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Unitarization Methods in meson-meson scattering and decays

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OUTLINE

- Introduction

- 2-body unitarization methods.

From more rigorous to rougher ↑ BUT ↓ From complicated (unfeasible sometimes) to simpler (but very successful)

<u>Examples:</u>

IAM single channel
IAM coupled channels

Chiral unitary approach $O(p^4)$
Chiral unitary approach $O(p^2)$

Chiral Perturbation Theory in the meson sector

Weinberg, Gasser & Leutwyler

π, K , η Goldstone Bosons
of the spontaneous
chiral symmetry breaking
$SU(N_f)_V \times SU(N_f)_A \rightarrow SU(N_f)_V$

$\leftrightarrow$

QCD degrees of freedom
at **low energies << $4\pi f$~1.2 GeV**

ChPT is the most general expansion in energies
of a lagrangian made only of pions, kaons and etas
compatible with the QCD symmetry breaking

Leading order parameters: breaking scale f_0 and masses

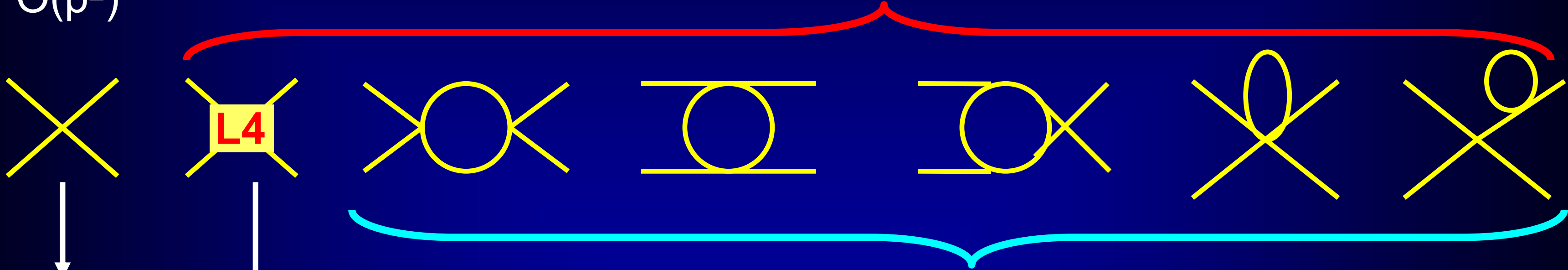
At 1-loop, QCD dynamics encoded in
chiral parameters: $L_1 \ldots L_8$
Determined from EXPERIMENT
leading $1/N_c$ behavior known from QCD

ChPT is the QCD Effective Theory
MODEL INDEPENDENT
but limited to low energies

- Full one-loop calculation:

$t_2(s,t,u)$
$O(p^2)$

$t_4(s,t,u)=O(p^4)$



f_π ,M_π
set the scale

Loops with O(p2) vertices. Divergent. Need
REGULARIZATION,scale μ, (or cutoff or whatever).

<u>Low Energy Constants (LECS) L_i</u>
Contain Underlying dynamics, QCD in this case
Absorb all loop divergences $\rightarrow$ regularization $\rightarrow$
LECS depend on Regulator

When calculation fully RENORMALIZED
finite up to a given order in energy/momenta,
regulator dependencen disappears from observables

Elastic two-body Unitarity Constraints: One channel

Partial wave UNITARITY

(On the real axis above threshold)

$$\mathrm{Im}\, t = \sigma |t|^2$$

$\sigma(s) = 2p_{CM}/\sqrt{s}$ KNOWN EXACTLY (kinematics)

EXACT unitarity not satisfied by ChPT series

(or any other series)

$$\mathrm{Im}\frac{1}{t} = -\sigma$$

$$|t| < \frac{1}{\sigma} \approx 1$$

Unitarity bound

Badly violated if ChPT series extrapolated to high energies or resonance region

How to fix that?

$$t \approx \frac{1}{\mathrm{Re}\, t^{-1} - i\sigma}$$

exactly unitary !!

We only need the Real part of 1/t (dynamics)

Different unitarization methods are just different approximations to Re(1/t)

2-body unitarization methods.

From
more rigorous
to rougher ↑ BUT ↓ From
complicated (unfeasible sometimes)
to simpler (but very successful)

Examples:

IAM single channel
IAM coupled channels

Chiral unitary approach $O(p^4)$
Chiral unitary approach $O(p^2)$

Of course, there are other variations,

Unitarization of ChPT. The Inverse Amplitude Method. One channel

Truong, Dobado, Herrero, Peláez...

Partial wave unitarity

(On the real axis above threshold)

$$\mathrm{Im}\, t = \sigma |t|^2$$

$$\mathrm{Im}\frac{1}{t} = -\sigma$$

$$t \approx \frac{1}{\mathrm{Re}\, t^{-1} - i\sigma}$$

exactly unitary !!

$$t \approx \frac{t_2^2}{t_2 - \mathrm{Re}\, t_4 - i\sigma t_2^2}$$

ChPT = series in p^2

$$t = t_2 + t_4 \ldots$$

exact perturbative unitarity

If t_4 defined to satisfy

$$\mathrm{Im}\, t_4 = \sigma |t_2|^2$$

provides ....

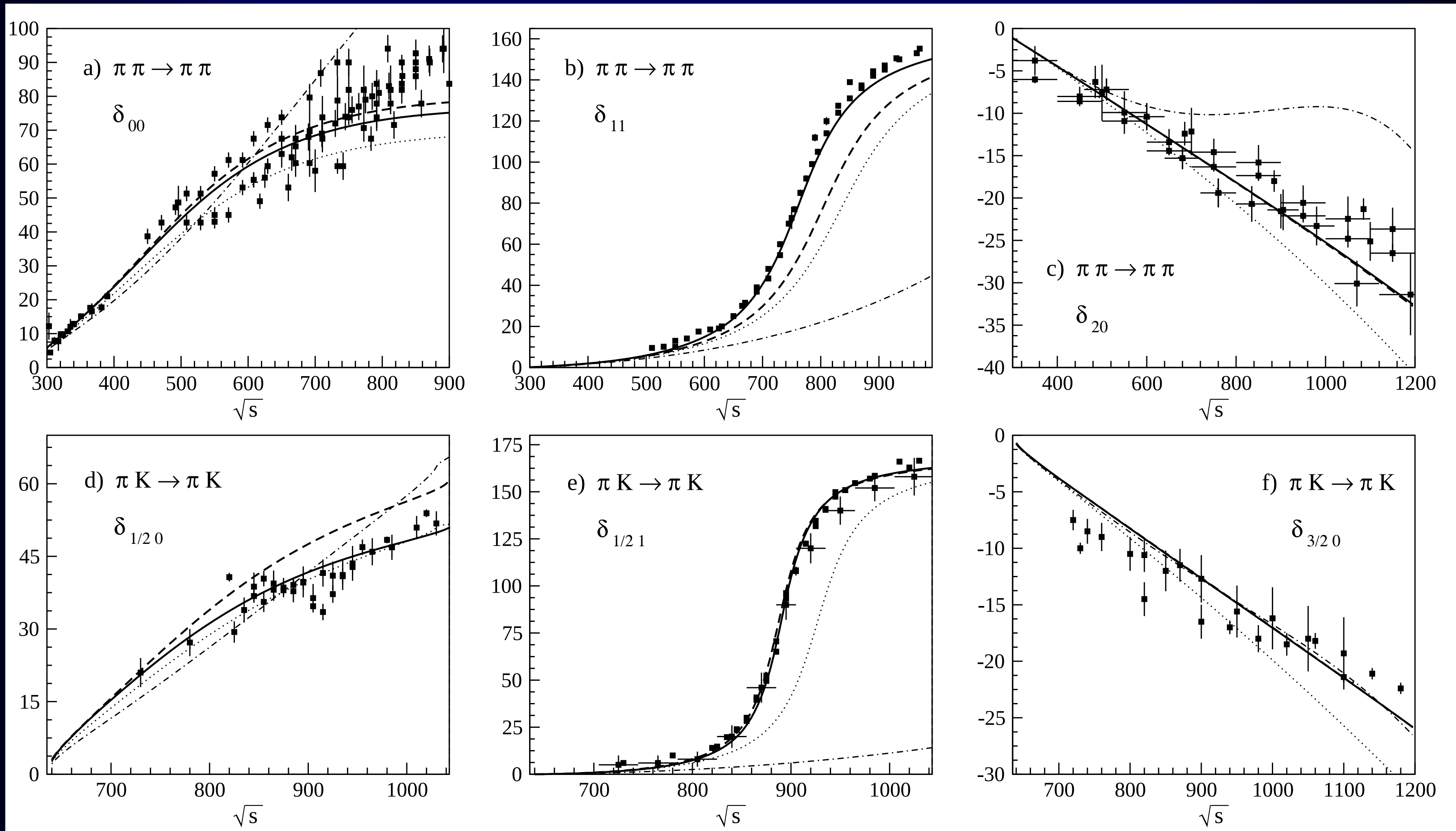
$$\mathrm{Re}\, t^{-1} \approx t_2^{-2}(t_2 - \mathrm{Re}\, t_4 + \ldots)$$

IAM

$$t \approx \frac{t_2^2}{t_2 - t_4}$$

Truong ‘89, Truong,Dobado,Herrero,’90, Dobado, JRP,‘93,‘96

Fit ππ and πK ELASTIC scattering data



Preliminary Update: J. Nebrera and JRP ‘09

The Inverse Amplitude Method: Dispersive Derivation: THE REAL THING

- We have just seen that, for physical s

$$\mathrm{Im}\frac{1}{t} = -\sigma \quad \text{and} \quad \mathrm{Im}\, t_4 = \sigma\, t_2^{\,2}$$

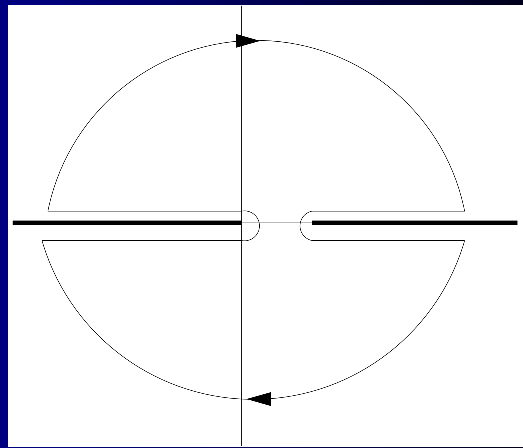
- Define $G \equiv \frac{t_2^{\,2}}{t}$,

$$\mathrm{Im}\, t_4 = \sigma\, t_2^{\,2} = -\mathrm{Im}\, G$$

- Write dispersion relations for G and t_4

$$t_{IJ}^{(4)} = b_0 + b_1 s + b_2 s^2 + \frac{s^3}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}\, t_{IJ}^{(4)}(s')ds'}{s'^3(s'-s-i\varepsilon)} + LC(t_{IJ}^{(4)}).$$

$$G(s) = G_0 + G_1 s + G_2 s^2 + \frac{s^3}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}\, G(s')ds'}{s'^3(s'-s-i\varepsilon)} + LC(G) + PC,$$



Subtraction Constants from ChPT expansion OK since s=0 $G(0)=t_2(0)-t_4(0)$

PHYSICAL cut EXACTLY Opposite to each other

Up to NLO ChPT Opposite to each other

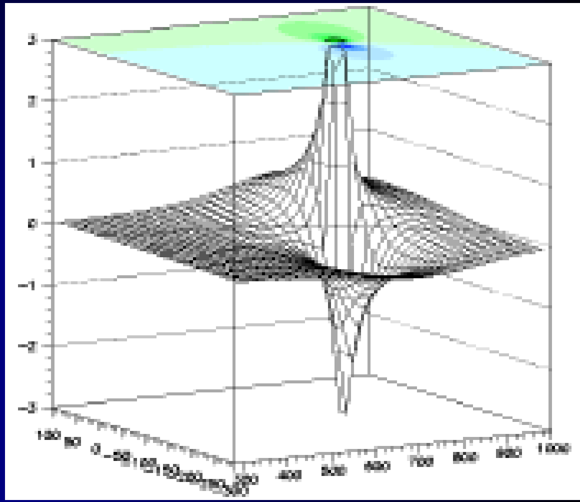
PC is $O(p^6)$ and we neglect it or use ChPT

All together…we find AGAIN

IAM

$$t \approx \frac{t_2^2}{t_2 - t_4}$$

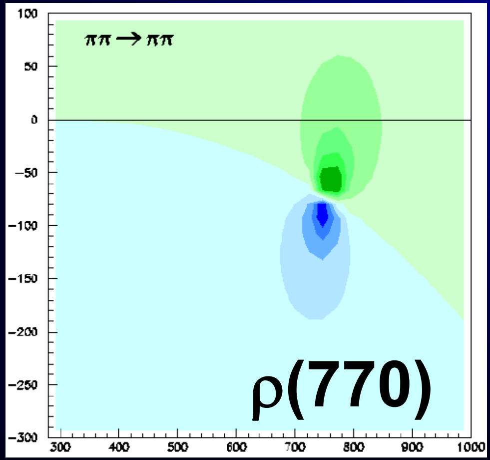
Truong '89, Truong,Dobado,Herrero,'90, Dobado JRP,'93,'96

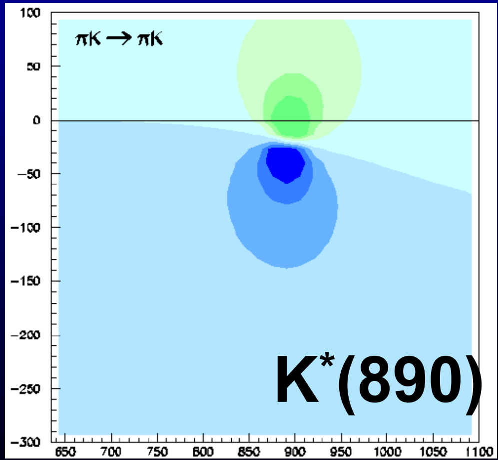


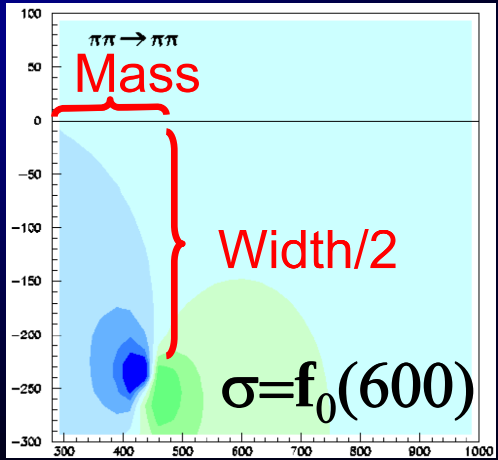
- EXTREMELY SIMPLE
- Unitarity + Chiral Low energy expansion
- Systematic extension to higher orders
- Originally obtained from dispersion relation
  This allows us to go to the complex plane.
- Dynamically Generates Poles of Resonances:
  $f_0(600)$ or "σ", ρ(770), κ(800), K*(892),

Dobado, Pelaez '96

$f_0(600)$ pole: 440-i245 MeV







IAM, one channel:

- Simultaneously resonances and low energy meson-meson scattering with parameters compatible with ChPT

Large N_c expansion

We cannot obtain the L_i from QCD, BUT their
1/Nc expansion, is known and Model Independent

($\times 10^{-3}$)	ChPT ($\mu=M_\rho$)	IAM fits	Large N_c SCALING
$2L_1 - L_2$	-0.6 ± 0.6	0.0 ± 0.2	$O(1)$
L_2	1.4 ± 0.3	1.2 ± 0.1	$O(N_c)$
L_3	-3.5 ± 1.1	-2.79 ± 0.14	$O(N_c)$
L_4	-0.3 ± 0.5	-0.36 ± 0.17	$O(1)$
L_5	1.4 ± 0.5	1.4 ± 0.5	$O(N_c)$
L_6	-0.2 ± 0.3	0.07 ± 0.08	$O(1)$
L_7	-0.4 ± 0.2	-0.44 ± 0.15	$O(1)$
L_8	0.9 ± 0.3	0.8 ± 0.2	$O(N_c)$

The qqbar meson masses $M=O(1)$ and their decay constants $f=O(\sqrt{N_c})$

Pions, kaons and etas states:

$$M \approx O(1), \Gamma \approx O(1/N_c)$$

Our IAM ChPT amplitudes **do not** have any other parameter hiding Nc dependence
like cutoffs, subtractions, etc...

We can thus study the Nc scaling of the resonances

LIGHT VECTOR MESONS

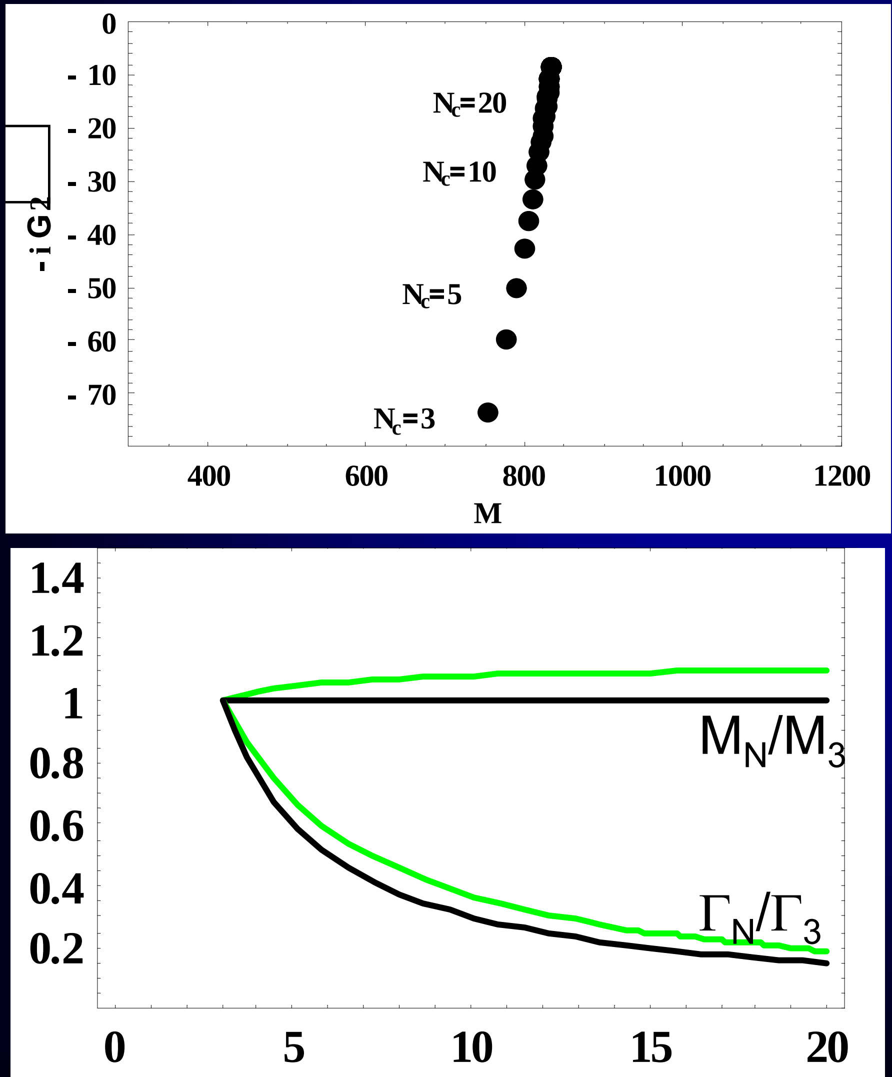
qqbar states:

$M \approx O(1), \Gamma \approx O(1/N_c)$

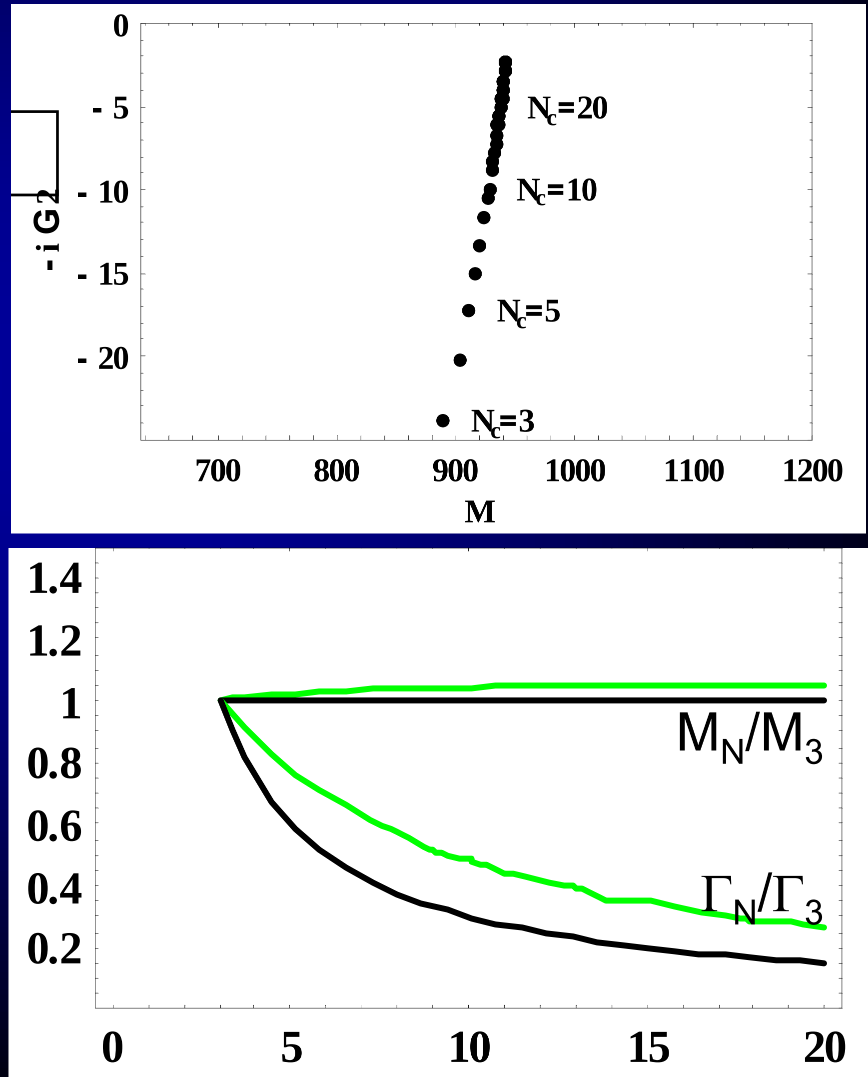
The IAM generates the expected N_c scaling of established qq states

JRP, Phys.Rev.Lett. 92:102001,2004

The ρ(770)



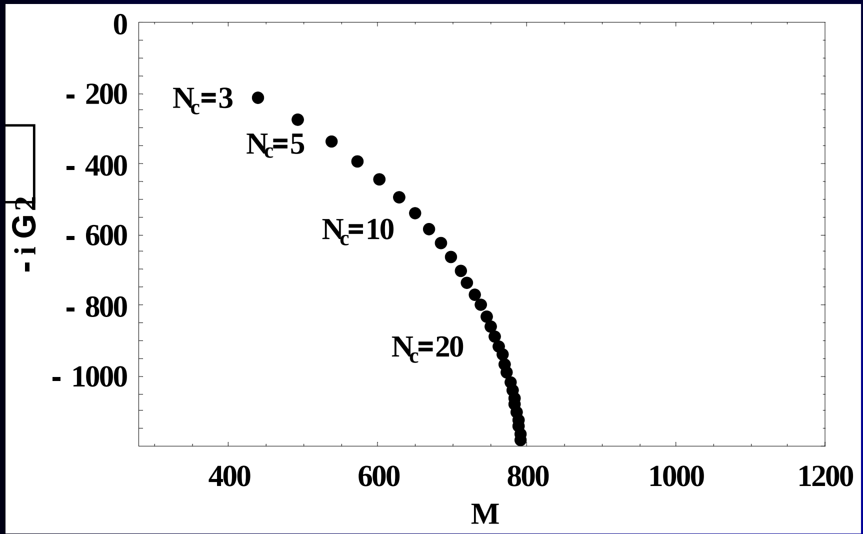
The K*(892)

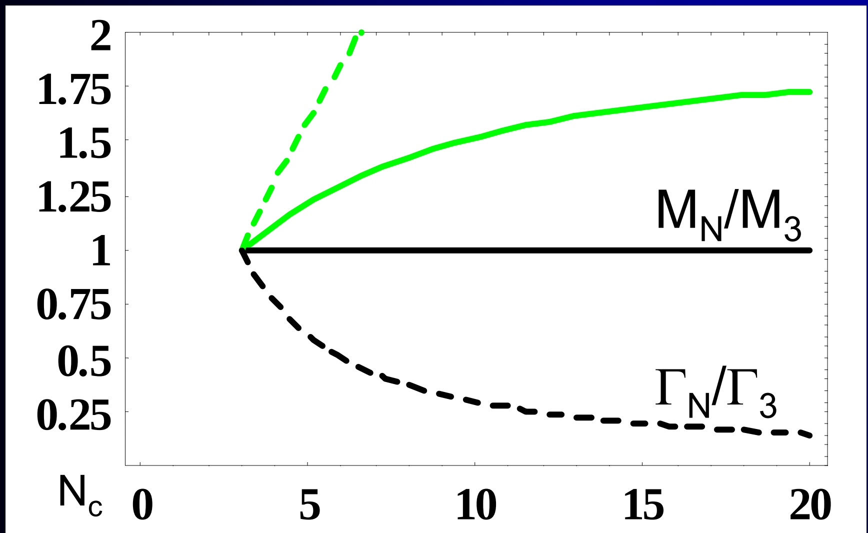


What about scalars ?

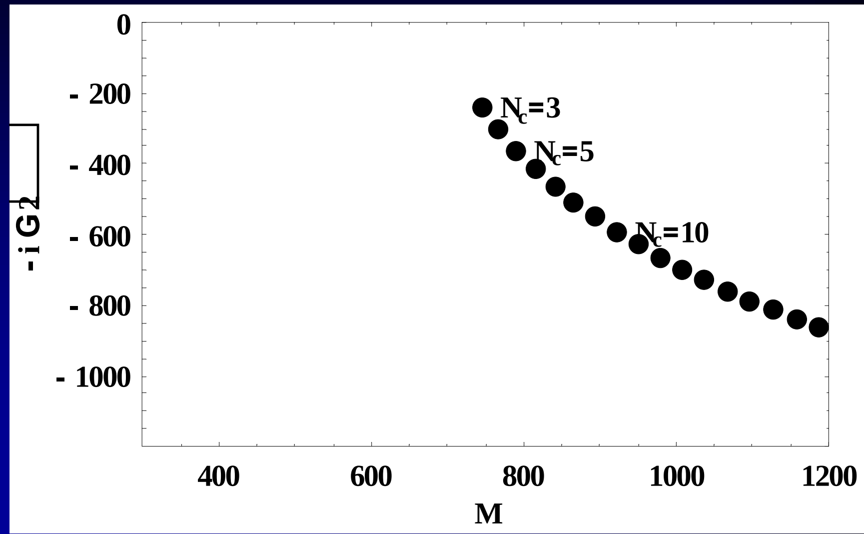
JRP, Phys.Rev.Lett. 92:102001,2004

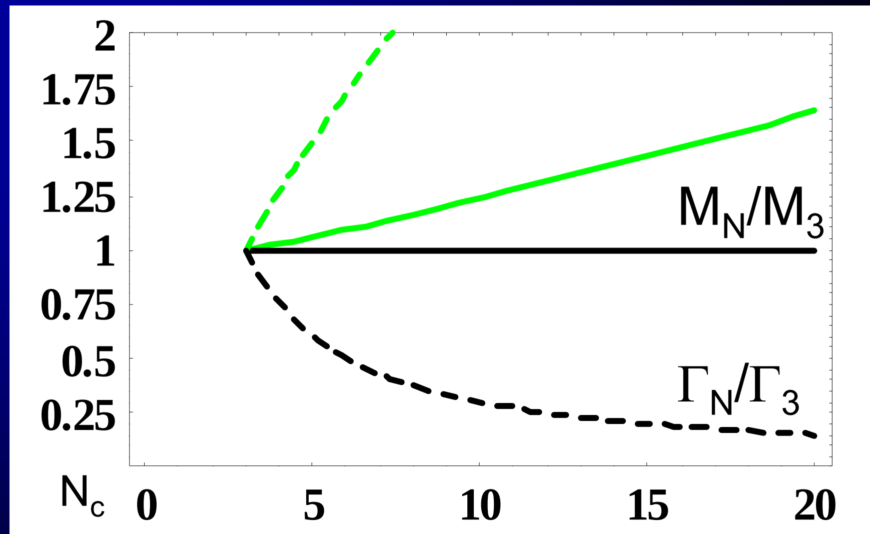
The σ (μ=770MeV)





The κ (μ=500MeV)





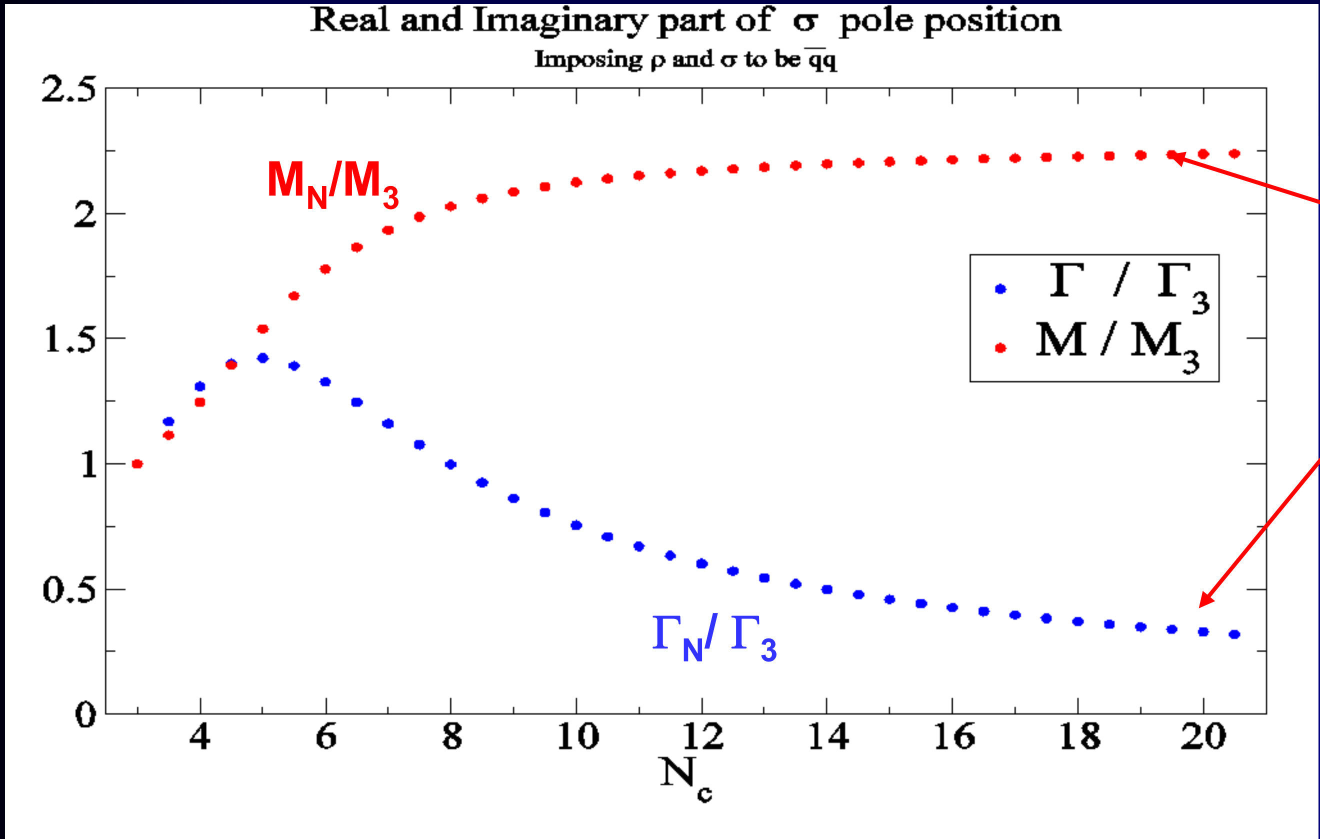
Similar results follow for the $f_0(980)$ and $a_0(980)$

Complicated by the presence of THRESHOLDS and except in a corner of parameter space for the $a_0(980)$

The sigma:

G. Ríos and JRP, Phys.Rev.Lett.97:242002,2006.

Large Nc behavior of UNITARIZED $\pi\pi\rightarrow\pi\pi$ TWO LOOP ChPT



quark-antiquark mixing may emerge at larger Nc at M≈1 GeV

The $f_0(600)$ still does NOT behave DOMINANTLY as quark-antiquark

BUT, from Nc>8 or 10, the $f_0(600)$ we might be seeing a quark-antiquark **subdominant** component whose large Nc mass is ≥ 1 GeV

Motivation for Chiral extrapolation

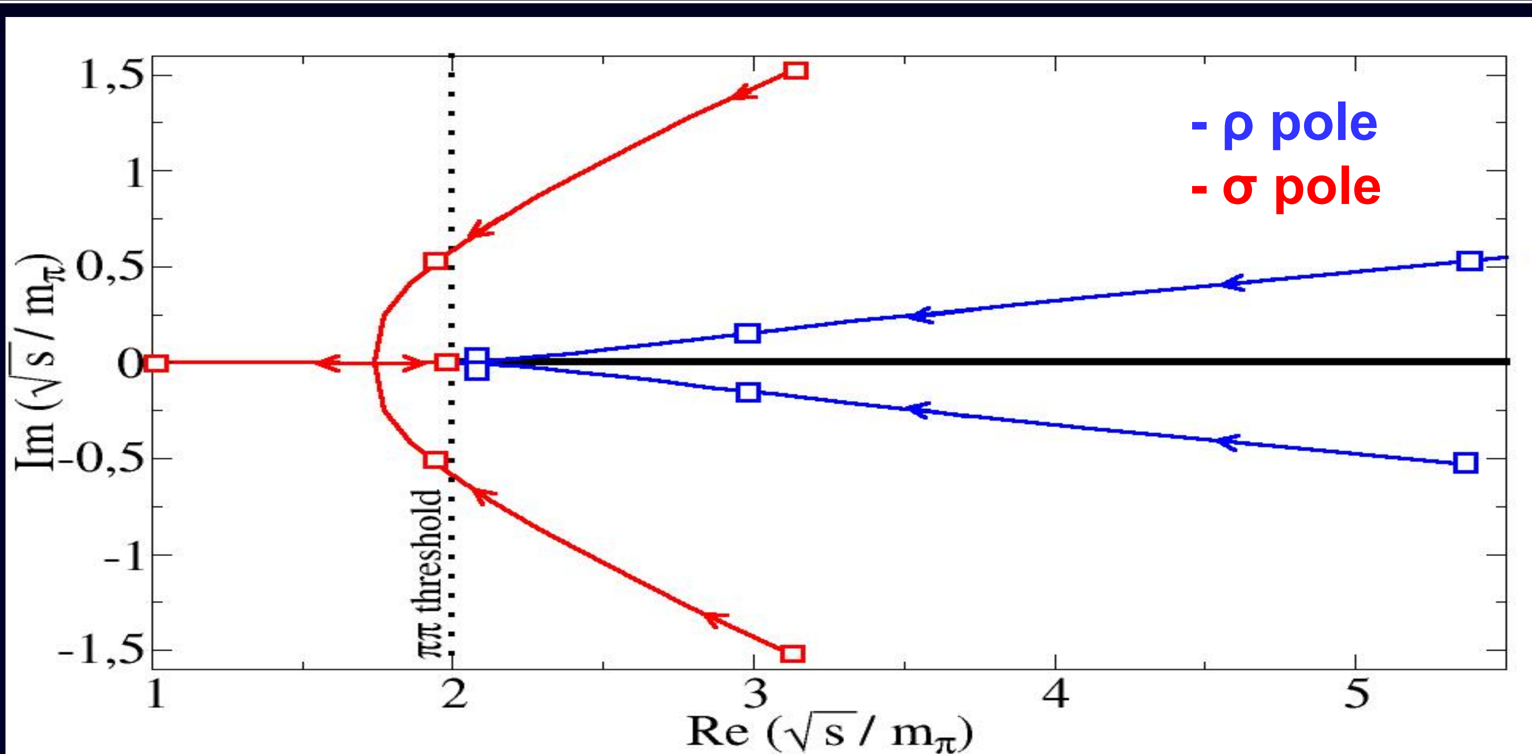
- The LATTICE provides rigorous and systematic QCD results in terms of quarks and gluons with growing interest in scattering and the scalar sector.

  Caveat: small, realistic, quark masses are hard to implement.

- Anthropic considerations...

ChPT provides the correct QCD dependence of quark masses as an expansion...

We can study the scalars in Unitarized ChPT for
larger quark masses
(chiral extrapolation)
and provide a reference for lattice studies



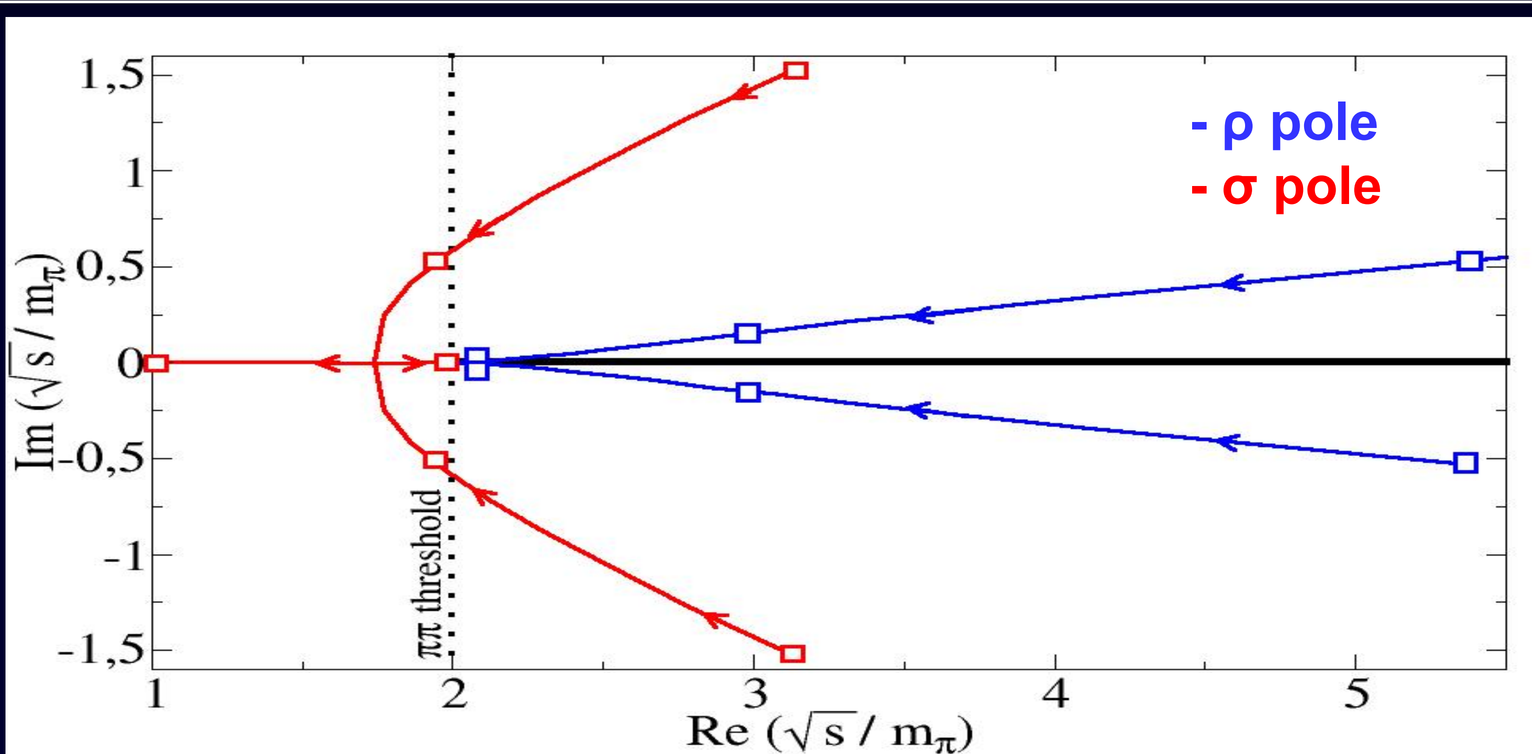
To follow the position relative to threshold: normalize to m_π units

The rho: Conjugate poles reach the real axis AT THRESHOLD:

- one pole in the 1st sheet (bound state).

- another in the 2nd sheet in almost the same position

Pole movements with increasing m_π



<u>The sigma:</u> 1) Conjugate poles reach the real axis BELOW threshold:

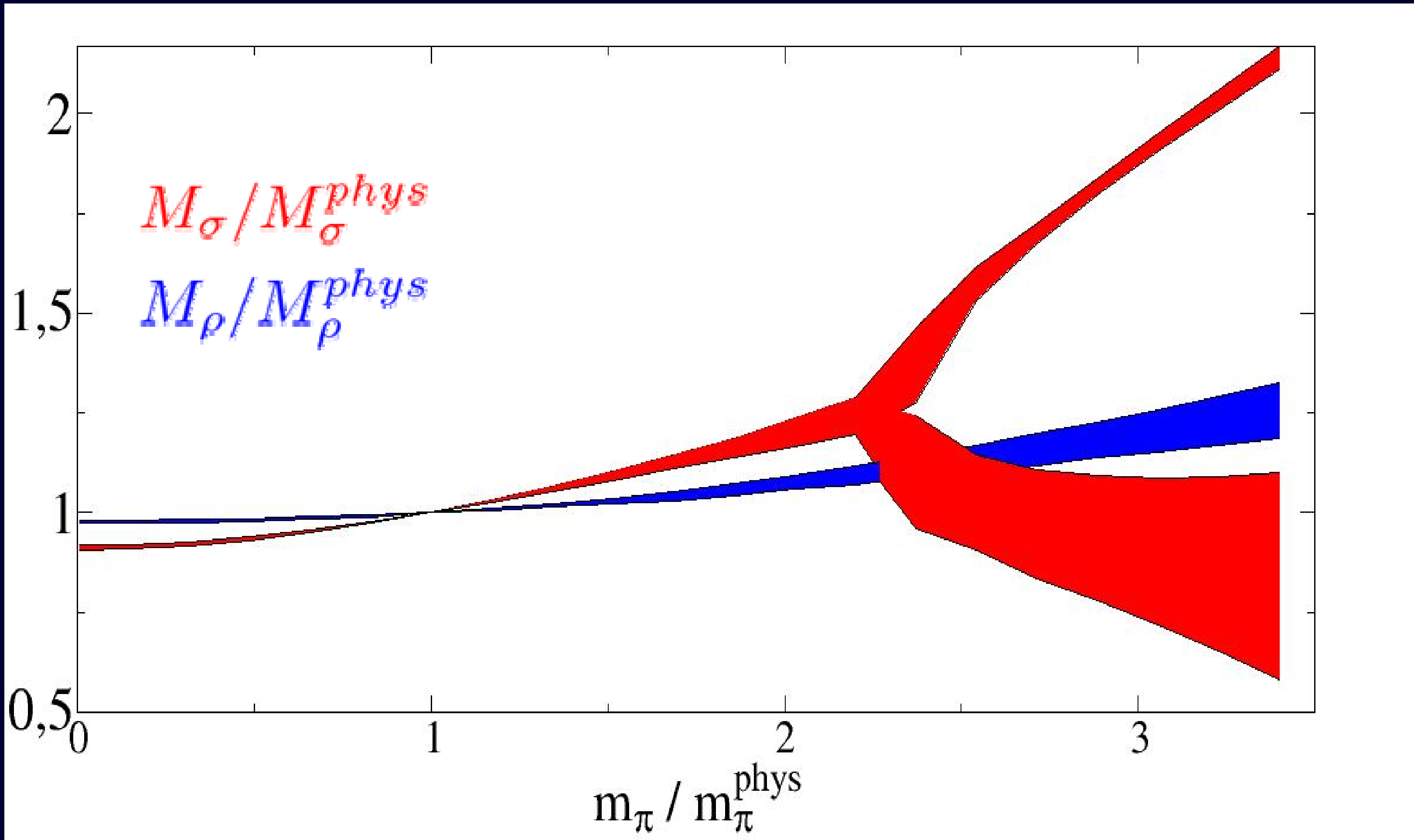
2) TWO real POLES on the 2nd sheet: “Splitting” typical of scalars.

3) One moves towards threshold until it jumps to the 1st sheet.
The other remains on the 2nd sheet in ASYMMETRIC position

If very asymmetric: sizable “molecular” component

Morgan, Pennington. PRD**48** (1993) 1185

Resonane mass m_π dependence



There is a “non-analyticity” in the sigma m_π dependence.

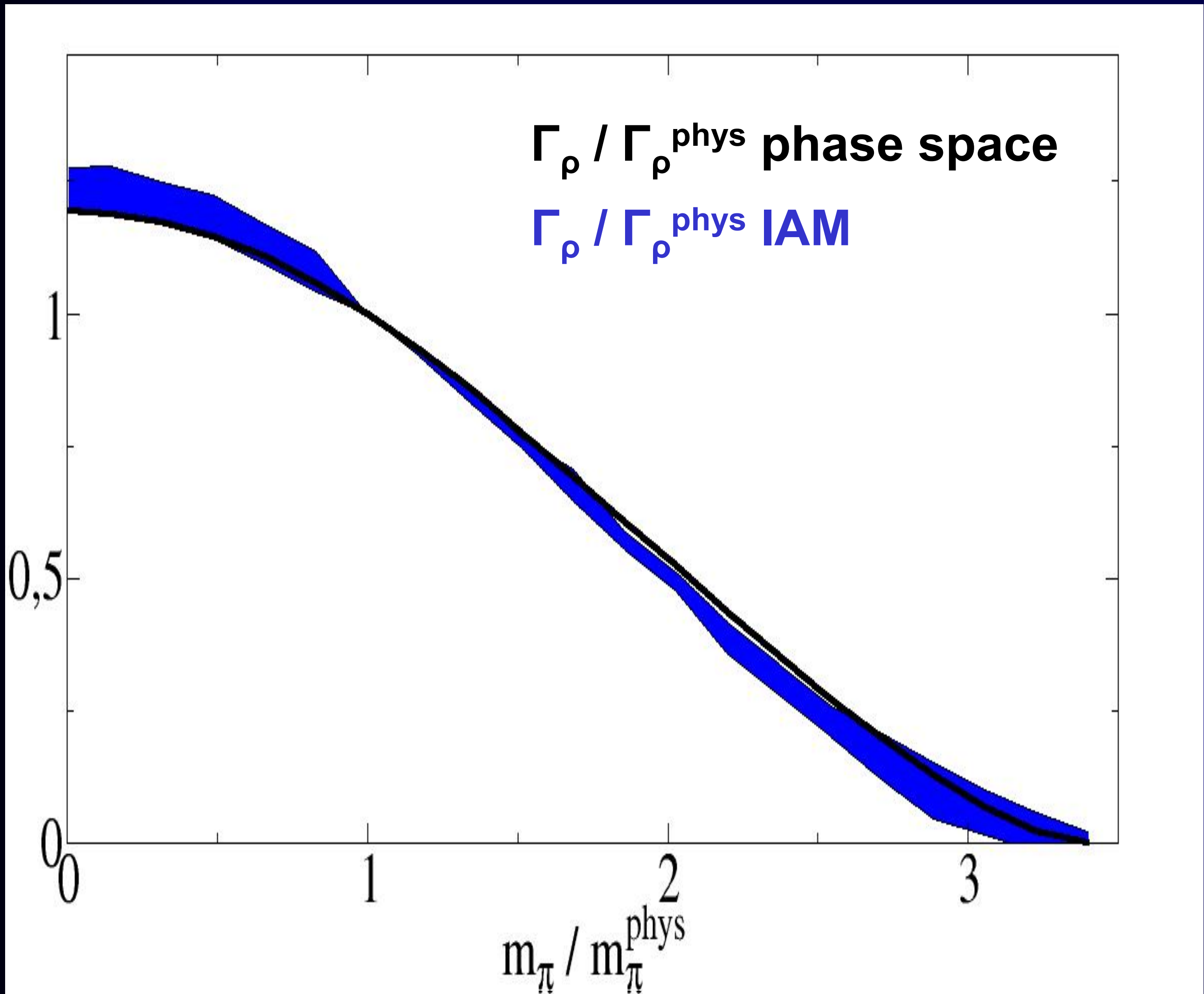
The rho mass grows slower than sigma

For a narrow vector particle (like the rho) the decay width is given by

$$\Gamma_\rho = \frac{g^2 |\vec{p}|^3}{6\pi M_\rho^2} \quad \begin{cases} \dfrac{|\vec{p}|^3}{M_\rho^2} & \text{Phase space} \\ g^2 & \text{Coupling to pions} \end{cases}$$

We can calculate the width variation due to phase space reduction and compare with our results. The difference gives the dependence of the coupling constant on the pion mass

Rho width m_π dependence vs. phase space

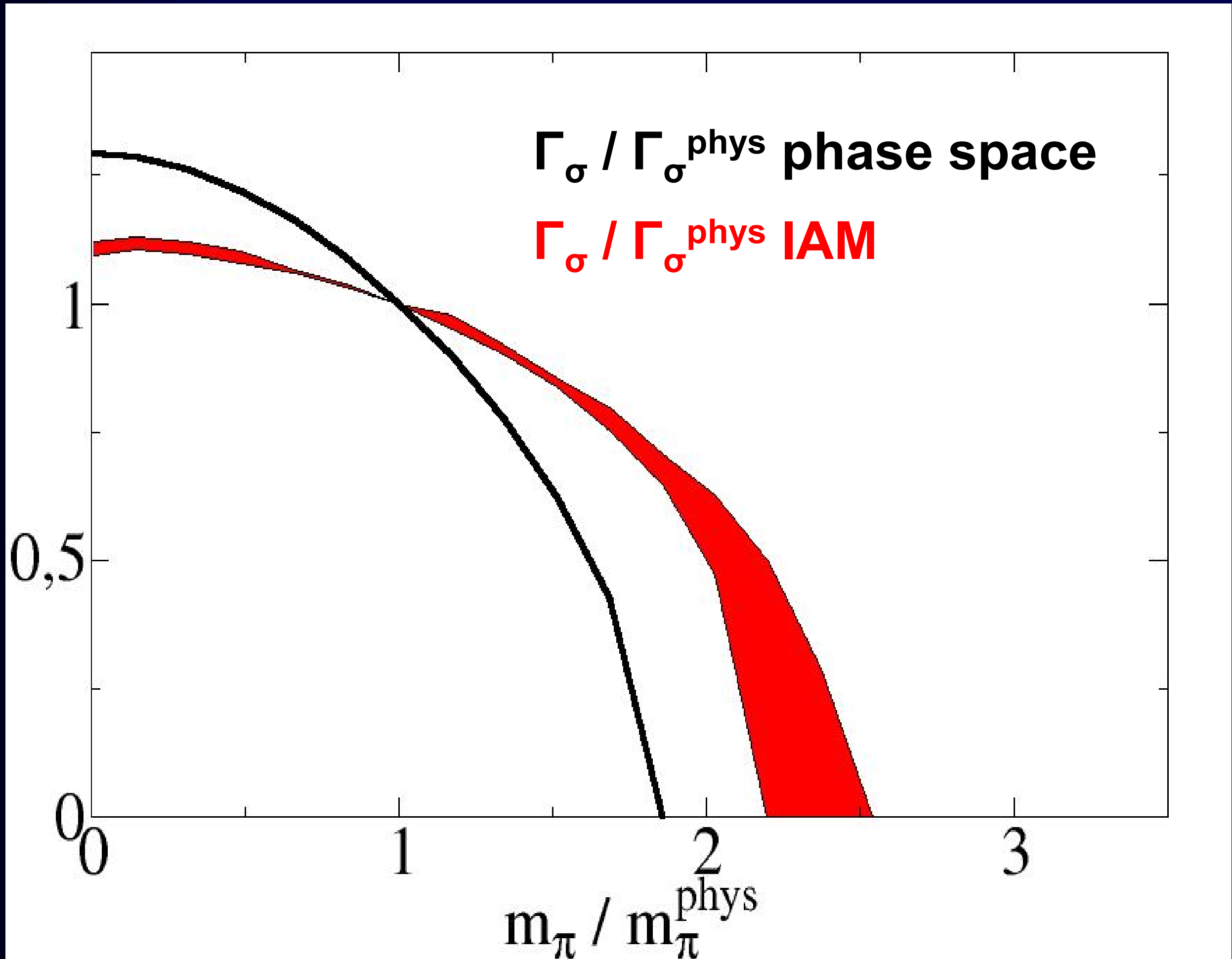


Width behavior explained by phase space

$\rho \rightarrow \pi\pi$ coupling almost independent of m_π

(assumption in some lattice calculations)

It **does not follow** the phase space decrease of a Breit-Wigner:



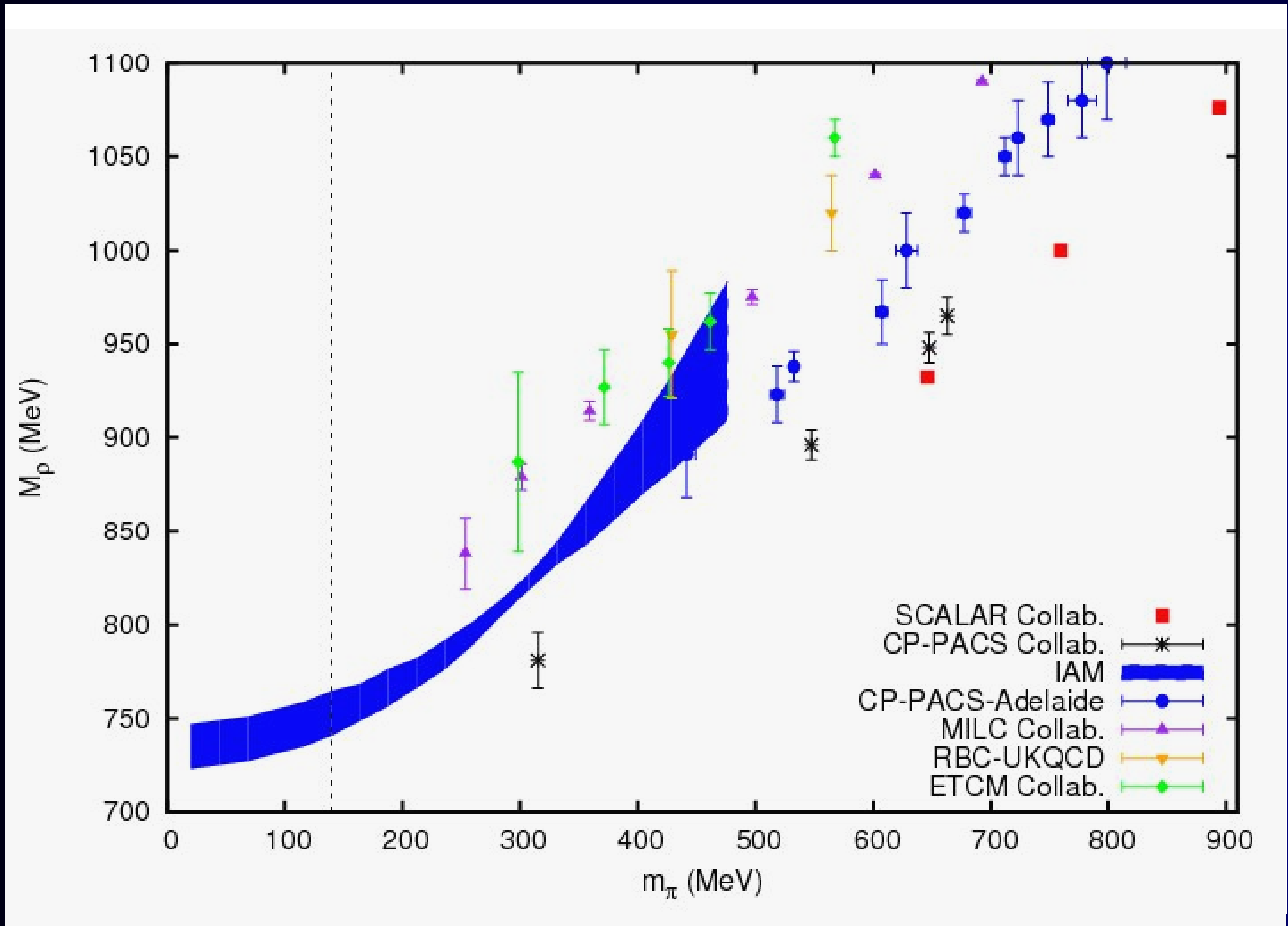
$$\Gamma_\sigma = \frac{g^2 |\vec{p}|}{8\pi M_\sigma}$$

Very bad approximation for a wide resonance as the sigma

g dependence on m_π

The **dynamics** of the sigma decay depends strongly on the pion(quark) mass (Recall that some pion-pion vertices in ChPT depend on the pion mass).

Comparison with lattice results for the rho



CAUTION!!!

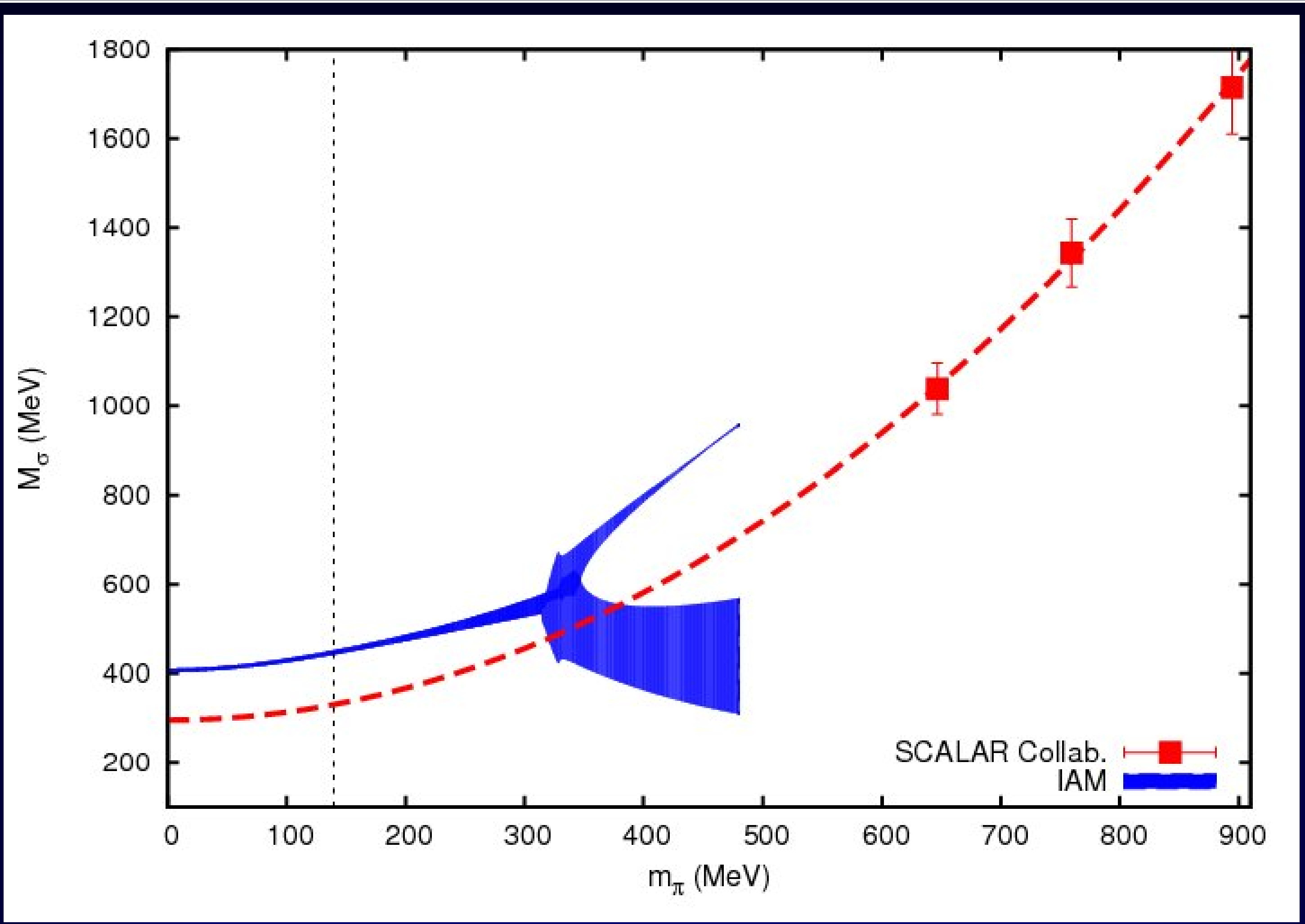
We give POLE MASS in complex plane

Lattice caveats:

Improved actions,
Lattice spacing...
Finite volume...
WIDTHLESS rho

The best would be to use ChPT on the lattice....future work

Comparison with lattice results for the sigma



<u>AGAIN CAUTION!!!</u>

We give POLE MASS
in complex plane + usual
lattice caveats

IMPORTANT REMARK
Extrapolations should take care
of known scalar mass "splitting"
non-analyticity

QCD LINK: Scalars in Unitarized Chiral Perturbation Theory

IAM, one channel:

- Simultaneously resonances and low energy meson-meson scattering with parameters compatible with ChPT

N_c behavior of light resonances

- quark-antiquark remarkably good for vectors
- SCALARS predominantly NOT quark-antiquark states

  SUBDOMINANT quark-antiquark component around 1.1 GeV.
  (Suggests mixing with heavier ordinary scalar nonet)

Quark mass dependence: lattice connection

- Good agreement for ρ. Coupling independence.
- Two mass branches for sigma

2-body unitarization methods.

From more rigorous to rougher ↑ BUT ↓ From complicated (unfeasible sometimes) to simpler (but very successful)

Examples:

IAM single channel
IAM coupled channels

Chiral unitary approach $O(p^4)$
Chiral unitary approach $O(p^2)$

Of course, there are other variations,

Unitarity and the Inverse Amplitude Method: Multiple channels–one loop

Oller, Oset, JRP, PRL80(1998)3452, PRD59,(1999)074001

Partial wave unitarity

(On the real axis above **all thresholds**)

$$\mathrm{Im}\, T = T\, \Sigma\, T^{*}$$

$$\mathrm{Im}\, T^{-1} = -\Sigma$$

$$T \approx \left(\mathrm{Re}\, T^{-1} - i\,\Sigma\right)^{-1}$$

exactly unitary !!

$$T = T_2 (T_2 - \mathrm{Re}\, T_4 - i\, T_2 \Sigma T_2)^{-1} T_2$$

To the DATA !!

ChPT = series in p^2

$$T = T_2 + T_4 \ldots$$

perturbative unitarity

$$\mathrm{Im}\, T_4 = T_2\, \Sigma\, T_2$$

provides ....

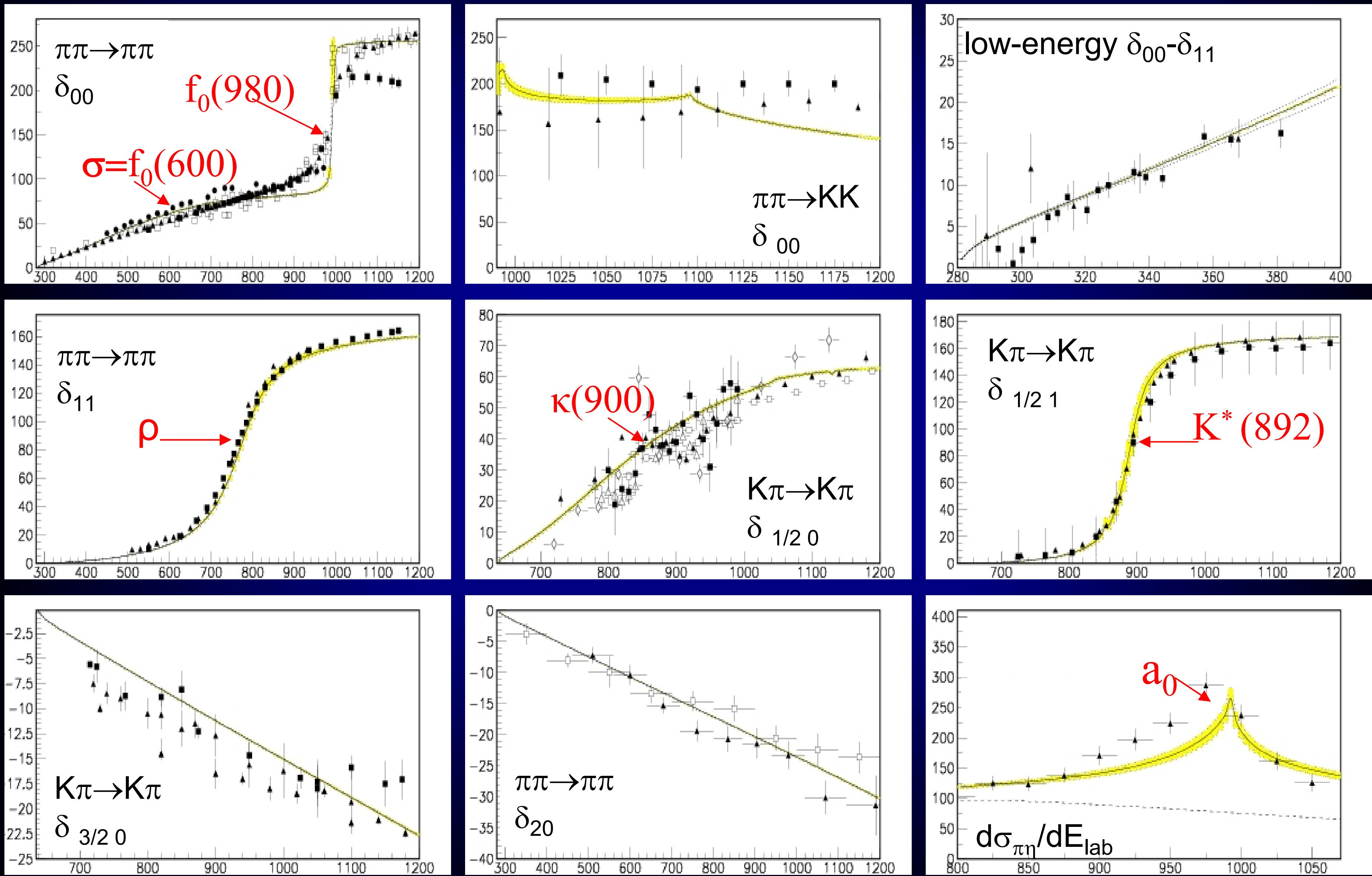
$$\mathrm{Re}\, T^{-1} \approx T_2^{-1} (T_2 - \mathrm{Re}\, T_4 + \ldots) T_2^{-1}$$

Coupled channel IAM

$$T \approx T_2 (T_2 - T_4)^{-1} T_2$$

One-loop ChPT IAM fit to meson-meson scattering
(+3% syst.)

Gómez-Nicola, JRP, Phys. Rev. D65:054009, (2002)

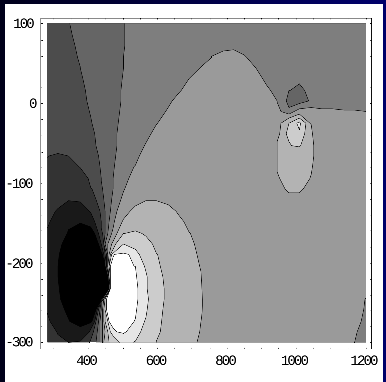


Coupled channel IAM

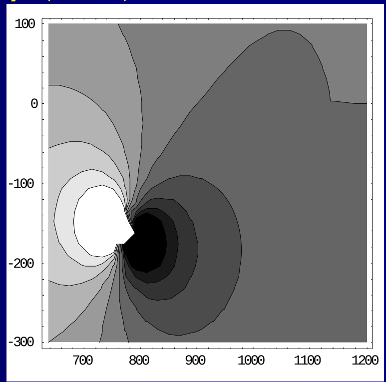
Oller, Oset, JRP, PRL80(1998)3452, PRD59,(1999)074001
A. Gomez Nicola and J.R.P.. PRD65:054009, (2002)

- With the full one-loop SU(3) unitarized ChPT, we GENERATE, the following resonances, not present in the ChPT Lagrangian, as poles in the second Riemann sheet

f0(600) and $f_0(980)$



ρ(770)

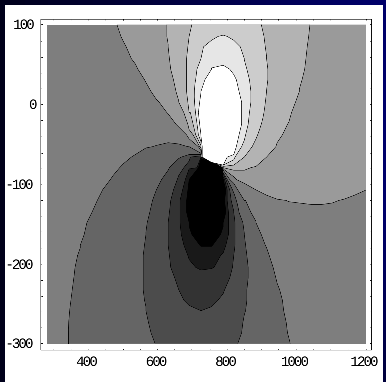


without a priori assumptions on on their existence or nature

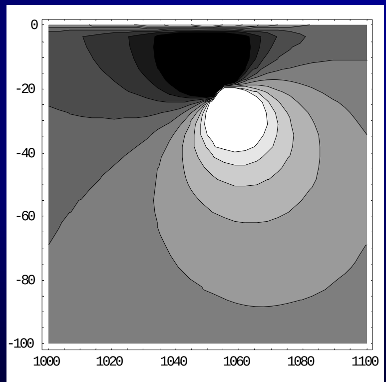
J.R.P, hep-ph/0301049. AIP Conf.Proc.660:102-115,2003

Brief review: Mod.Phys.Lett.A19:2879-2894,2004

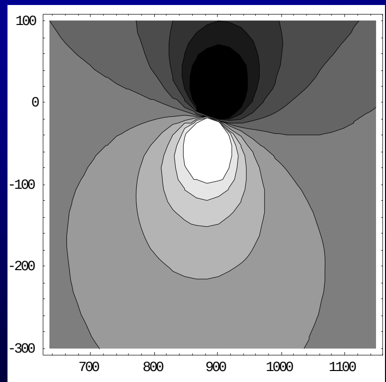
κ



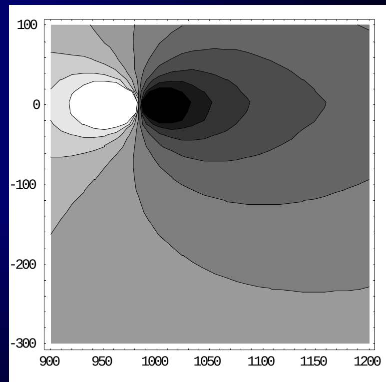
a_0



K*



ϕ (octet)



Gómez-Nicola, JRP, Phys. Rev. D65:054009, (2002)

- MINUIT fit :
  - Incompatible sets of Data.
    Customarily add systematic error: 1%, 3%, 5%

    } Identical curves but variation in parameters

  - Final error: MINUIT error + Systematic error

	ChPT($\mu=M_\rho$)	IAM fit (+3%)	IAM fits
L_1	0.4± 0.3	0.561± 0.008	0.6± 0.1
L_2	1.35± 0.3	1.211± 0.001	1.2± 0.1
L_3	-3.5±1.1	-2.79± 0.02	-2.79± 0.14
L_4	-0.3± 0.5	-0.36± 0.02	-0.36± 0.17
L_5	1.4± 0.5	1.39± 0.02	1.4± 0.5
L_6	-0.2± 0.3	0.07± 0.03	0.07± 0.08
L_7	-0.4± 0.2	-0.444± 0.03	-0.44± 0.15
L_8	0.9± 0.3	0.78± 0.02	0.8± 0.2

Fairly good agreement with existing LECS

Simultaneous description of low energy and resonances

Fully renormalized and with parameters compatible with ChPT.

Caveats (At Bastian Kubis' request)

- Strong correlations for LECS. (Also in ChPT)
- Other acceptable solutions with different LECS
  We can impose constraints on the LECS when fitting
- No existing dispersive derivation for coupled channels (yet?)
- Incorrect left cut analytic structure.

  Old problem in coupled channel approach (Faddeev, Bjorken...)

  Fortunately: Numerically small
  Can be made correct order by order, but very complicated
- Full one-loop calculation needed. Complicated functions

However coupled channels essential for a0(980) and f0(980).
But for FSI in decays the analytic structure is different
and the relevant one is the right or unitarity cut

⇒ SIMPLIFY

UNITARIZATION !!

2-body unitarization methods.

From
more rigorous
to rougher

BUT

From
complicated (unfeasible sometimes)
to simpler (but very successful)

Examples:

IAM single channel
IAM coupled channels

Chiral unitary approach $O(p^4)$
Chiral unitary approach $O(p^2)$

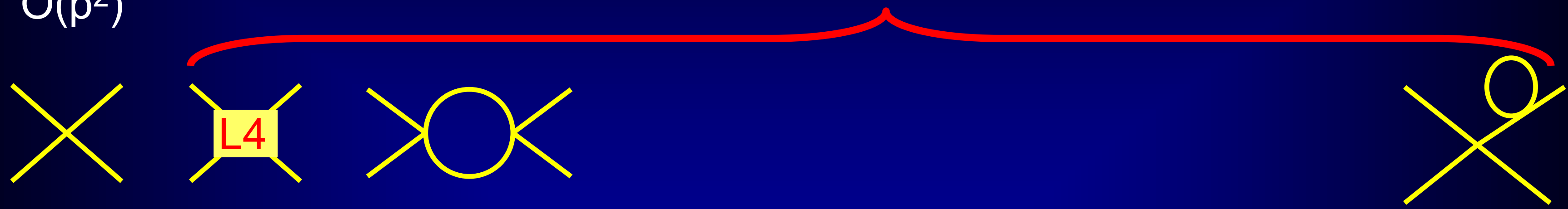
Of course, there are other variations,

ChPT meson -meson one-loop Calculation

- If we do not care about left cuts:

$t_2(s,t,u)$
$O(p^2)$

$t_4(s,t,u)=O(p^4)$



But now the divergences are not fully absorbed in the LECS

Supurious parameter (regulator) dependence

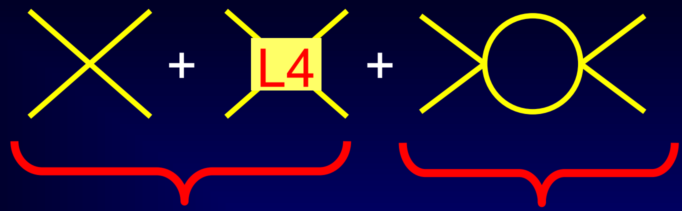
- If you can live with that, then... why not....

forget pure tadpoles

but keeping those in mass and f renormalization

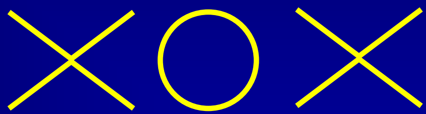
Simply use physical masses and constants.

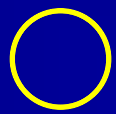
O(p^4) Chiral Unitary Approach

t ~  $= t_2 + t_4^{tree} + t_2\, G\, t_2$

$t_2 + t_4^{tree}$

Shown to factorize
(up to tadpoles neglected or absorbed in mass and decay constants)

 $= t_2\, G\, t_2$

with G=  $$= i \int \frac{dq}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\varepsilon} \frac{1}{(s-q^2) - M^2 + i\varepsilon}$$

so that Im G=σ and Re t= $t_2 + t_4^{tree} + t_2\, \mathrm{Re}\, G\, t_2 + ..$.

Inverting... $$\mathrm{Re}\frac{1}{t} = \frac{1}{t_2}\left(1 - \frac{\mathrm{Re}\, t_4}{t_2} + \ldots\right)$$ and recalling the exactly unitary amplitude

$$t = \frac{1}{\mathrm{Re}\, 1/t - i\sigma} = \frac{t_2^{\;2}}{t_2 - t_4^{\;tree} - t_2 G t_2}$$

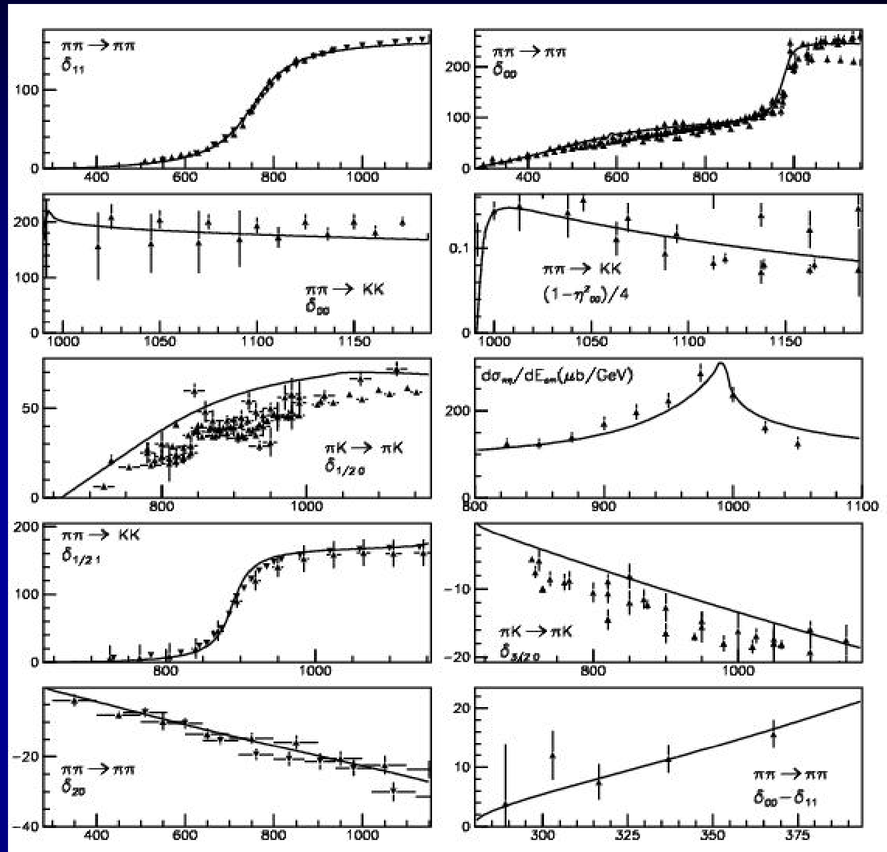
O(p⁴) Chiral Unitary Approach

Oller, Oset, Peláez
PRL80(1988)3452
PRD59,(1999)074001

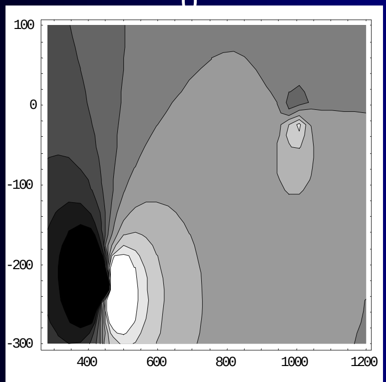
REMARKABLE RESULTS!!

All Resonances: σ,κ,ρ , a_0, f_0, K*, Φ_8

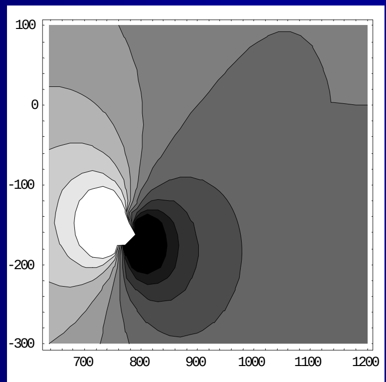
with their associated poles!!



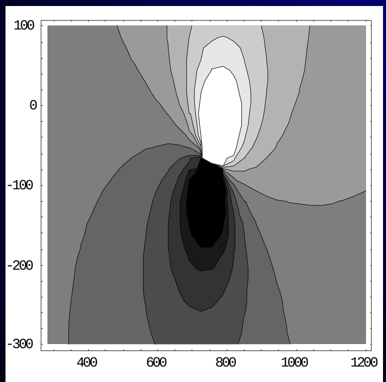
σ and f_0



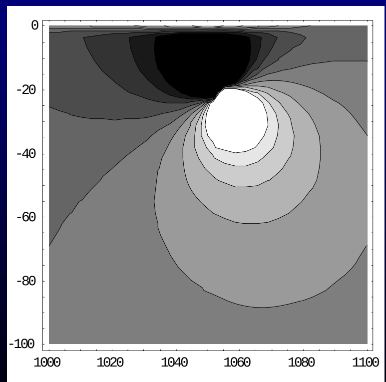
κ



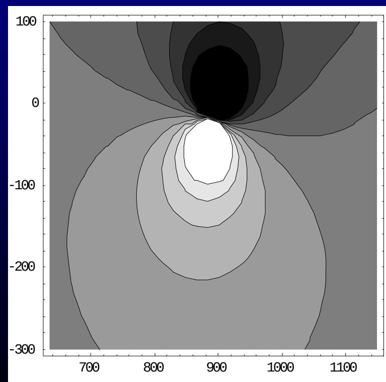
ρ



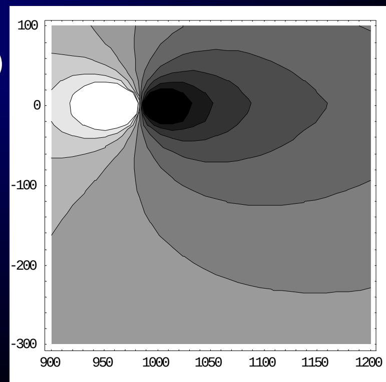
a_0



K*



Φ(octet)



$O(p^4)$ Chiral unitary approach: Pros and Cons.

Cons. (Again at B.Kubis request)

- No left cuts
- Spurious parameters. In principle, one per channel
  Variations in the literature: Cutoff, dimensional regularization scale, subtraction constants ... choose favorite
  Luckily, with natural choices of regulator, it can be reduced to one parameter

Pros.

- Simple. Just tree level calculations but for G function
- Satisfies coupled channel unitarity exactly.
- Reproduces LO ChPT and the numerically largest part of NLO
- Generates both scalar and vector resonances with their widths
- Surprisingly works with LECS numerically similar to those in ChPT
  Because uncertainties in LECS large and what we dropped is numerically small

2-body unitarization methods.

From more rigorous to rougher ↑ BUT ↓ From complicated (unfeasible sometimes) to simpler (but very successful)

Examples:

IAM single channel

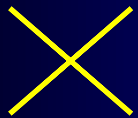
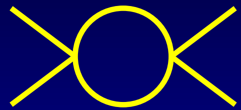
IAM coupled channels

Chiral unitary approach $O(p^4)$

Chiral unitary approach $O(p^2)$

Of course, there are other variations,

- We have dropped so many terms.... who cares one more!

t ~  +  $= t_2 + t_2\, G\, t_2$

$$t = \frac{t_2}{1 - G t_2}$$

Single channel

$$T = (1 - T_2 G)^{-1} T_2$$

Coupled channels

Now without Counterterms!

Really?

Actually, NO. There is still the “spurious” regulator in the G integral.

It can play the role of a combination of parameters, and mimic the energy dependence of the dropped terms if it si sufficiently soft

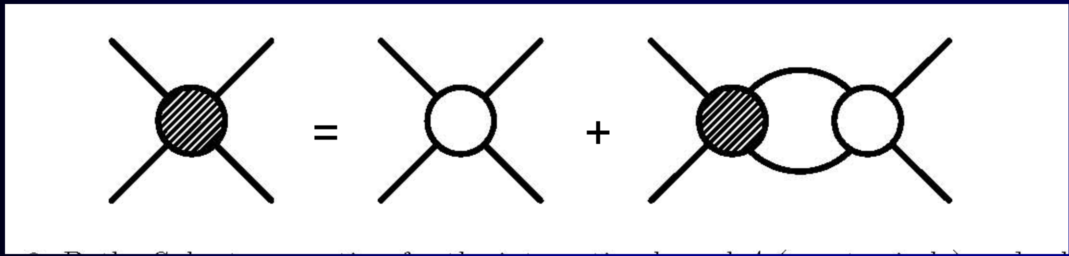
But it is not assured that you can play this game with the same natural regulator in all channels simultaneously.

Surprisingly, this enough to generate all the light scalars!! Oller, Oset 1996

You could generate the vectors too, but need another NON-NATURAL regulator

The Bethe Salpeter Interpretation (Oller, Oset, JRP, Nieves, RUiz Arriola...)

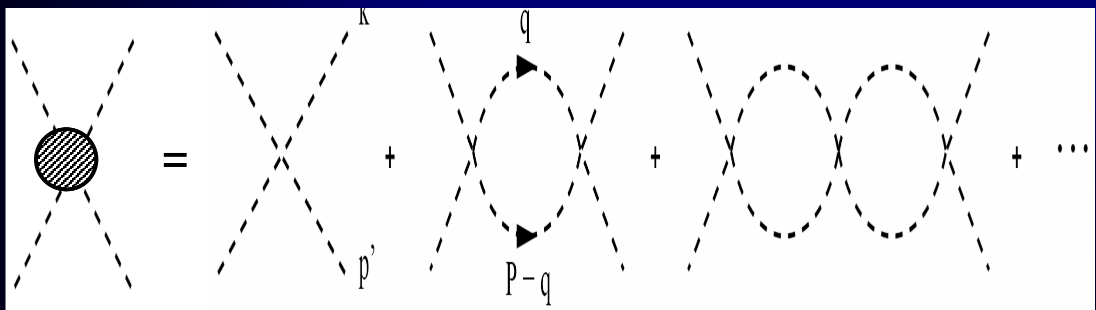
The same result can be obtained from a BS equation $T = V + \int V\overline{G}T$



... if we use the T_2 ChPT matrix as the kernel and we use factorization inside the integral, then

$T = T_2 + T_2 G T$ solving iteratively $T = T_2 + T_2 G T_2 + T_2 G T_2 G T_2 + \ldots$

Effectively, one is summing this series of diagrams



Summing the geometric series

$$T = (1 - T_2 G)^{-1} T_2$$

Meson-meson unitarization summary and Variations

Truong,Dobado, Herrero, Pelaez, Oller, Oset, Meissner, Kaiser, Weise, Ramos, Vicente-Vacas, Nieves, Ruiz-Arriola, Lutz,...

IAM: Use of full ChPT series. Fully renormalized. No spurious parameters
Extensions up to two loops. LECS compatible with ChPT

- One channel: Dispersive derivation. Analytic structure correct.
  Only elastic resonances. Scalars: σ, κ. Vectors: ρ, K*.
  CONNECTION WITH QCD
- Coupled channels: NO dispersive derivation. Left cuts messy, but small.
  Light Scalar Nonet: σ, κ, a_0, f_0. Vectors: ρ, K*, octet Φ

Provides further justification/rigour for the next one

Chiral Unitary Approach: No tadpoles no crosses graphs. No left cuts
Spurious regulator dependence.
Bethe-Salpeter interpretation. Also N/D derivation

- $O(p^4)$: "LECS" compatible with ChPT and natural regulator
  Light Scalar Nonet: σ, κ, a_0, f_0. Vectors: ρ, K*, octet Φ
- $O(p^2)$: No "LECS". With Natural regulator only
  light Scalar Nonet: σ, κ, a_0, f_0.

$O(p^2)$+ explicit high resonances (vectors, axials...) by hand
Also N/D Methods, etc...

(all of the above, from $O(p^4)$ on:) Used in decays !!!