid:

$$B(r) = B_{\max} \left( \frac{r_0}{r} \right)$$

(within acceptance)



Recall bend:

$$\delta\theta = \frac{0.3 Bl}{P(\text{GeV})} \quad \text{where } Bl \text{ is in T.m}$$

$$\therefore \theta' = \theta + \frac{\theta_0^2}{\theta} \quad (\text{within acceptance})$$

$$\therefore \Delta\theta' = \left(1 - \frac{\theta_0^2}{\theta^2}\right) \Delta\theta : \theta_0 \text{ measures } r_0 B_{\max}$$

No-focus constraint  $\Rightarrow \underline{\theta_0 \leq \theta_{\min}}$

(lets  $G\emptyset$  produce the target image)

Sets scale for torus size,  $B_{\max}$

Example:  $\theta_0 = 4^\circ$

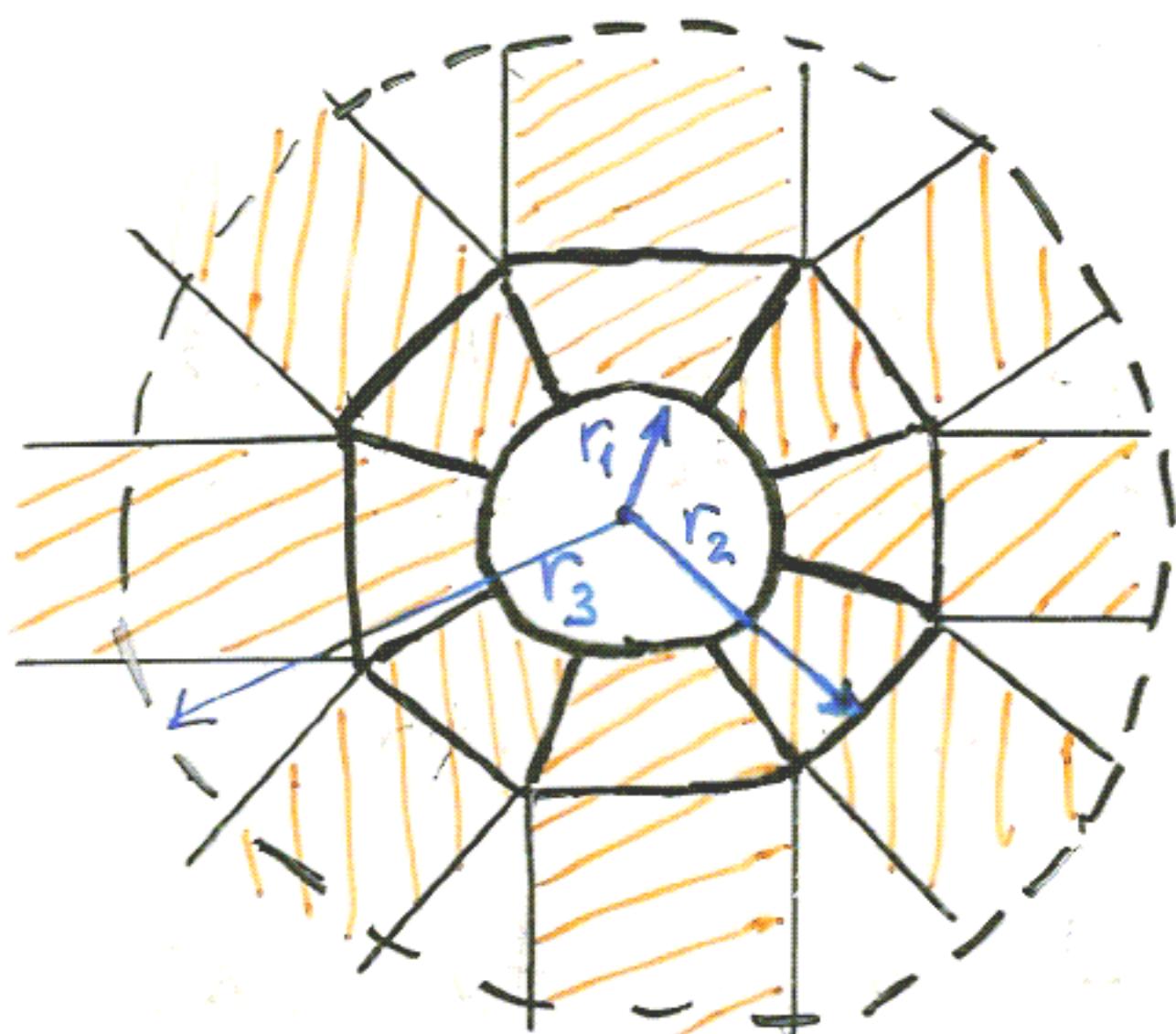
$$[8^\circ, 11^\circ] \rightarrow [4^\circ, 10.5^\circ]$$

$\frac{\Delta\theta}{\theta_0} : \underline{32\%}$       95%      \*3!

This is an over-estimate because

- 1) there will be some cost in  $\Delta\phi$
- 2) the coils may block access to very low angles like  $4^\circ$ .

- Coil geometry



$r_1$  = beam keep-out zone

$r_2$  = radius of inner coil boundary

$r_3$  = start of acceptance of collimator

Assume acceptance starts when

$$\frac{\Delta \phi}{2\pi} = 0.5$$

$$\Rightarrow r_3 = 2r_2 \quad (\text{after some tri-} \cdot)$$

... and  $r_2$  depends on  $r_1$  and the amount of Copper needed

Recall for an ideal solenoid:

$$B_{\max} = \frac{\mu_0 I}{2\pi r_0} = \frac{\mu_0}{2\pi r_0} j (r_2^2 - r_1^2) \pi$$

where  $j$  is the current density of copper coils :

$$\underline{500 \text{ A/cm}^2} \quad (\text{P. Brindza})$$

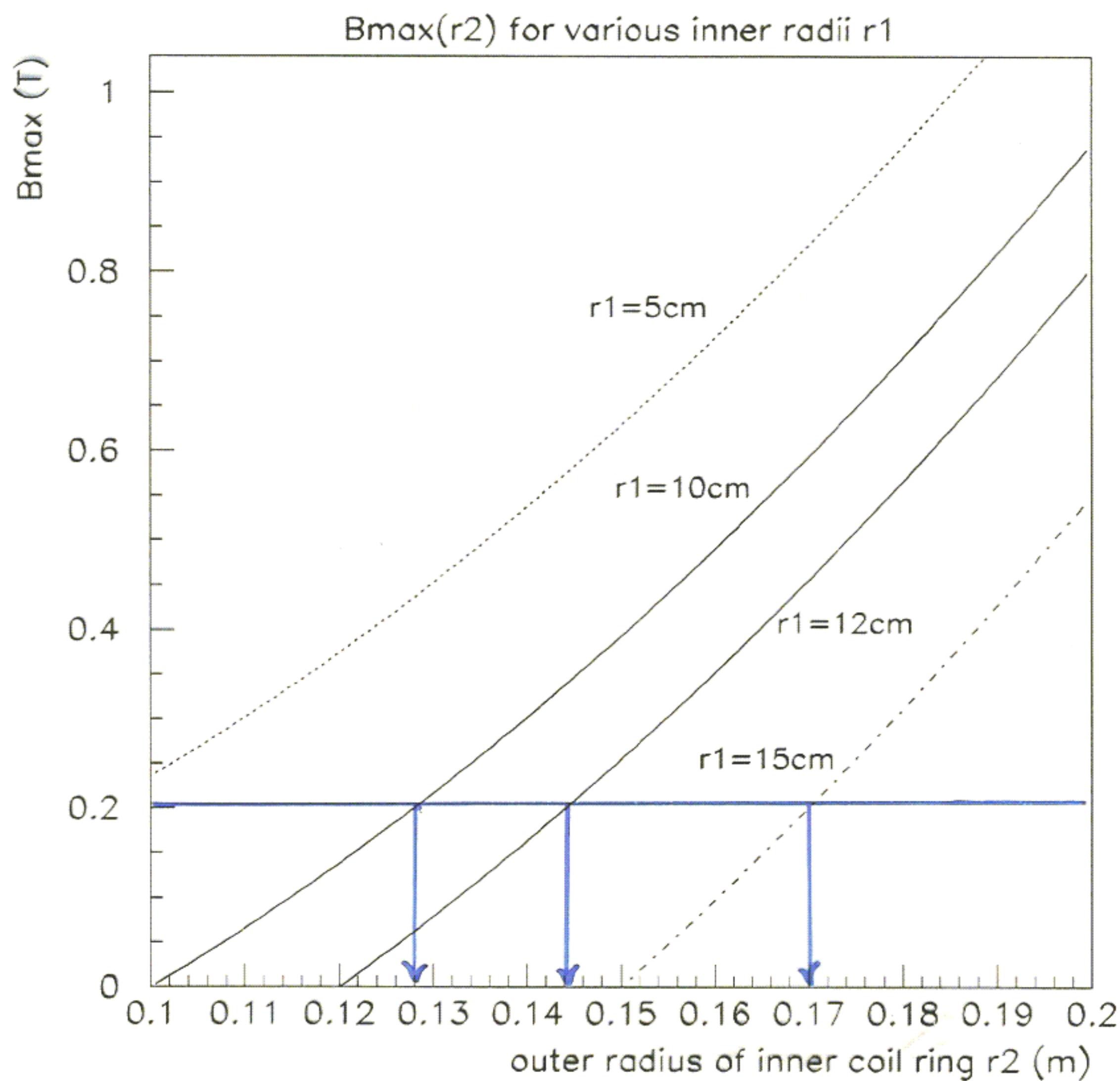
$\theta_0 = 4^\circ$  would require  $l \cdot B_{\max} = 3 \text{ kG}$  to capture  $4^\circ$  electrons.

Try:  $l = \underline{1.5 \text{ m}}$ ,  $B_{\max} = \underline{2 \text{ kG}}$

This sets scale for  $r_0 \sim l \theta_0 = \underline{10 \text{ cm}}$

Solve for  $r_2$  given  $r_1$

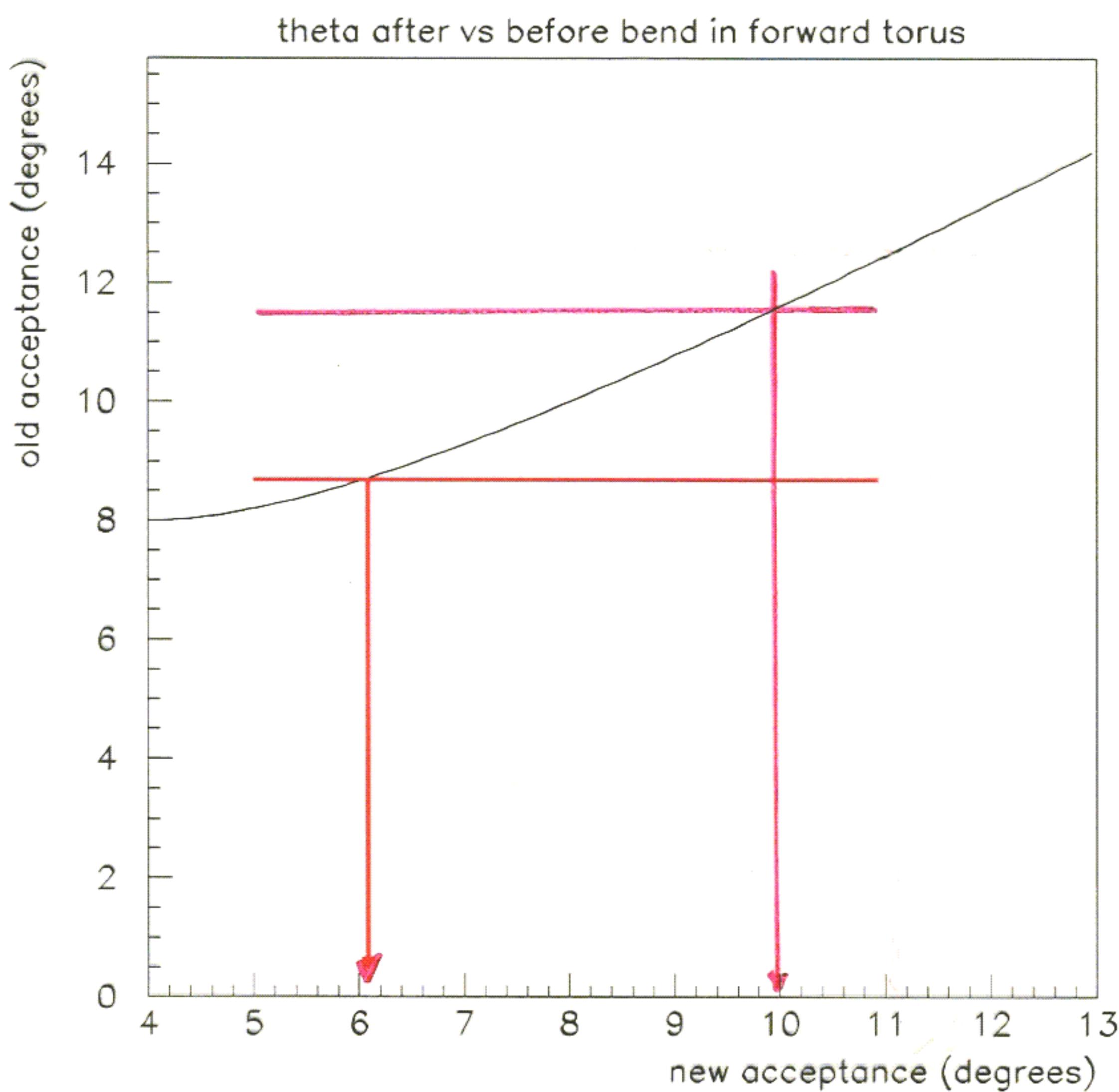
<u><math>r_1</math></u>	<u><math>r_2</math></u>	<u><math>r_3</math></u>
10cm	13 cm	26cm
12cm	14.5cm	29cm
15cm	17 cm	34cm



Take a favorable case for study:

$$r_1 = 10\text{cm} \quad r_2 = 13\text{cm} \quad r_3 = 26\text{cm}$$

$$B_m = 2\text{ kG} \cdot \frac{r_0}{r}, \quad r_0 = 10\text{cm}, \quad l = 15$$



$$\left. \begin{aligned} \frac{\Delta \Theta}{\Theta} &= 50\% \\ \frac{\Delta \phi}{2\pi} &= 50\% \end{aligned} \right\} \quad \boxed{\text{Combined factor } \underline{1.14} \text{ improvement}}$$