

# Exotic and excited-state radiative transitions in charmonium from lattice QCD

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In collaboration with:  
Jo Dudek, Robert Edwards, David Richards  
and the *Hadron Spectrum Collaboration*

# Outline

- Introduction and motivation
- Method
- Result highlights and interpretations
- Conclusions



# Broader Picture

Develop Lattice QCD Techniques

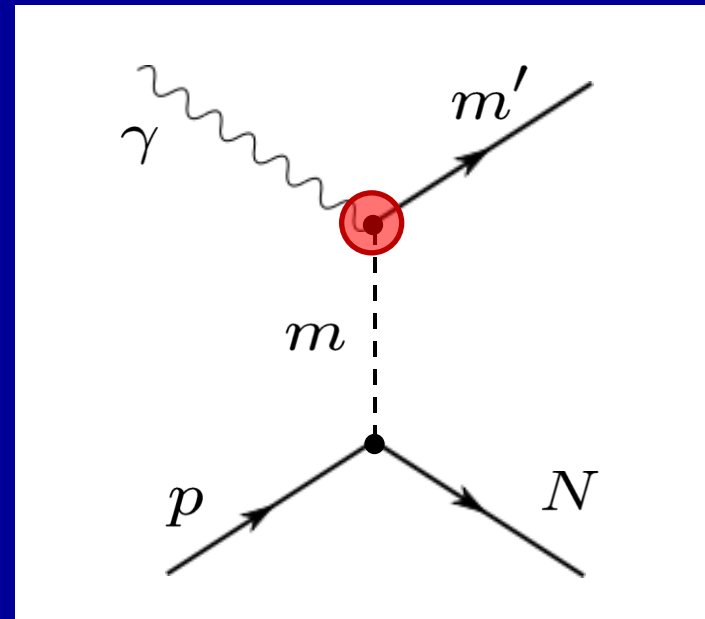
Test in the charmonium system

Apply to lighter mesons...

Photoproduction at GlueX

Exotic  $1^{-+}$ ?

For progress on the light meson spectrum  
see Jo Dudek's talk from Tuesday



# Spectroscopy on the lattice

Calculate **energies** and **matrix elements (Z)** from correlation functions of meson interpolating fields

$$O = \bar{\psi}(x) \Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \dots \psi(x)$$

$$C(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2 E_n} \langle 0 | O_i(0) | n \rangle \langle n | O_j(0) | 0 \rangle$$

$$\xrightarrow{t \rightarrow \infty} \frac{Z_i^{(0)} Z_j^{(0)*}}{2 E_0} e^{-E_0 t}$$

$$Z_i^{(n)} \equiv \langle n | O_i | 0 \rangle$$

# Variational Method

Consider a large basis of operators  $\rightarrow$  matrix of correlators  $C_{ij}(t)$

Generalised eigenvector problem:

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

Eigenvalues  $\rightarrow$  energies

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

$(t \gg t_0)$

Eigenvectors  $\rightarrow$  optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

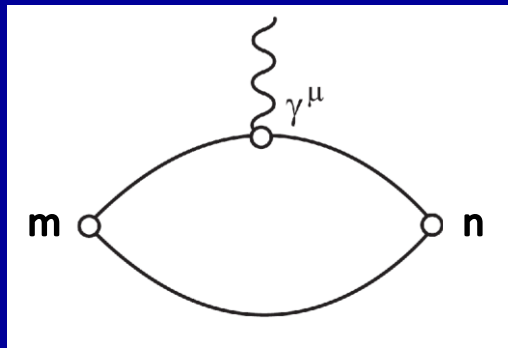
$Z^{(n)}$  related to eigenvectors

# Photocouplings on the Lattice

Calculate from 3-point correlators:

$$C_{ij}(t_f, t, t_i) = \langle 0 | O_i(t_f) \bar{\psi}(t) \gamma^\mu \psi(t) O_j(t_i) | 0 \rangle$$

$$\sim \sum_n \sum_m \langle 0 | O_i(0) | n \rangle \langle n | \bar{\psi} \gamma^\mu \psi | m \rangle \langle m | O_j(0) | 0 \rangle \\ \times e^{-E_n(t_f - t)} e^{-E_m(t - t_i)} / (2E_n 2E_m)$$



Known from 2-point analysis

What we want: parameterize in terms of multipoles ( $\sim$  form factors) and known factors

# Multipoles

$$\langle n | \bar{\psi} \gamma^\mu \psi | m \rangle \sim \sum_k K_{k,n,m}^\mu(\vec{p}_i, \vec{p}_f) F_{k,n,m}(Q^2, t)$$

Multipoles

$$J_i = J_f \otimes k \quad (k > 0)$$

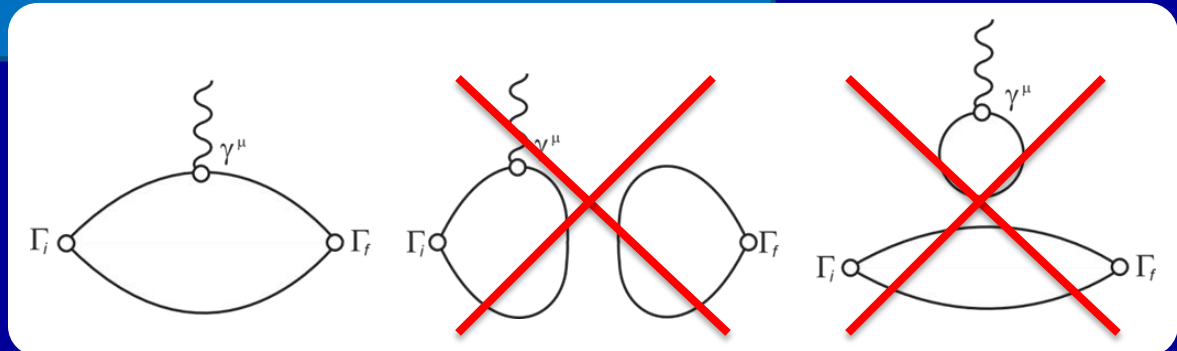
Experimentally measure multipoles at  $Q^2 = 0$

Discrete momenta on the lattice



# Charmonium radiative transitions

- Caveats:
  - Quenched (no quark loops; no light quarks at all)
  - One lattice spacing ( $a_t^{-1} = 6.05$  GeV)
  - One volume ( $L_s \approx 1.2$  fm)
  - Only connected diagrams



Only a selection; results and details in  
Dudek, Edwards & CT, PR **D79** 094504 (2009)

Also: Dudek et al PR **D77** 034501 (2008); Dudek & Rrapaj PR **D78** 094504 (2008)

# Exotic $1^{-+}$ – Vector $1^{--}$

Spectrum analysis:  $1^{-+} \eta_{c1}$  state found at 4300(50) MeV

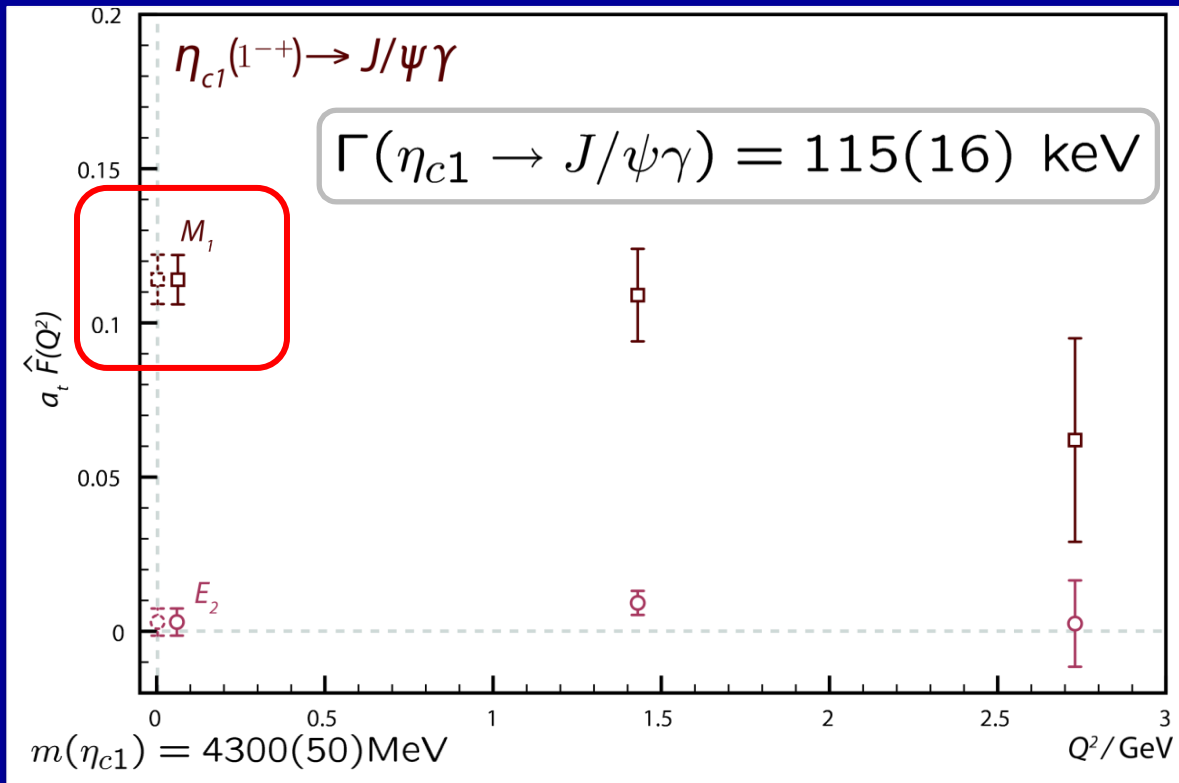
Exotic quantum numbers – can't be fermion-antifermion pair

Can't be a molecular/multi-quark state in **quenched** lattice calc.

→ Strongly suggests a hybrid

What about radiative transitions?

# Exotic $1^{--}$ – Vector $1^{--}$



$M_1$  multipole dominates

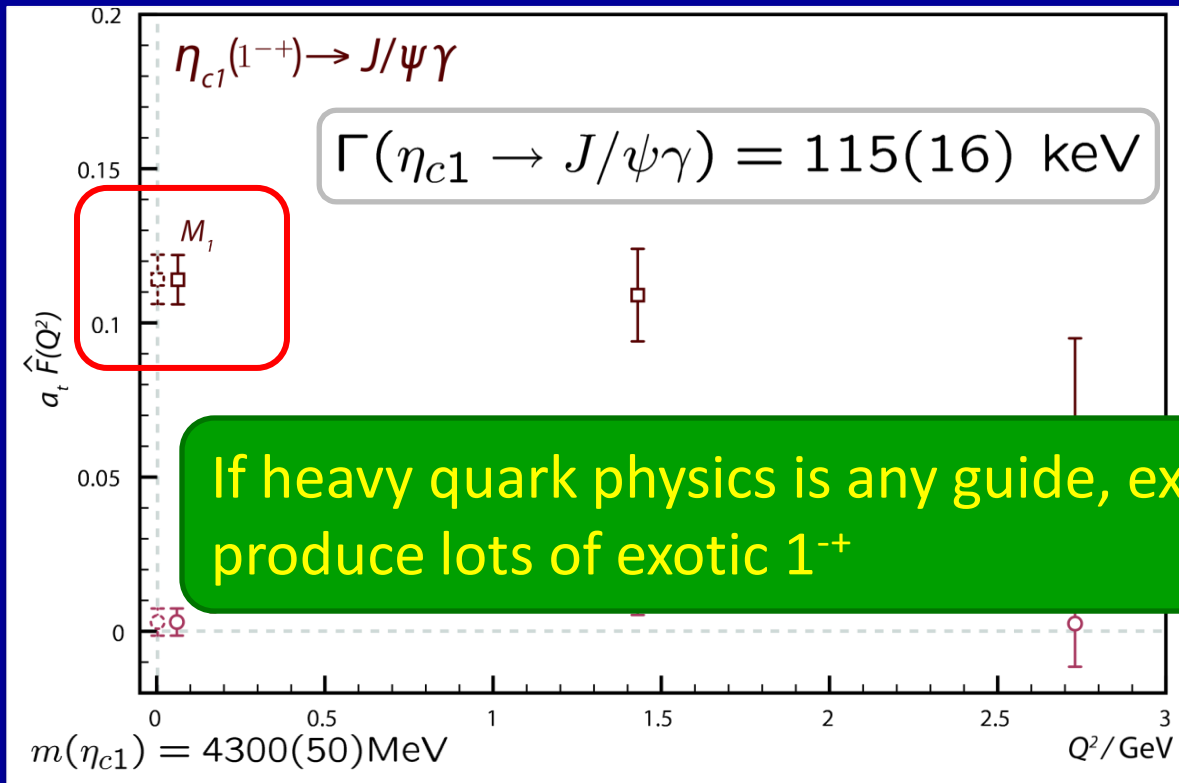
Same scale as many measured conventional charmonium transitions

BUT very large for an  $M_1$  transition

$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$

- Usually  $M_1 \rightarrow$  spin flip (e.g.  $^3S_1 \rightarrow ^1S_0$ )  $\rightarrow 1/m_c$  suppression
- Spin-triplet hybrid  $\rightarrow$  extra gluonic degrees of freedom  
 $\rightarrow M_1$  transition without spin flip  $\rightarrow$  not suppressed

# Exotic $1^{-+}$ – Vector $1^{--}$



$M_1$  multipole dominates

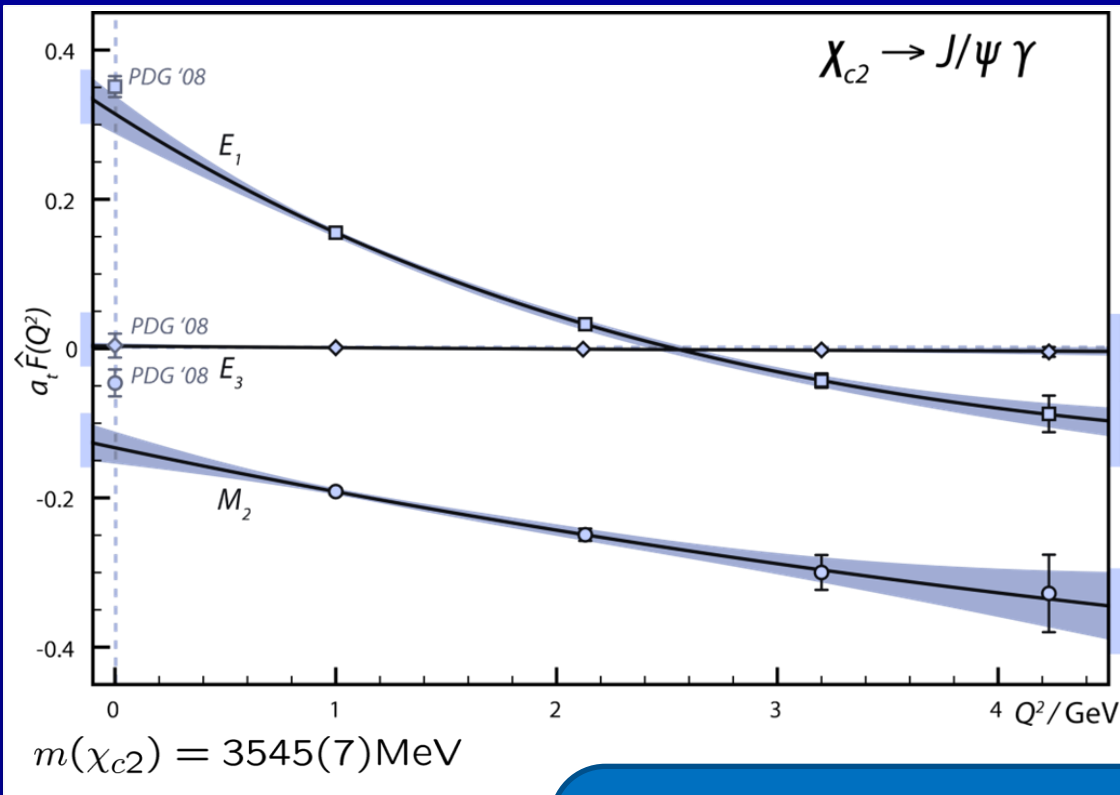
Same scale as many measured conventional charmonium transitions

If heavy quark physics is any guide, expect GlueX to produce lots of exotic  $1^{-+}$

$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$

- Usually  $M_1 \rightarrow$  spin flip (e.g.  $^3S_1 \rightarrow ^1S_0$ )  $\rightarrow 1/m_c$  suppression
- Spin-triplet hybrid  $\rightarrow$  extra gluonic degrees of freedom  
 $\rightarrow M_1$  transition without spin flip  $\rightarrow$  not suppressed

# Tensor $2^{++}$ – Vector $1^-$

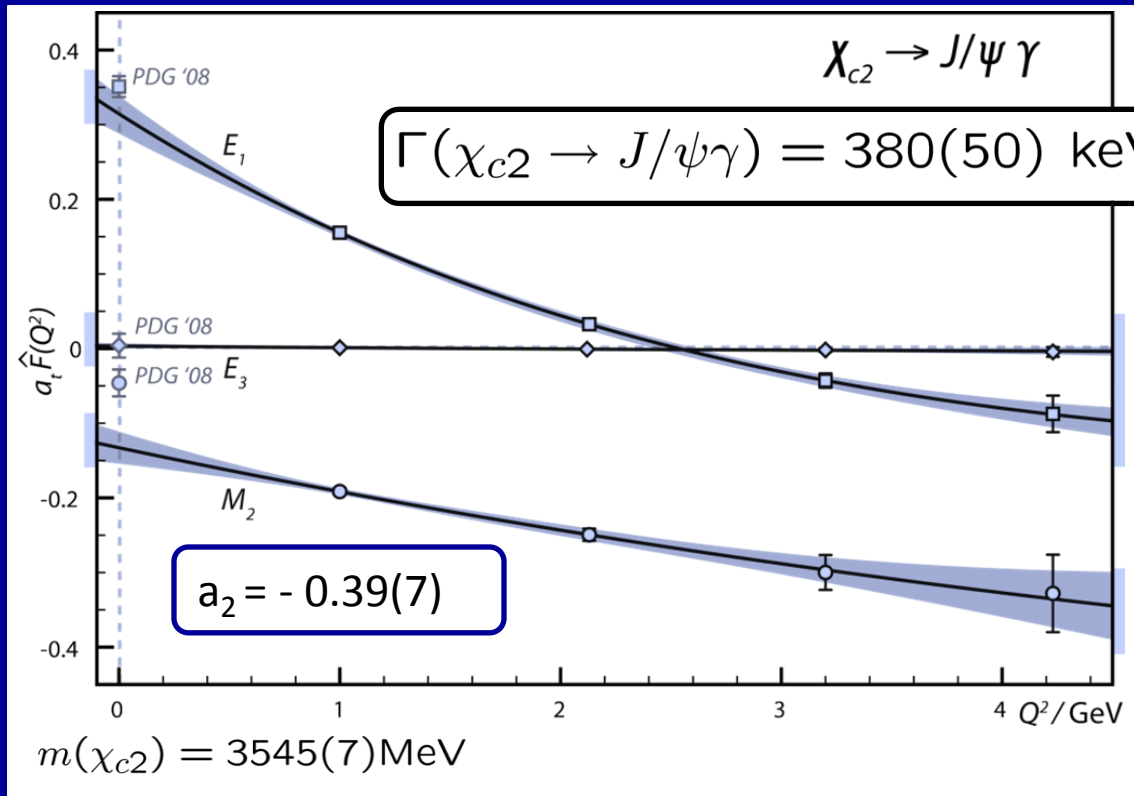


$E_1, M_2, E_3$

- Lattice: discrete set of allowed momenta
- Can't calculate at  $Q^2 = 0$  and so extrapolate:

$$F_k(Q^2) = F_k(0) \left(1 + \lambda Q^2\right) e^{-\frac{Q^2}{16\beta^2}}$$

# Tensor $2^{++}$ – Vector $1^-$



$E_1, M_2, E_3$

PDG08: 406(31) keV

Quark models ( $1^3P_2$ )  
~ 290 – 420 keV

$$a_2 = M_2 / \sqrt{E_1^2 + M_2^2 + E_3^2}$$

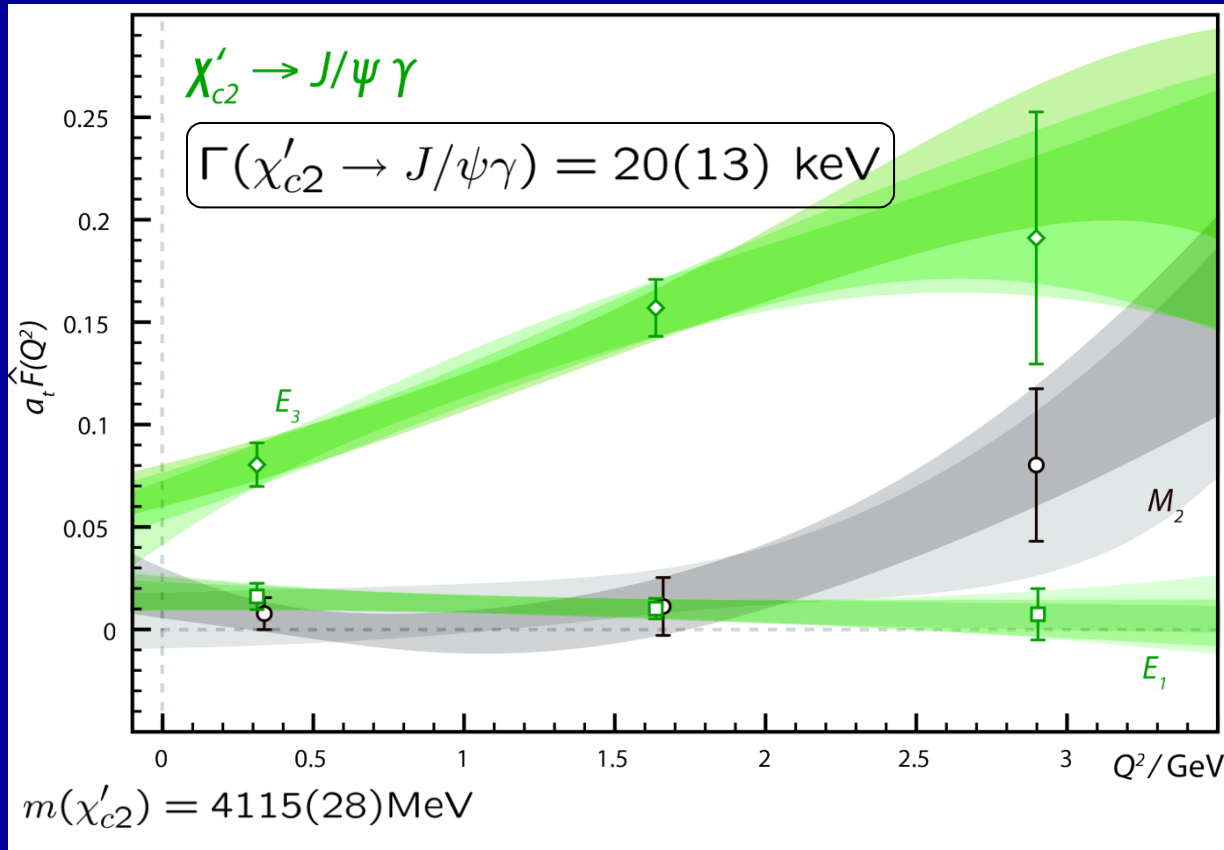
PDG08: -0.13(5)

CLEO: -0.079(19)

[CLEO arXiv:0910.0046]

- Same hierarchy as expected:  $|E_1(0)| > |M_2(0)| \gg |E_3(0)|$
- Ratio  $|M_2/E_1|$  is considerably larger than experiment

# Tensor $2^{++}$ – Vector $1^-$

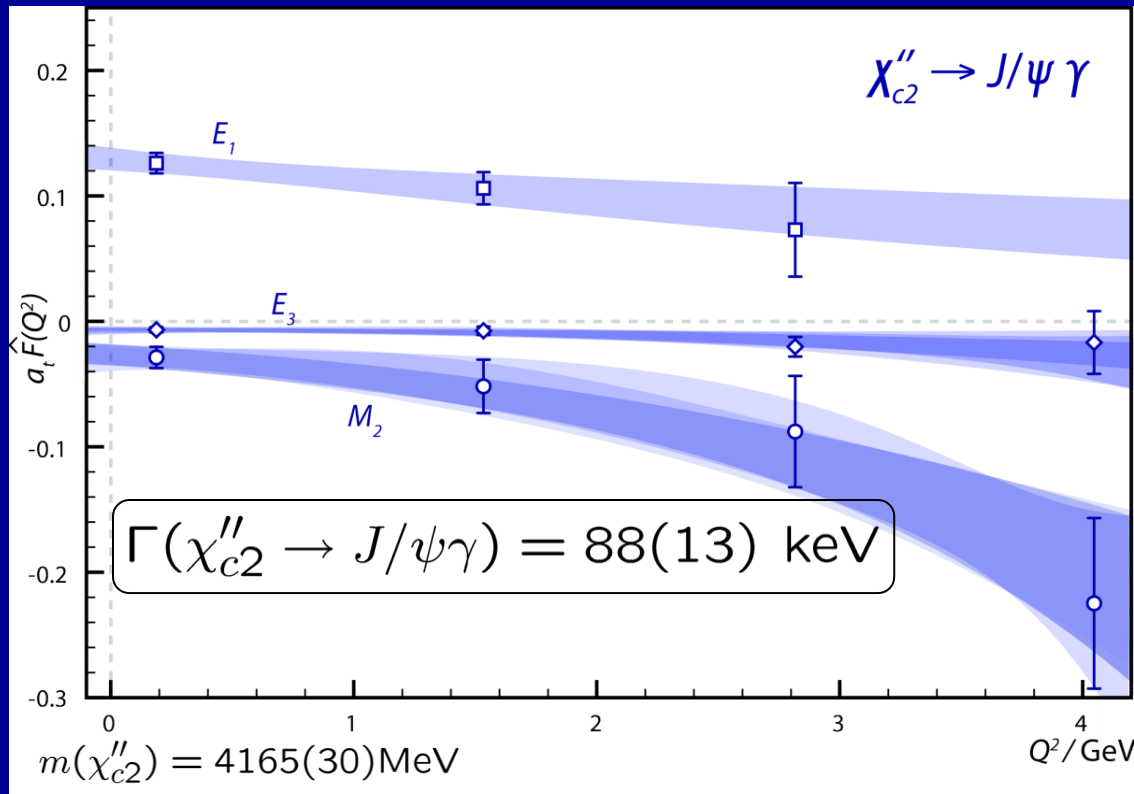


$E_1, M_2, E_3$

Completely different hierarchy!

$|E_3(0)| > |M_2(0)|, |E_1(0)|$

# Tensor $2^{++}$ – Vector $1^-$



$E_1, M_2, E_3$

Quark models ( $2^3P_2$ )  
 $\sim 50 - 80 \text{ keV}$

Reverted to expected hierarchy:

$$|E_1(0)| > |M_2(0)| \gg |E_3(0)|$$



# Tensor $2^{++}$ – Vector $1^-$

Interpretation: **single quark transition model**

In general:  $J_i = J_f \otimes k \quad (k > 0)$   
 $E_1, M_2, E_3 \quad (k = 1, 2, 3)$

If only a single quark is involved ( ${}^3P_2 \rightarrow {}^3S_1$ ):

$$j = 3/2 \rightarrow j = 1/2$$

$$k = 1, 2 \text{ only and } E_3 = 0$$

$$|E_1(0)| > |M_2(0)| \gg |E_3(0)|$$

If instead tensor is  ${}^3F_2$  ( ${}^3F_2 \rightarrow {}^3S_1$ ):

$$j = 5/2 \rightarrow j = 1/2$$

$$k = 2, 3 \text{ only and } E_1 = 0$$

$$|E_3(0)| > |M_2(0)| \gg |E_1(0)|$$

# Tensor $2^{++}$ – Vector $1^-$

Interpretation: **single quark transition model**

In general:  $J_i = J_f \otimes k \quad (k > 0)$   
 $5 \otimes 3 = 5 + 1 + 3 + 3$

**Interpretation:**

If only

$$\chi_{c2} = 1^3P_2$$

$$\chi'_{c2} = 1^3F_2$$

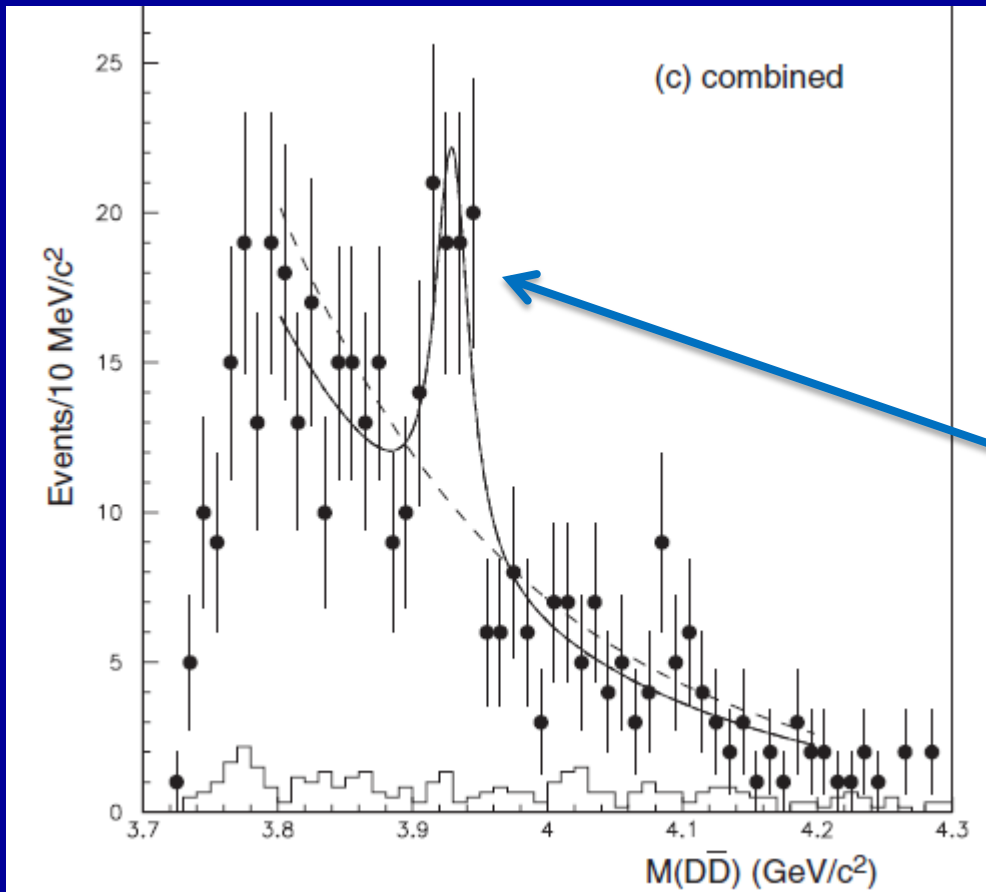
$$\chi''_{c2} = 2^3P_2$$

If instead

Supported by spectrum analysis

$$|E_3(0)| > |M_2(0)| \gg |E_1(0)|$$

# Tensor $2^{++}$ – Vector $1^-$



Belle

$\gamma\gamma \rightarrow D\bar{D}$

$\chi'_{c2} \sim 3930$  MeV

Needs lattice calc of  
two-photon coupling

Belle [PRL 96 082003 (2006)]

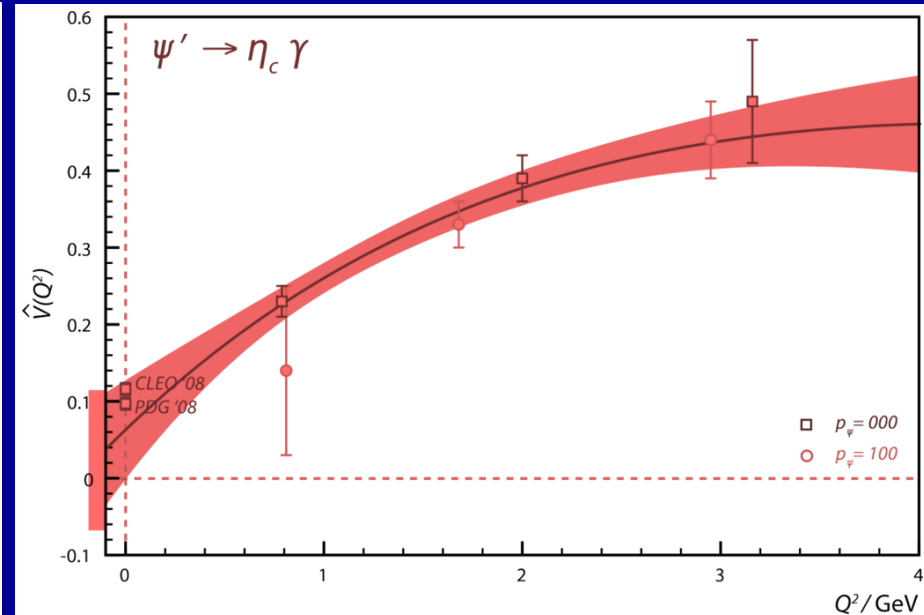
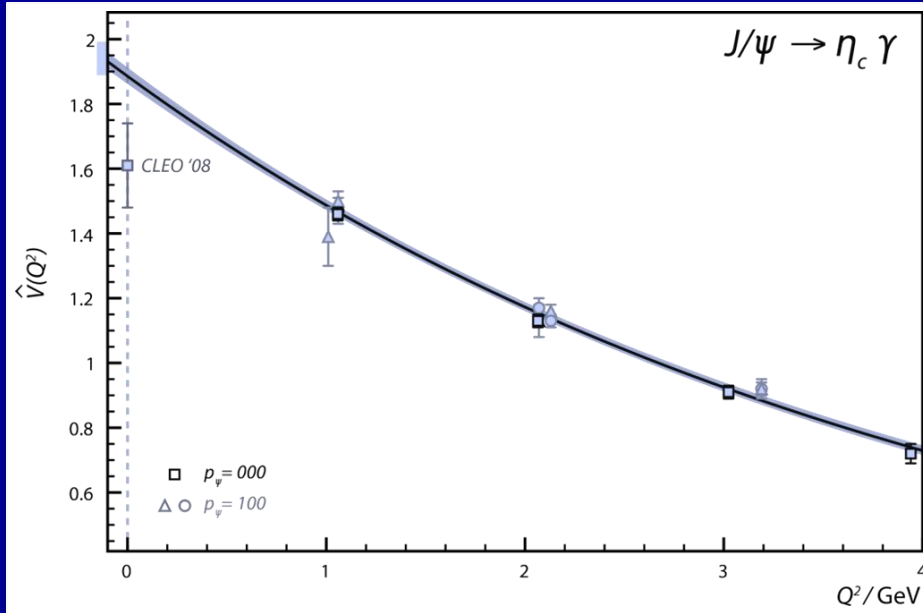
# Vector $1^-$ – Pseudoscalar $0^-$

Spectrum results [PR **D77** 034501 (2008), PR **D78** 094504 (2008) ]:

Level	Mass / MeV	Suggested state	Model assignment
0	3106(2)	$J/\psi$	$1^3S_1$
1	3746(18)	$\psi'(3686)$	$2^3S_1$
2	3846(12)	$\psi_3(3^{--})$	lattice artifact
3	3864(19)	$\psi''(3770)$	$1^3D_1$
4	4283(77)	$\psi('4040')$	$3^3S_1$
5	4400(60)	$Y$	hybrid

# Vector $1^-$ – Pseudoscalar $0^-$

Only  $M_1$

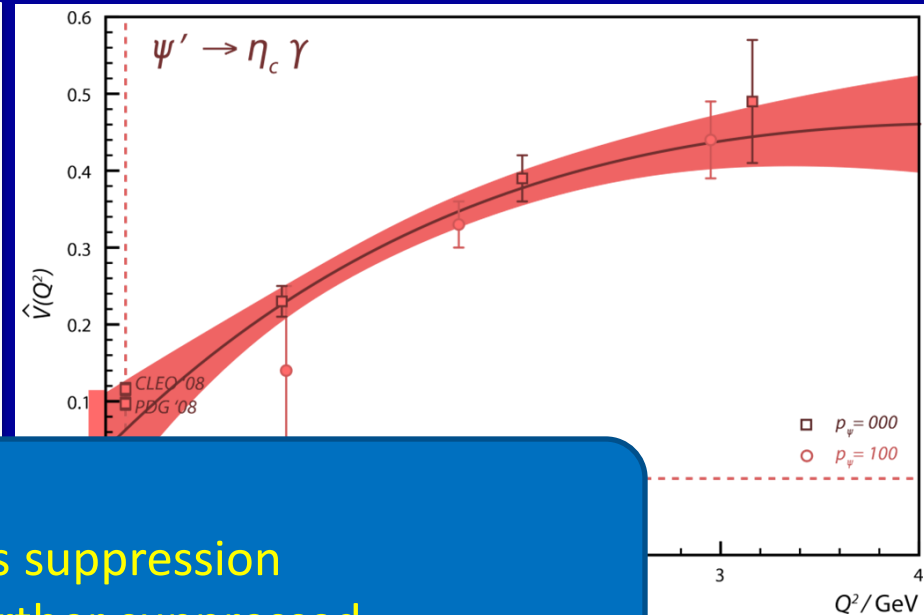
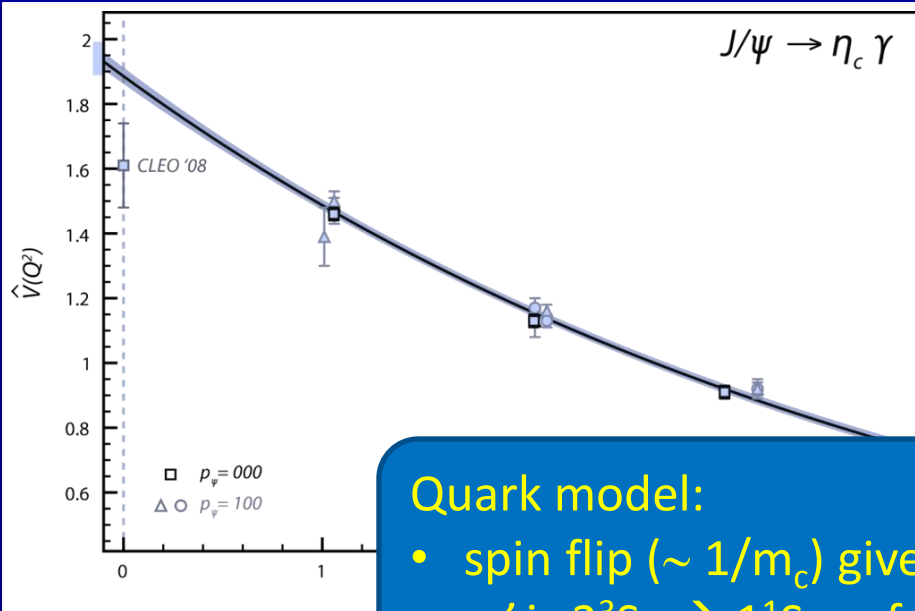


$\Gamma / \text{keV}$	Lattice	Exp.	Barnes, Godfrey, Swanson		Eichten et. al.
			'NR'	'GI'	
$J/\psi \rightarrow \eta_c \gamma$	2.51(8)	1.85(29) (CLEO-c)	2.9	2.4	1.92
$\psi' \rightarrow \eta_c \gamma$	0.4(8)	0.95(16) (PDG08) 1.37(20) (CLEO-c)	4.6, 9.7	9.6	0.91

[CLEO PRL 102 011801 (2009)]

# Vector $1^-$ – Pseudoscalar $0^+$

Only  $M_1$



Quark model:

- spin flip ( $\sim 1/m_c$ ) gives suppression
- $\psi'$  is  $2^3S_1 \rightarrow 1^1S_0$  – further suppressed

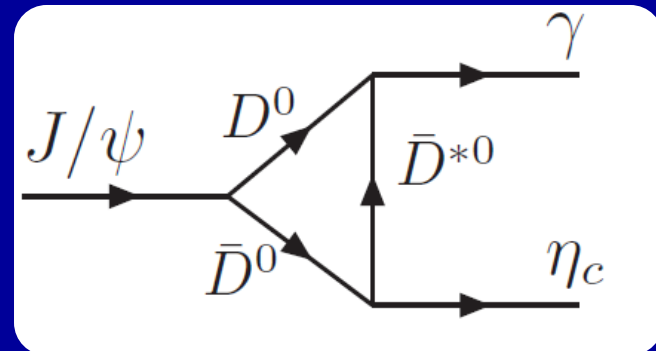
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[CLEO PRL 102 011801 (2009)]

# Vector $1^-$ – Pseudoscalar $0^-$

## Loops

Li & Zhao [PL B670 55 (2008)]:  
loop contributions in  
vector – pseudoscalar transitions



→ Large loop contrib ( $\sim 10$  keV) to  $\psi' \rightarrow \eta_c \gamma$

Interferes with  $\sim 10$  keV bare amp. →  $\sim 1$  keV (in line with experiment)

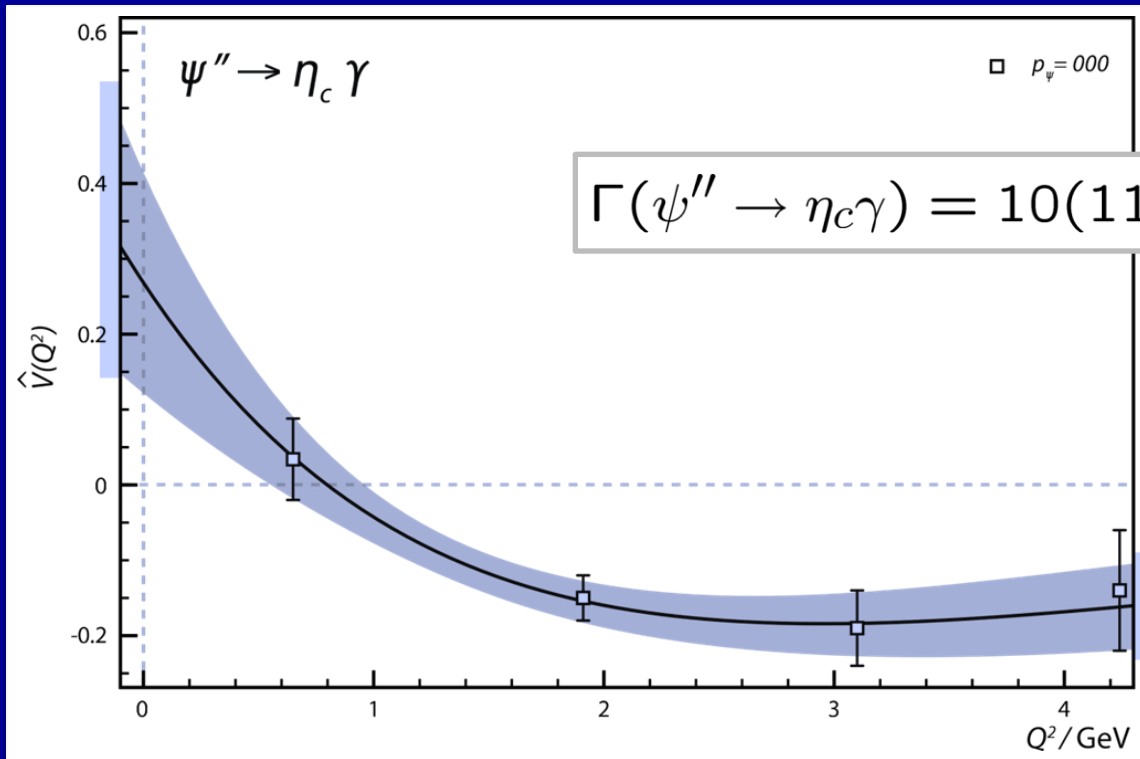
**But** no room for large loop contrib with  $0.4(8)$  keV from quenched lattice

Actually loops calc has uncertainties from couplings and phases  
E.g. use  $\Gamma(D^{*0} \rightarrow D^0 \gamma) \sim 800$  keV for  $g_{D^* D \gamma}$  c.f. QM  $\approx 30$  keV

→ **Not** incompatible with lattice results

# Vector $1^-$ – Pseudoscalar $0^-$

Only  $M_1$

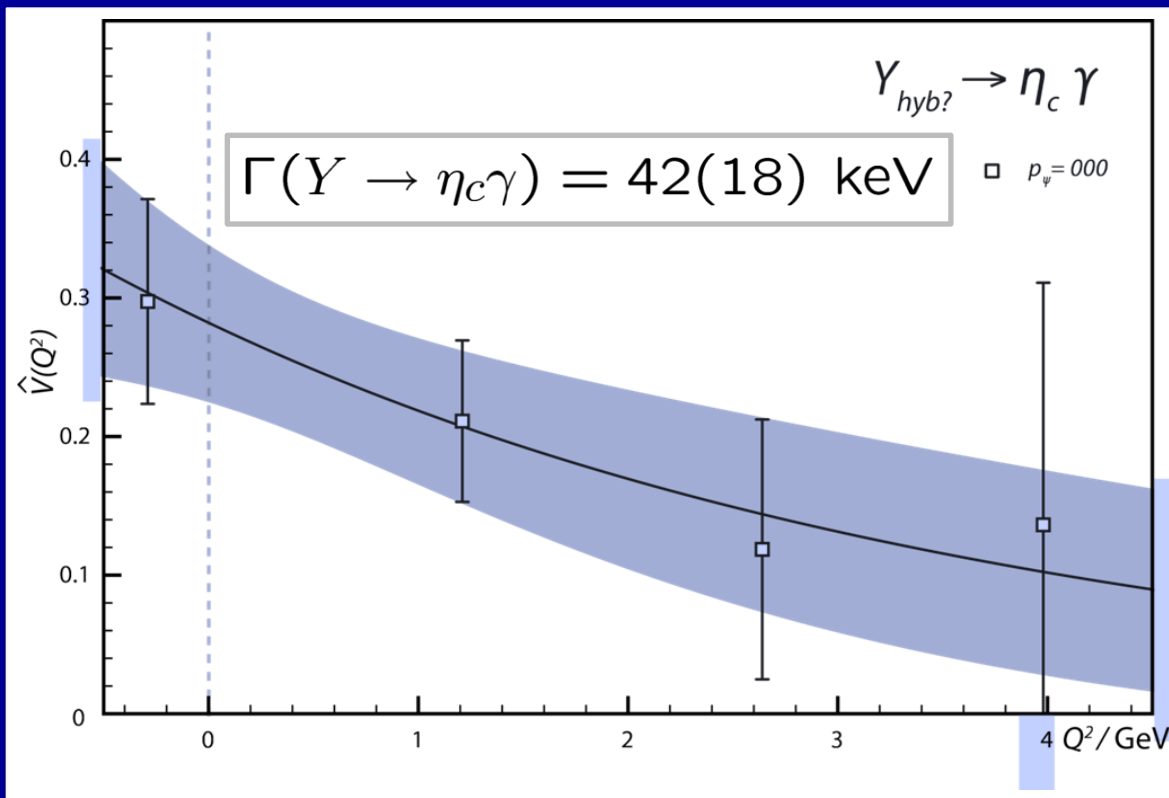


Quark model:  $1^3D_1 \rightarrow 1^1S_0$  has same leading  $Q^2$  behaviour as  $2^3S_1 \rightarrow 1^1S_0$



# Vector $1^-$ – Pseudoscalar $0^-$

Only  $M_1$



Much larger than other  
 $1^- \rightarrow 0^- M_1$  trans

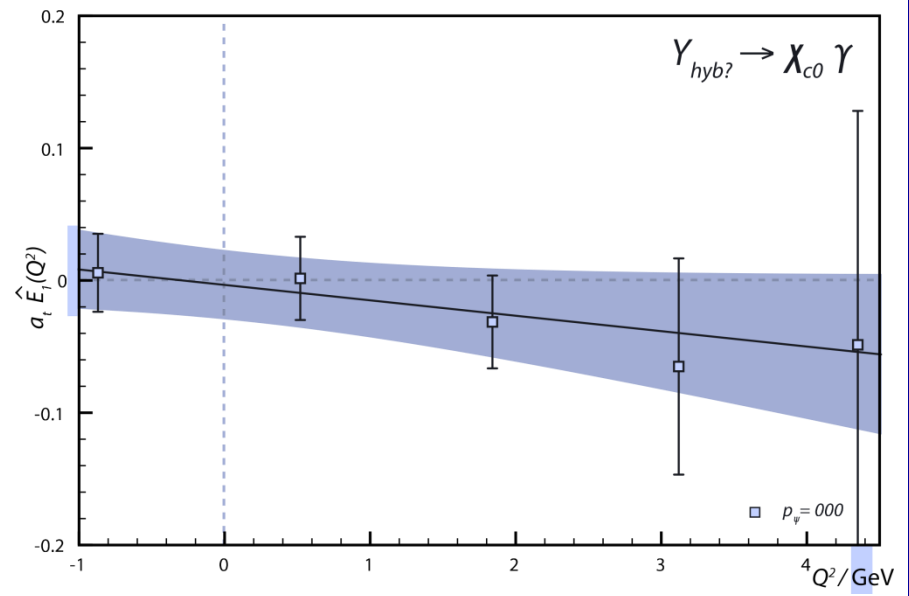
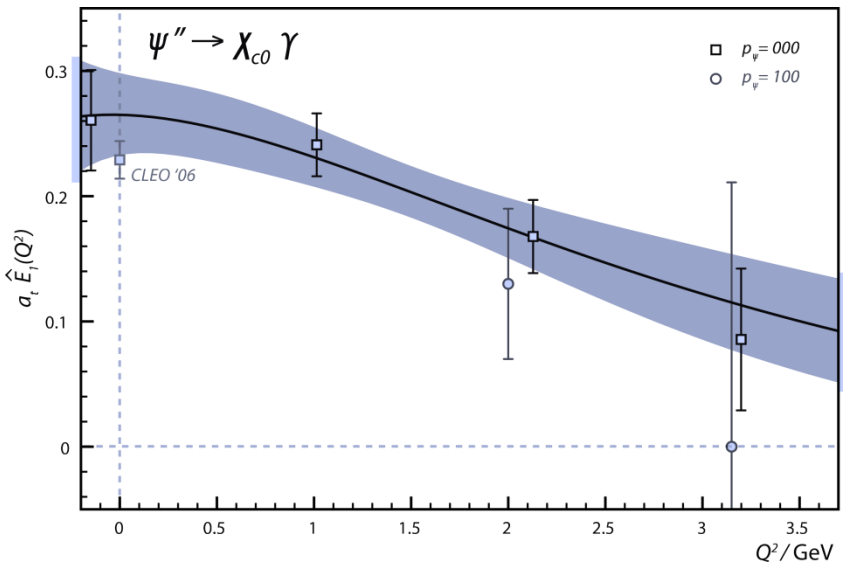
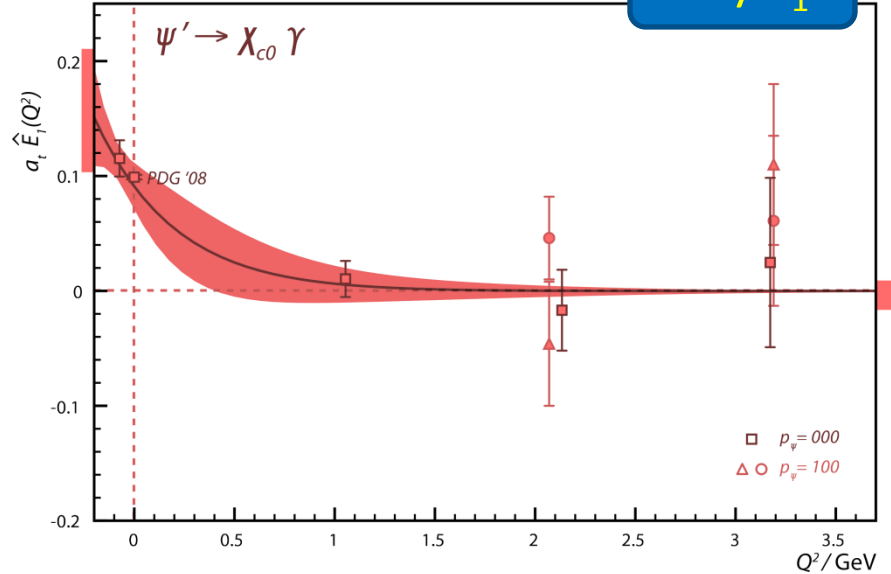
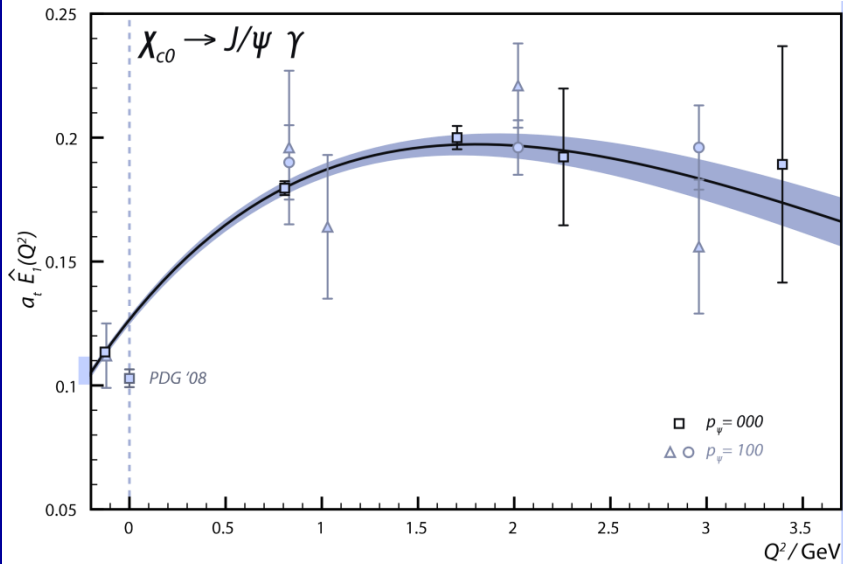
Spectrum analysis  
suggests a vector  
hybrid (spin-singlet)

Analogous to  $1^+$   
hybrid to vector trans:  
 $M_1$  with no spin flip

c.f. flux tube model 30 – 60 keV

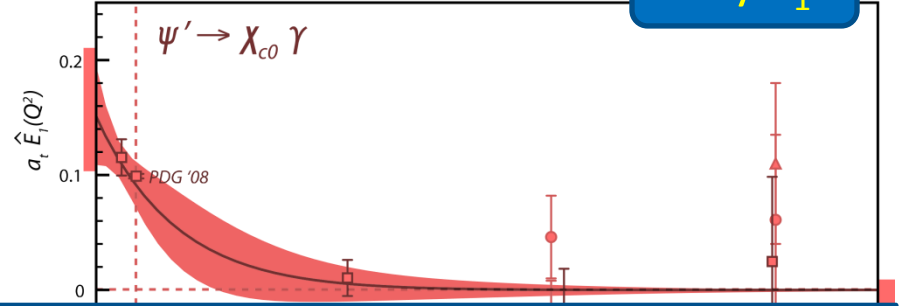
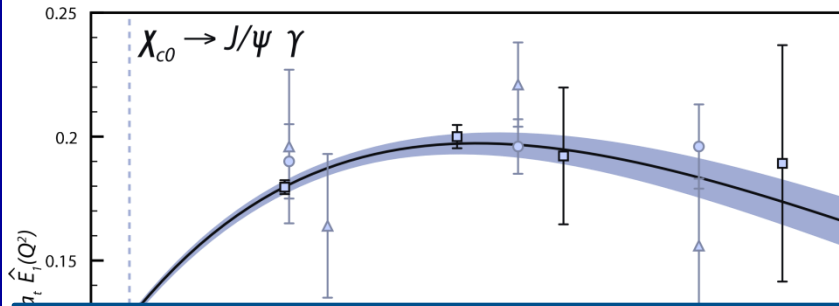
# Scalar $0^{++}$ – Vector $1^{--}$

Only  $E_1$

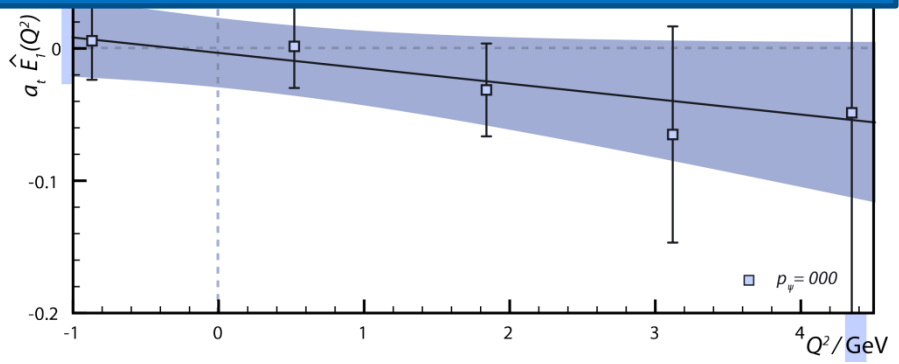
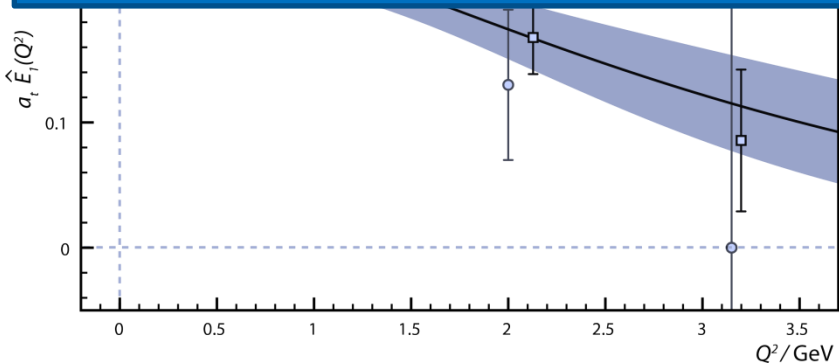


# Scalar $0^{++}$ – Vector $1^{--}$

Only  $E_1$



$\Gamma$ / keV	Lattice	Exp. (PDG08)	Barnes, Godfrey, Swanson 'NR'	Barnes, Godfrey, Swanson 'GI'	Eichten et. al.
$\chi_{c0} \rightarrow J/\psi(1^3S_1)\gamma$	199(6)	131(14)	152	114	120, 105
$\psi'(2^3S_1) \rightarrow \chi_{c0}\gamma$	26(11)	30(2)	63	26	46, 38
$\psi''(1^3D_1) \rightarrow \chi_{c0}\gamma$	265(66)	199(26)	403	213	287
$\psi''(3^3S_1) \rightarrow \chi_{c0}\gamma$			0.27	0.63	
$Y \rightarrow \chi_{c0}\gamma$	$\lesssim 20$				



# Summary and Outlook

## Charmonium Summary

- **Method successful:** first calc. of excited meson rad. trans. on lattice
- **Hybrid photocoupling is large:**  $\Gamma(\eta_{c1} \rightarrow J/\psi\gamma) \sim 100 \text{ keV}$
- $M_1$  transitions:  $\psi \rightarrow \eta_c\gamma$
- Non-exotic **vector hybrid candidate**  $\Gamma(Y \rightarrow \eta_c\gamma) = 42(18) \text{ keV}$
- $E_1, M_2, E_3$  multipoles;  $2^3P_2, 1^3F_2$  states in  $\chi_{c2} \rightarrow J/\psi\gamma$
- Comparison with quark models

## Outlook

- Systematically improvable
- Apply to lighter mesons (unquenched calc.)