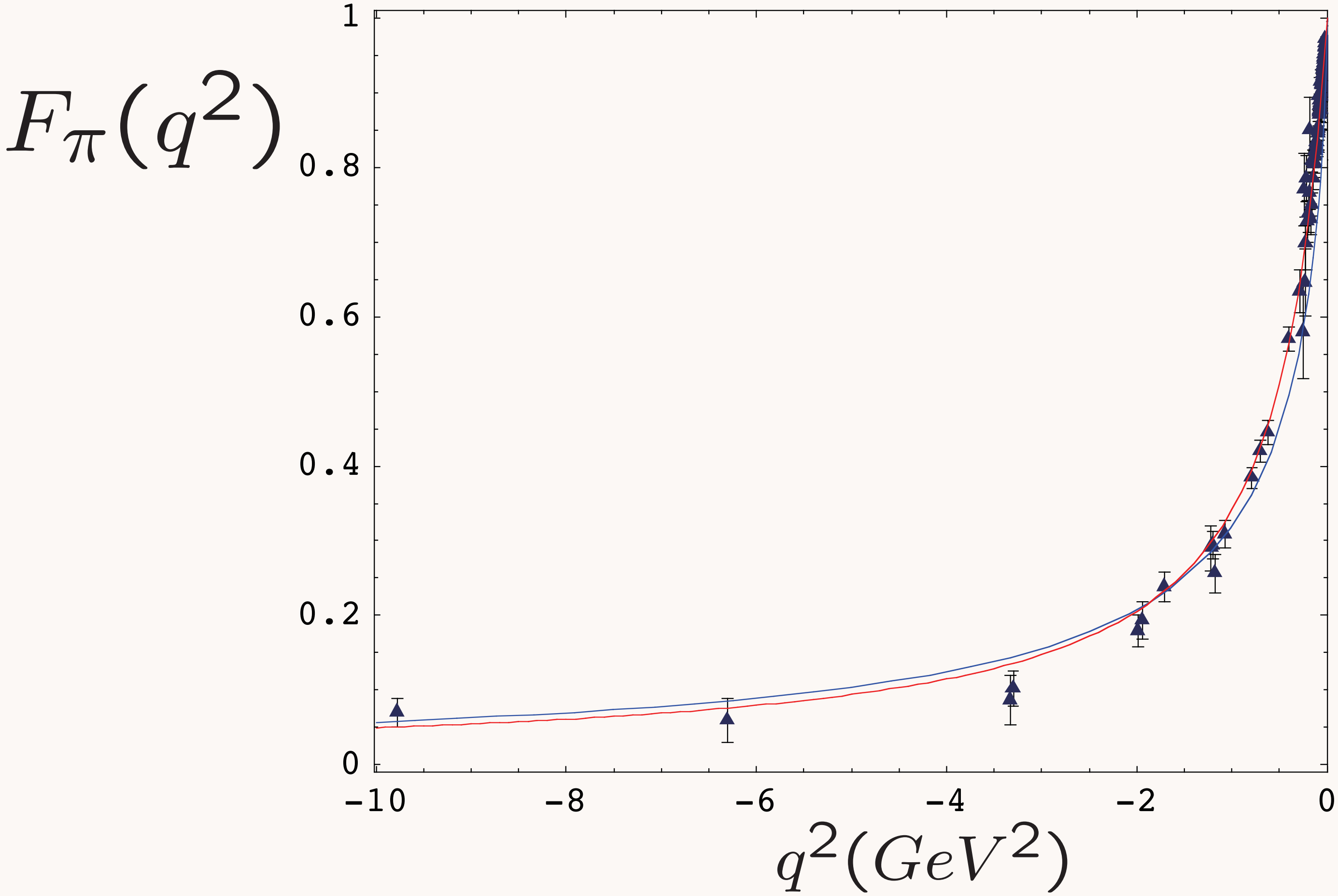
Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

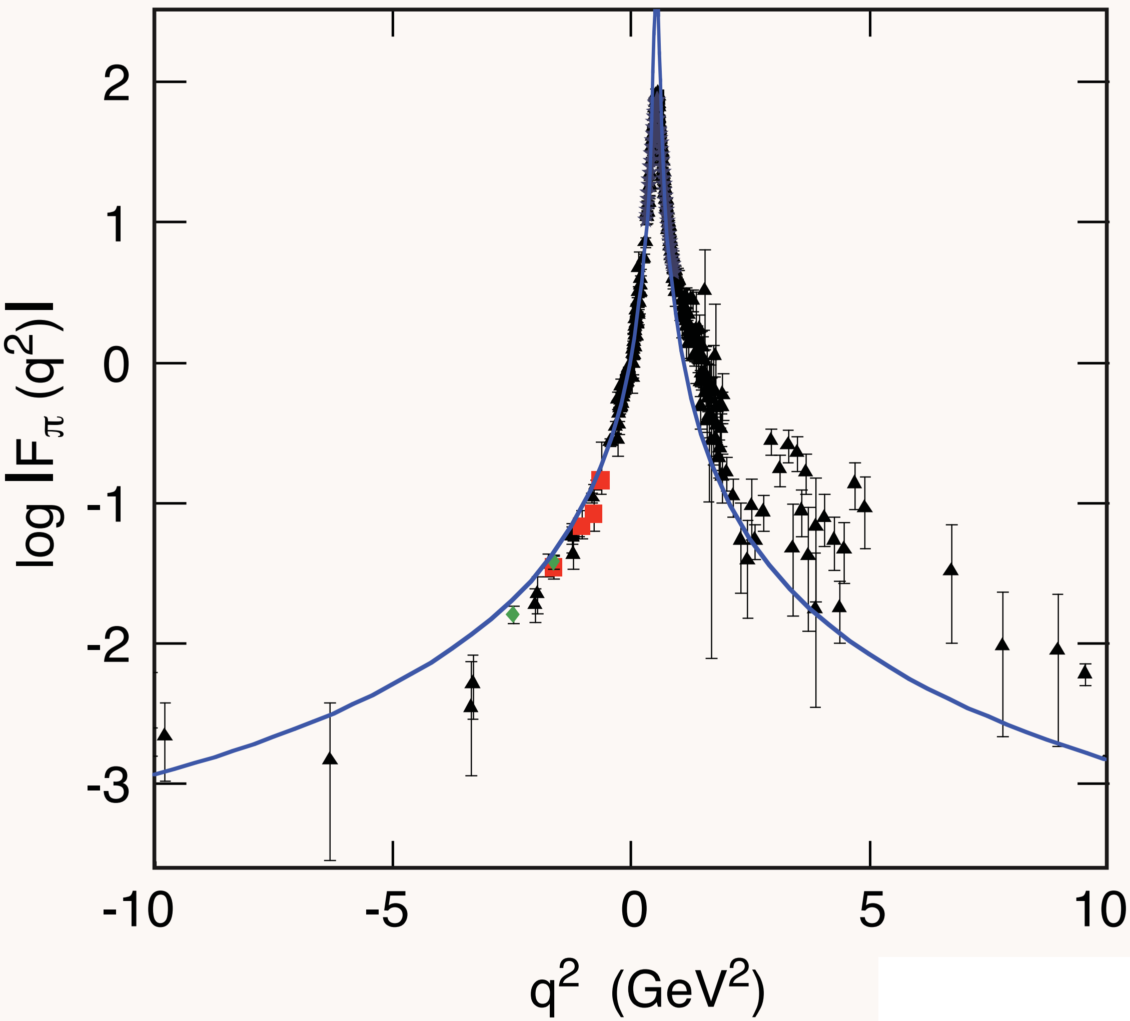
One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

- Analytical continuation to time-like region $q^2 \to -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N, \quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3,$$

$$\cdots$$

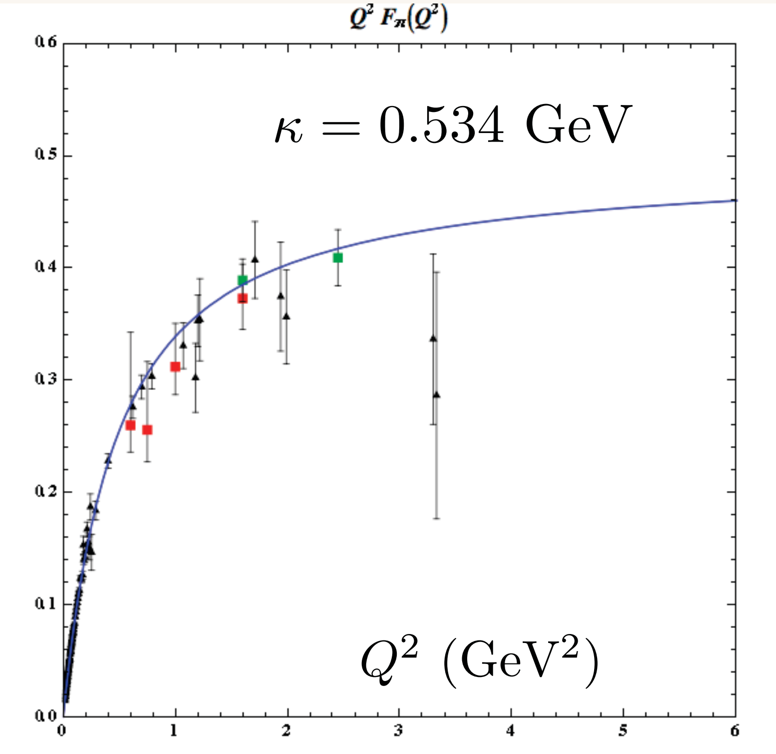
$$F(Q^2) = \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \cdots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.$$

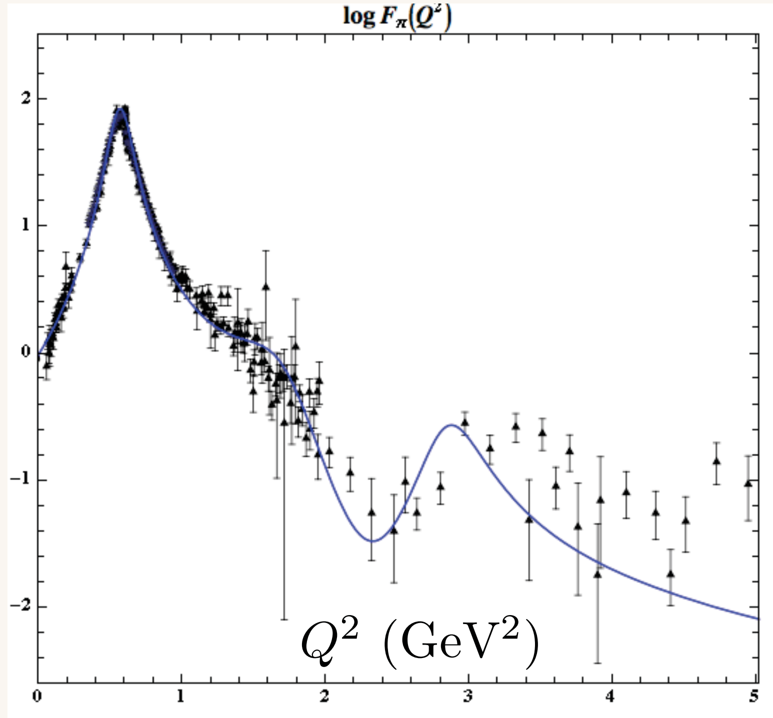
- For large Q^2:

$$F(Q^2) \to (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

Spacelike and timelike pion form factor

Preliminary

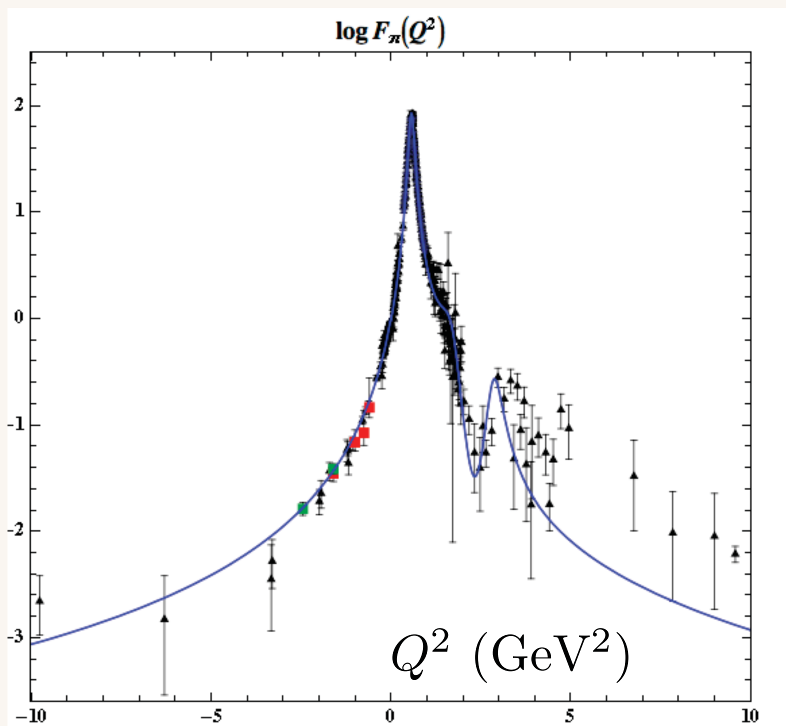




$$|\pi>= \psi_{q\bar{q}}|q\bar{q}> + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}>$$

$$\Gamma_\rho = 120 \text{ MeV}, \ \Gamma'_\rho = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 15\%$$



Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^{\,2} = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\left(bqx\right) \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} \, J(Q^2, z) \, |\Phi_{\pi^+}(z)|^2 ,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde{\psi}_{q\overline{q}/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}}$$

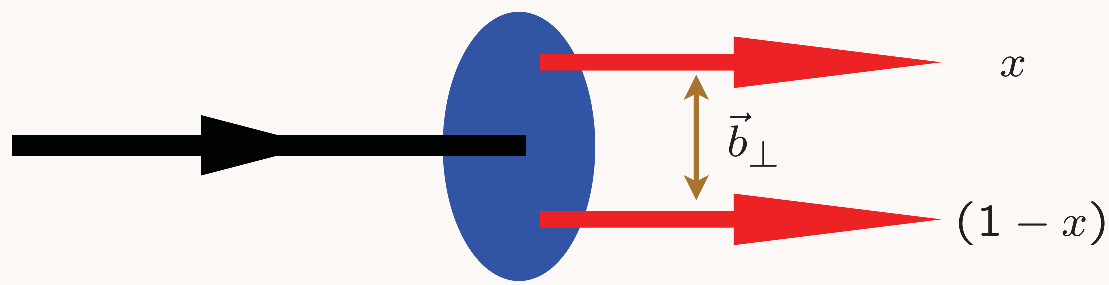
with $\zeta = z, \; 0 \leq \zeta \leq \Lambda_{\rm QCD}$

LF(3+1) $\longleftrightarrow$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}}\,\phi(\zeta)$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2,z) \left|\Phi_\pi(z)\right|^2,$$

Abidin & Carlson

where $H(Q^2,z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2,z)$

$$H(Q^2,z) = 2\int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

Identical to LF Holography obtained from electromagnetic current

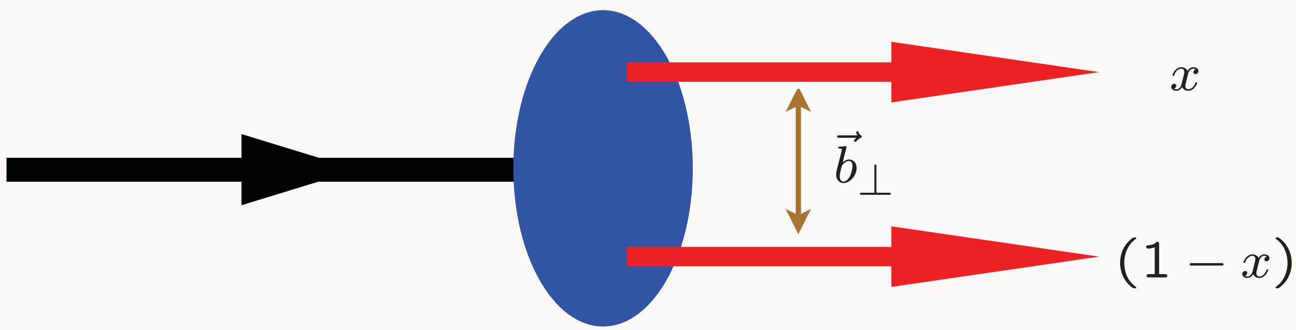
Light-Front Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\phi(\zeta)=\mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

G. de Teramond, sjb

soft wall
confining potential:

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

Change variables

$$(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}}$$

$$+ \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H^{LF}_{QCD}$$

QCD Meson Spectrum

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b^2_\perp$$

$$[-\frac{d^2}{d\zeta^2} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

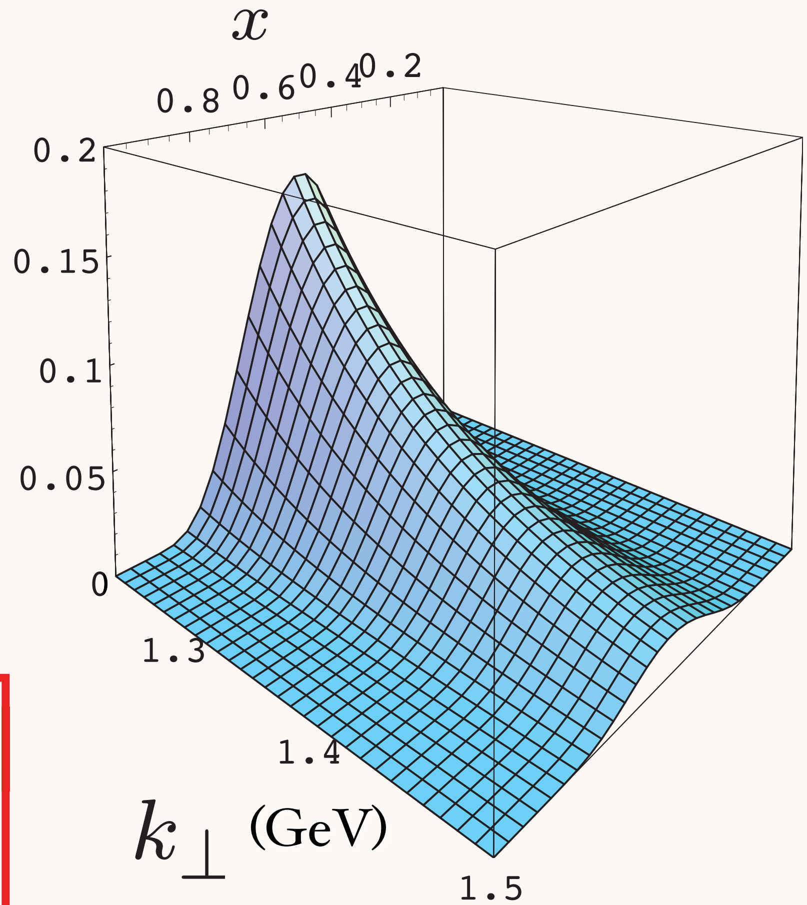
Confining AdS/QCD potential

Semiclassical first approximation to QCD

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

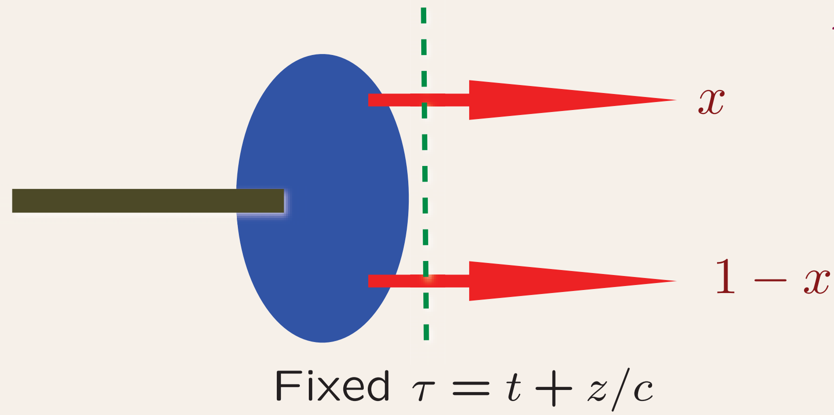
Connection of Confinement to TMDs

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

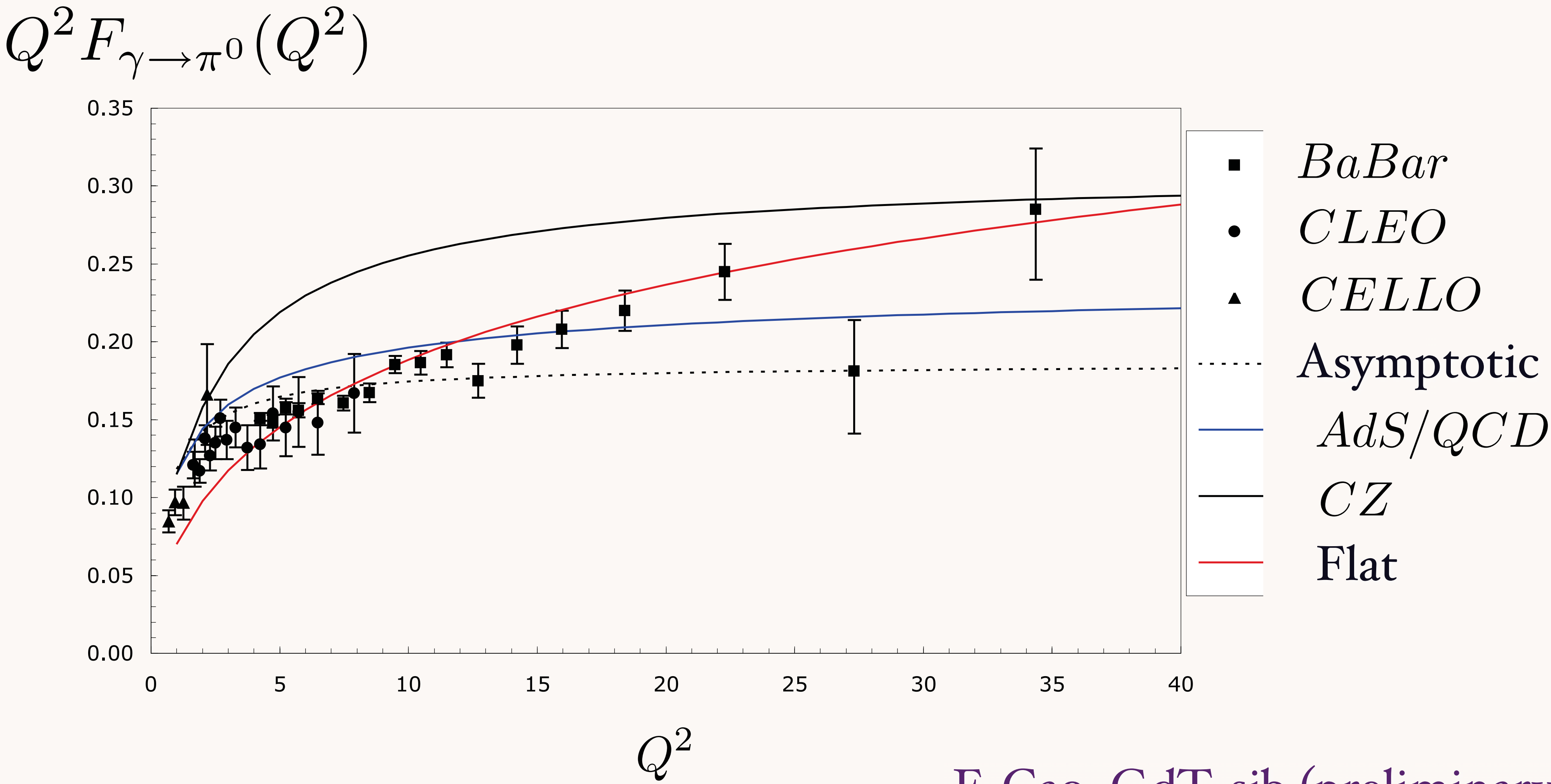
$$\text{Lattice (I)} \; < \xi^2 >_\pi = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \; < \xi^2 >_\pi = 0.269 \pm 0.039$$

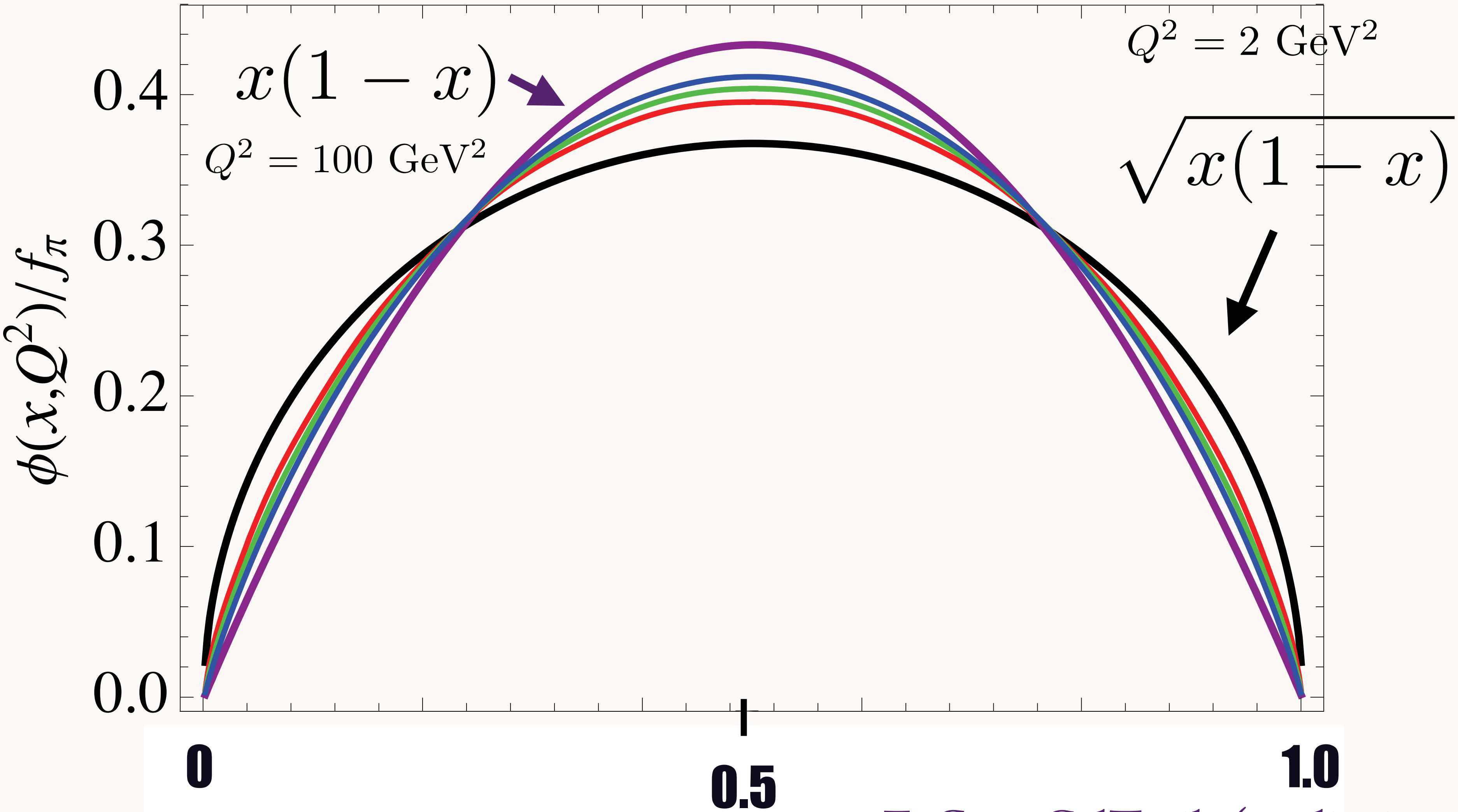
Braun et al.

Photon-to-pion transition form factor



F. Cao, GdT, sjb (preliminary)

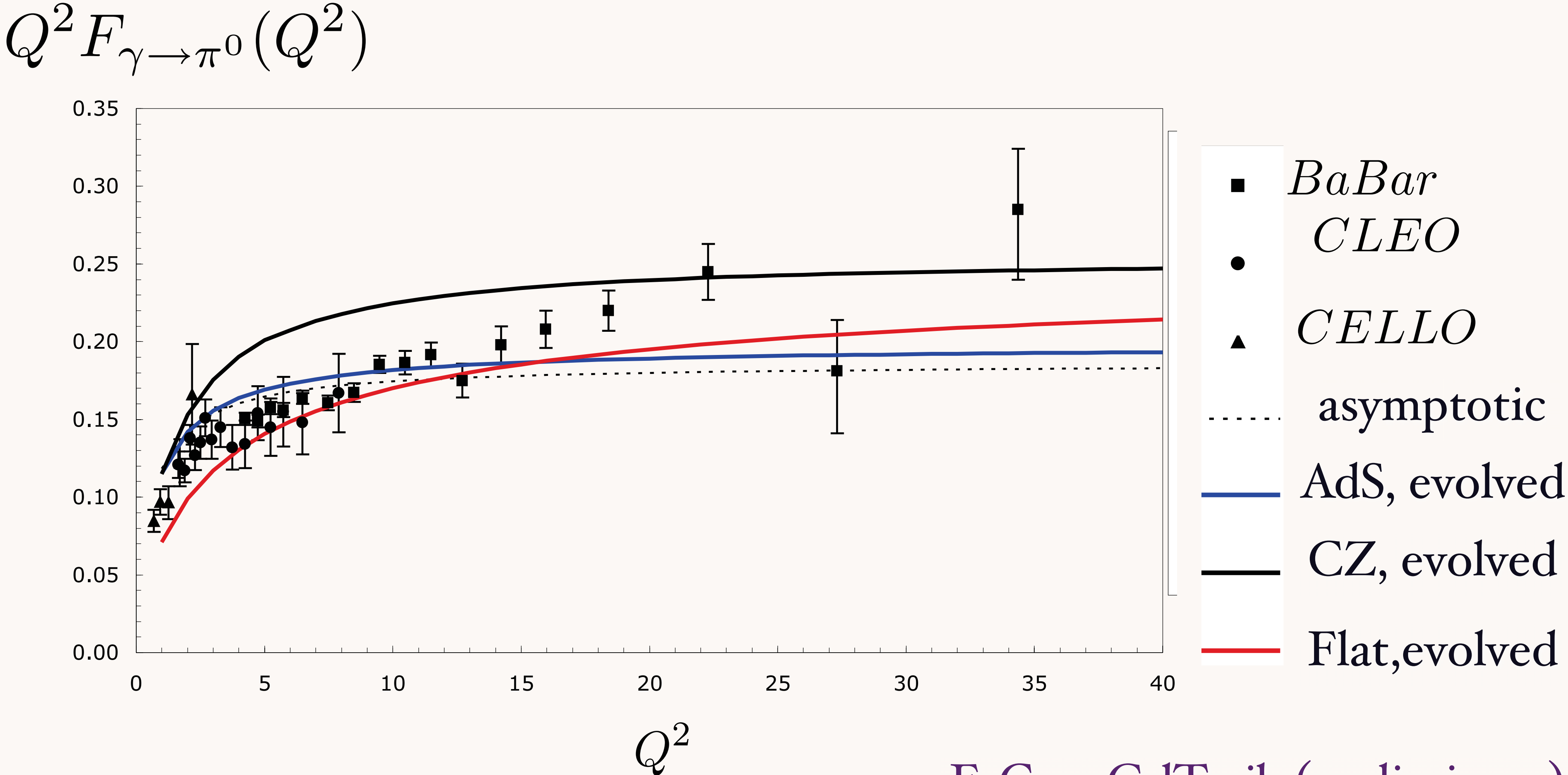
ERBL Evolution of Pion Distribution Amplitude



F. Cao, GdT, sjb (preliminary)

NLO: Diehl, Kroll

Photon-to-pion transition form factor with ERBL evolution

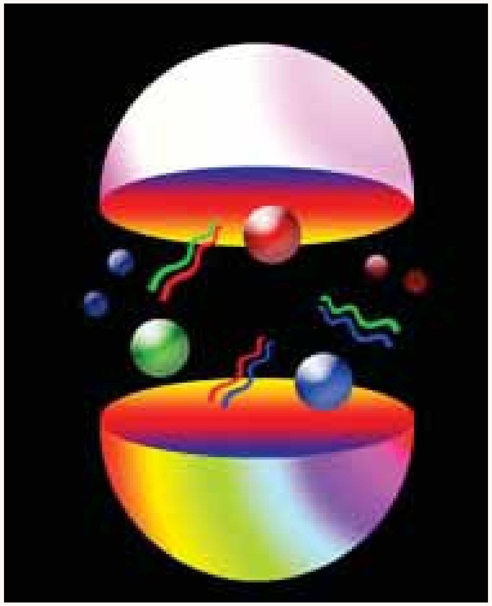


F. Cao, GdT, sjb (preliminary)

- Baryons Spectrum in "bottom-up" holographic QCD

GdT and Sjb hep-th/0409074, hep-th/0501022.

Baryons in AdS/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g}\, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.$$

Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_L(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{L+\frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_L(\zeta), \Pi_L^\dagger(\zeta)\right] = \frac{2L+1}{\zeta^2}\gamma_5.$$

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i .$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} \left[J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].$$

  Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD} \right) = 0,$$

  given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k} \Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k} \Lambda_{\rm QCD},$$

  with a scale independent mass ratio.

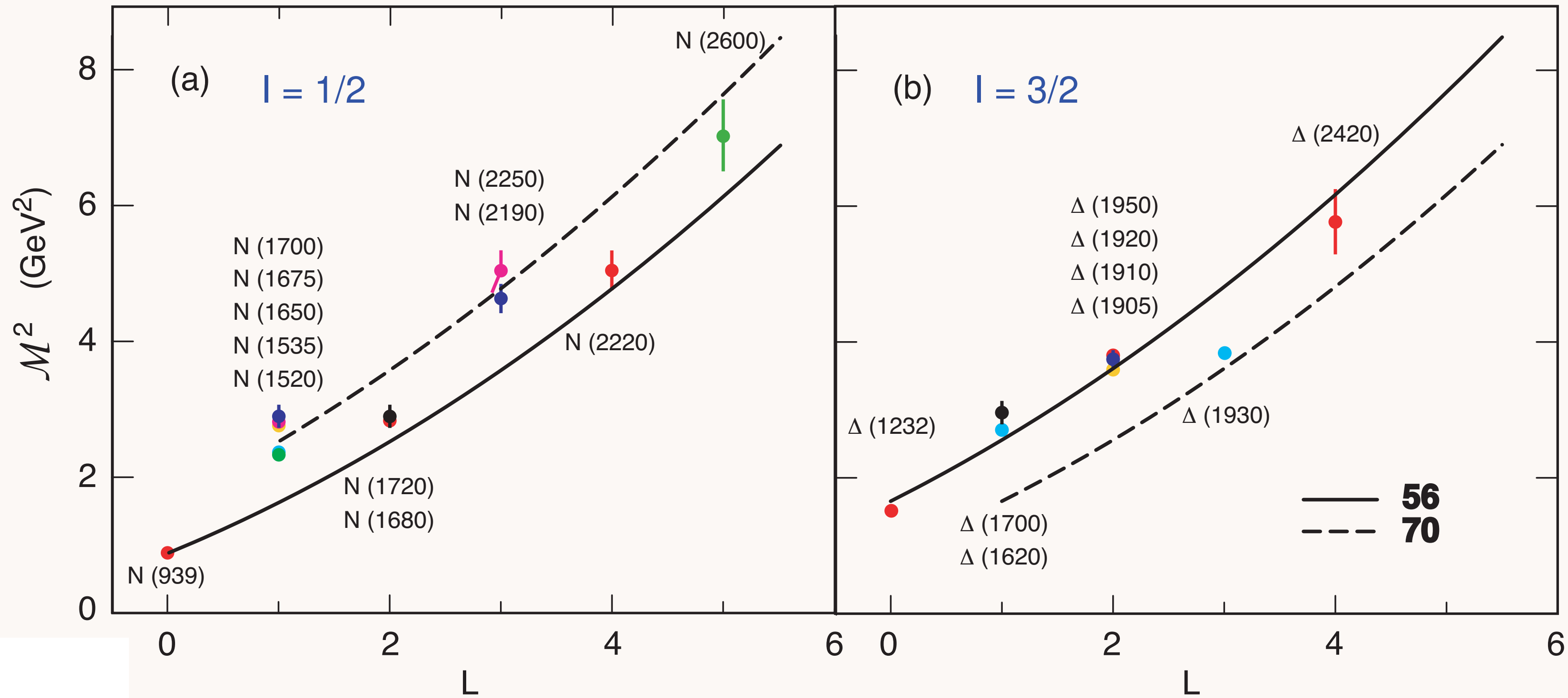


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

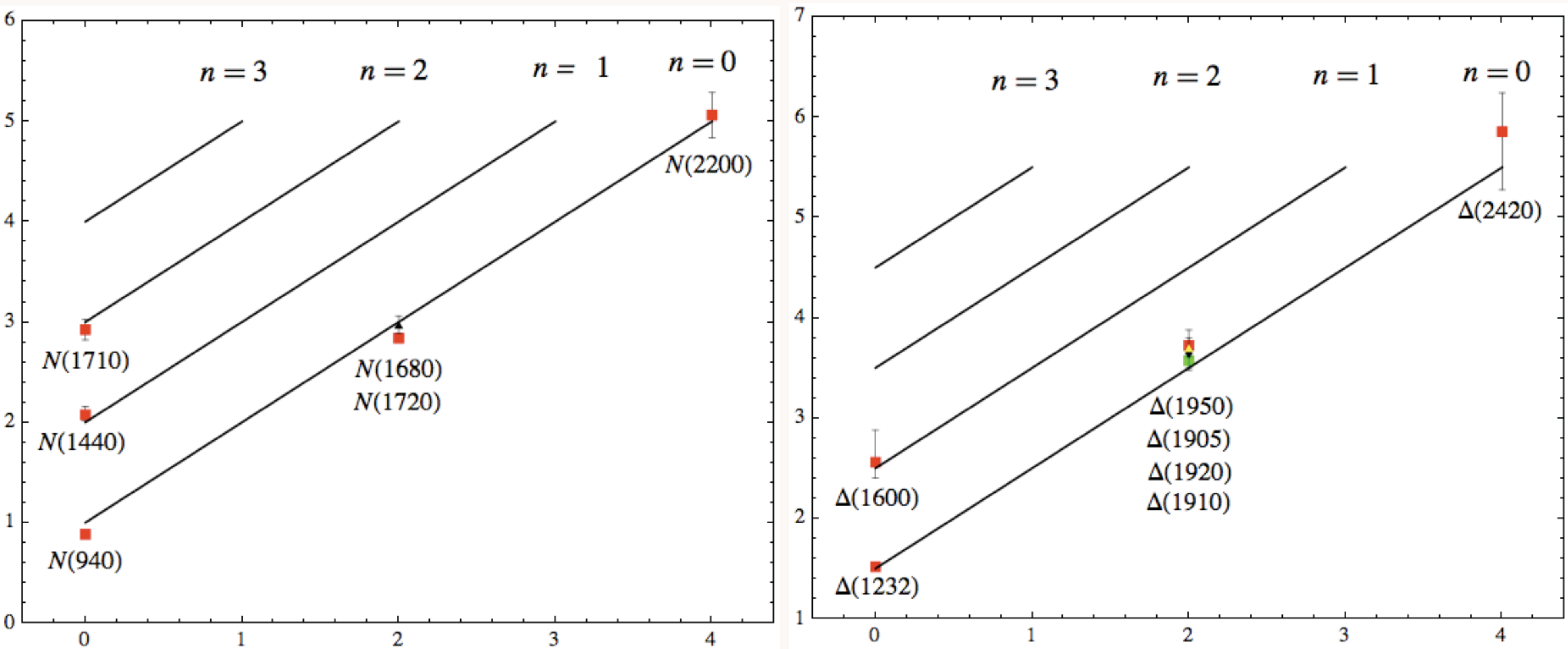
$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$\mathcal{M}^2$



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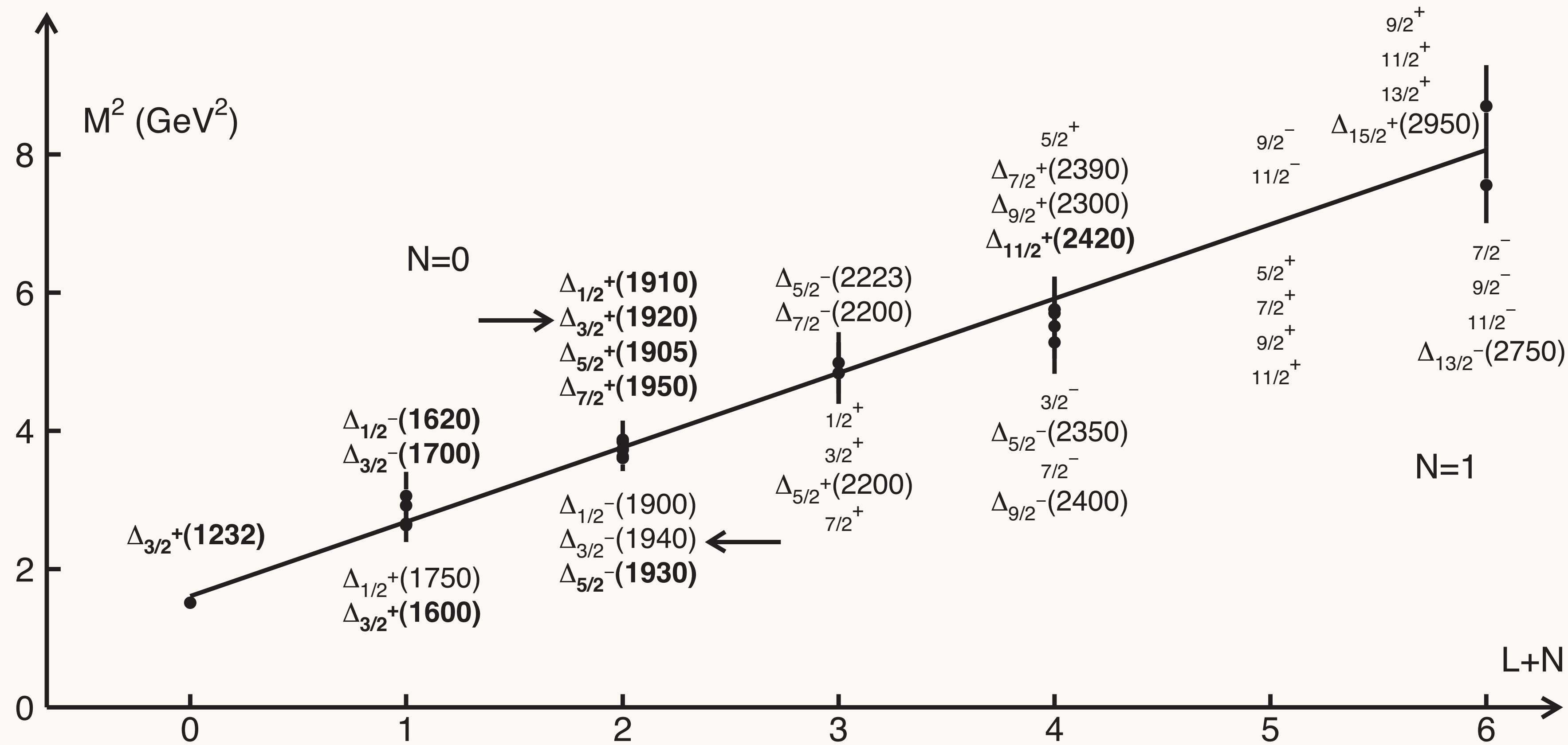
Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV



E. Klempt *et al.*: Δ^* resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

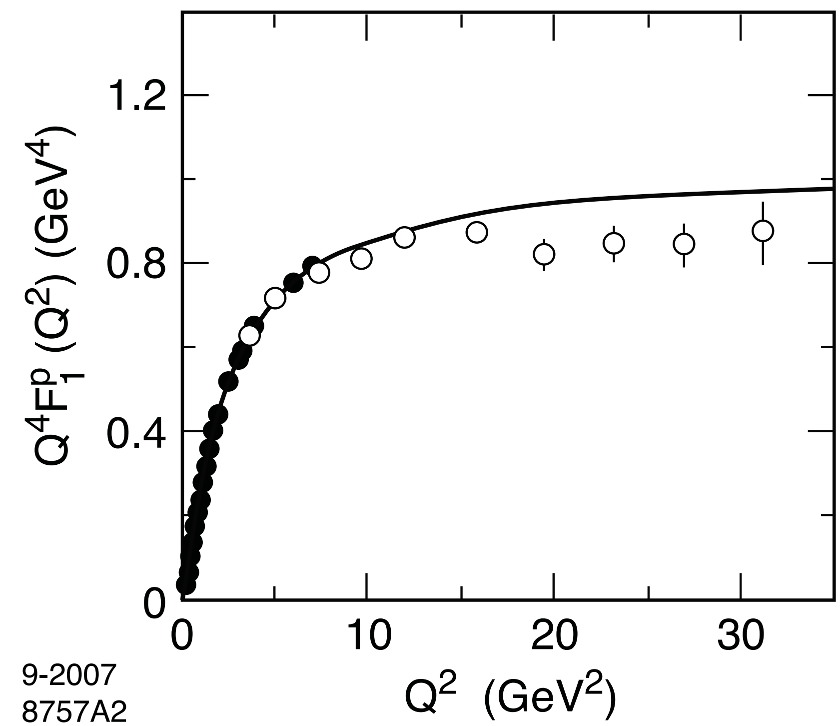
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

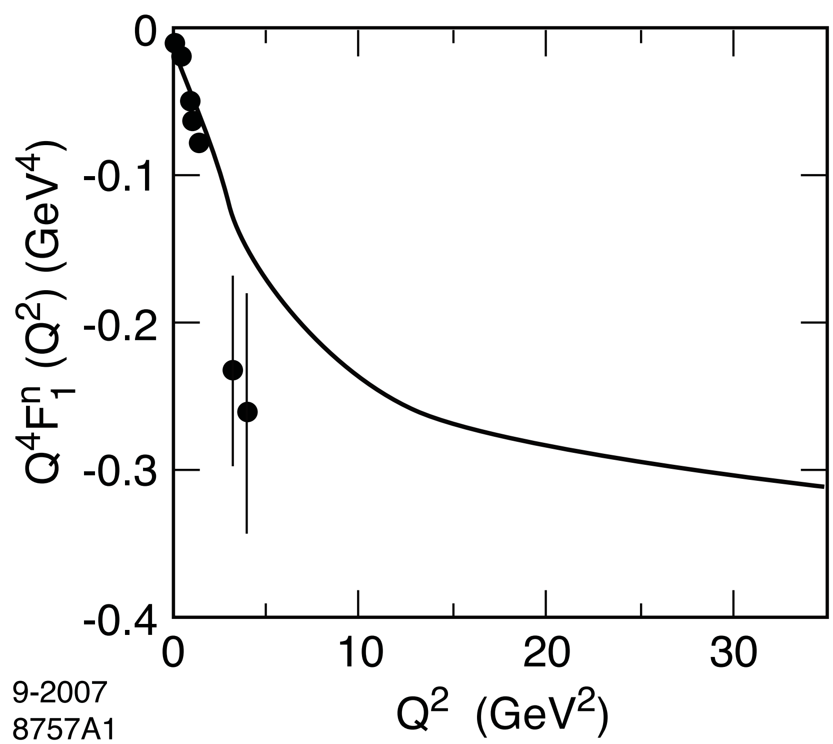
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

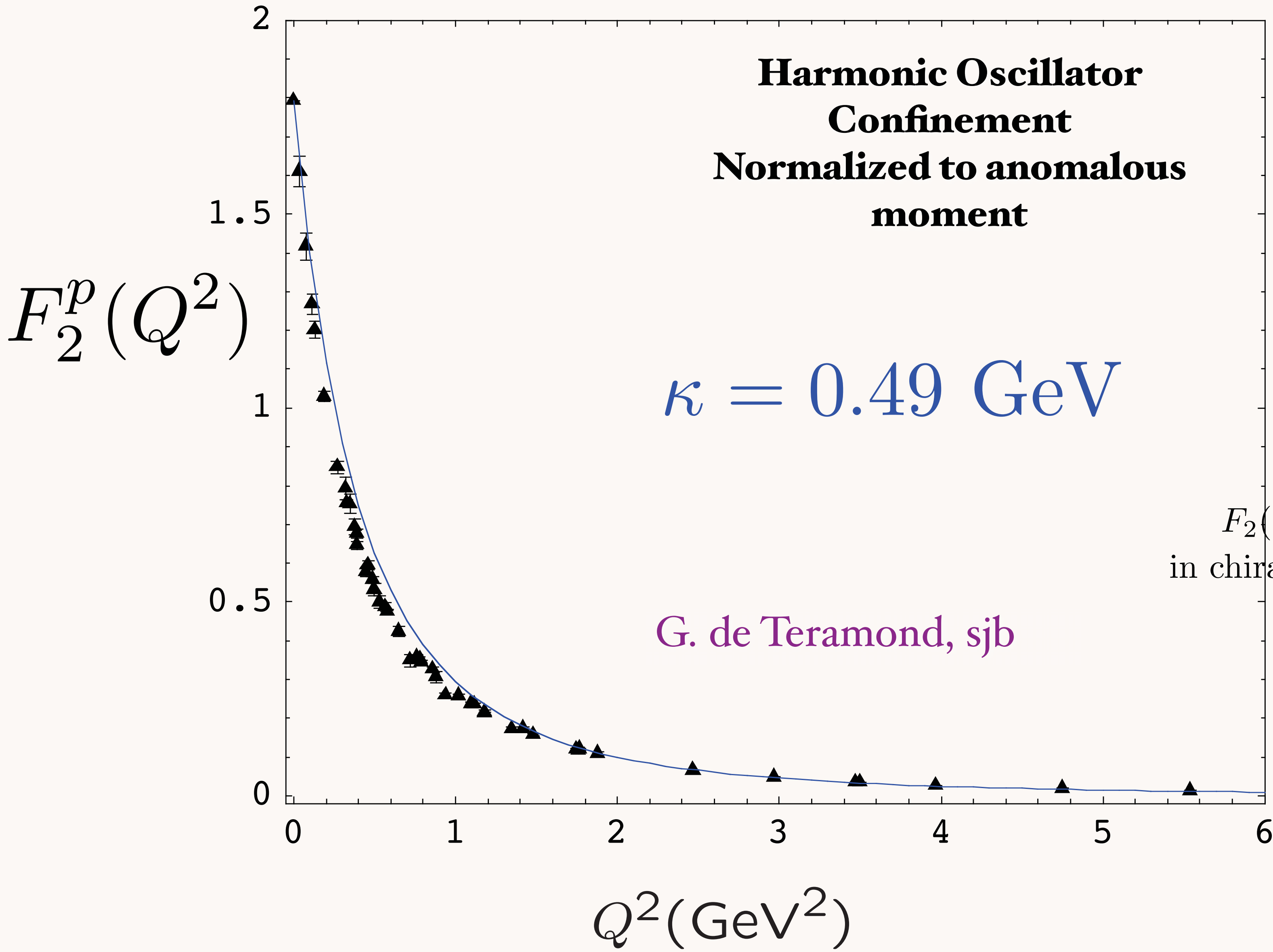


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L=1$ and $L=0$ LFWFs



AdS/QCD No chiral divergence!

$F_2(Q^2) = 1 + \mathcal{O}\frac{Q^2}{m_\pi m_p}$
in chiral perturbation theory

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g}\; e^{\phi(z)} \frac{1}{g_5^2} G^2 \qquad \text{where } \sqrt{g} = \left(\frac{R}{z}\right)^5 \text{ and } \phi(z) = +\kappa^2 z^2$$

Define an effective coupling $g_5(z)$

$$S = -\frac{1}{4}\int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2$$

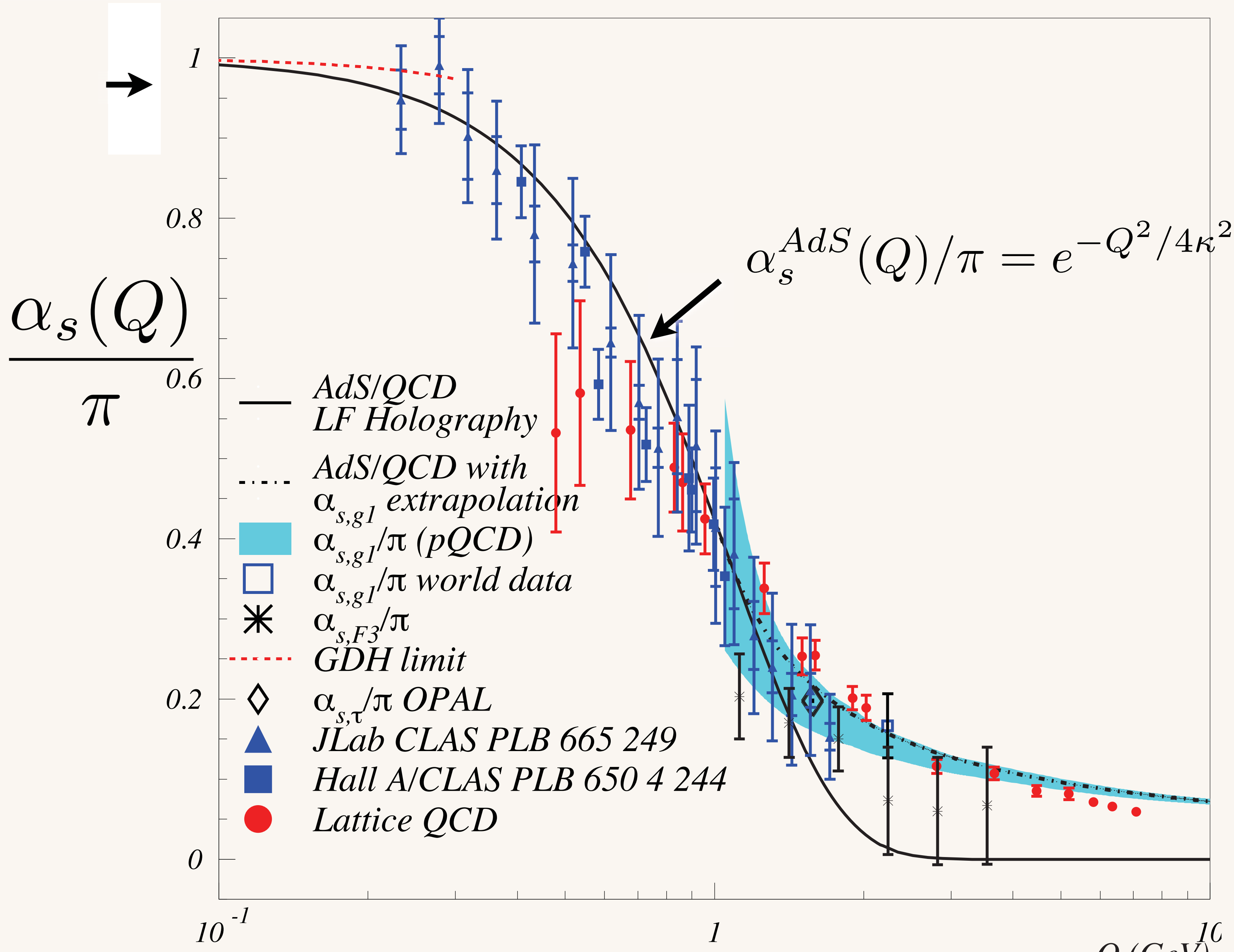
$$\text{Thus } \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \text{ or } g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

Light-Front Holography: $z \to \zeta = b_\perp \sqrt{x(1-x)}$

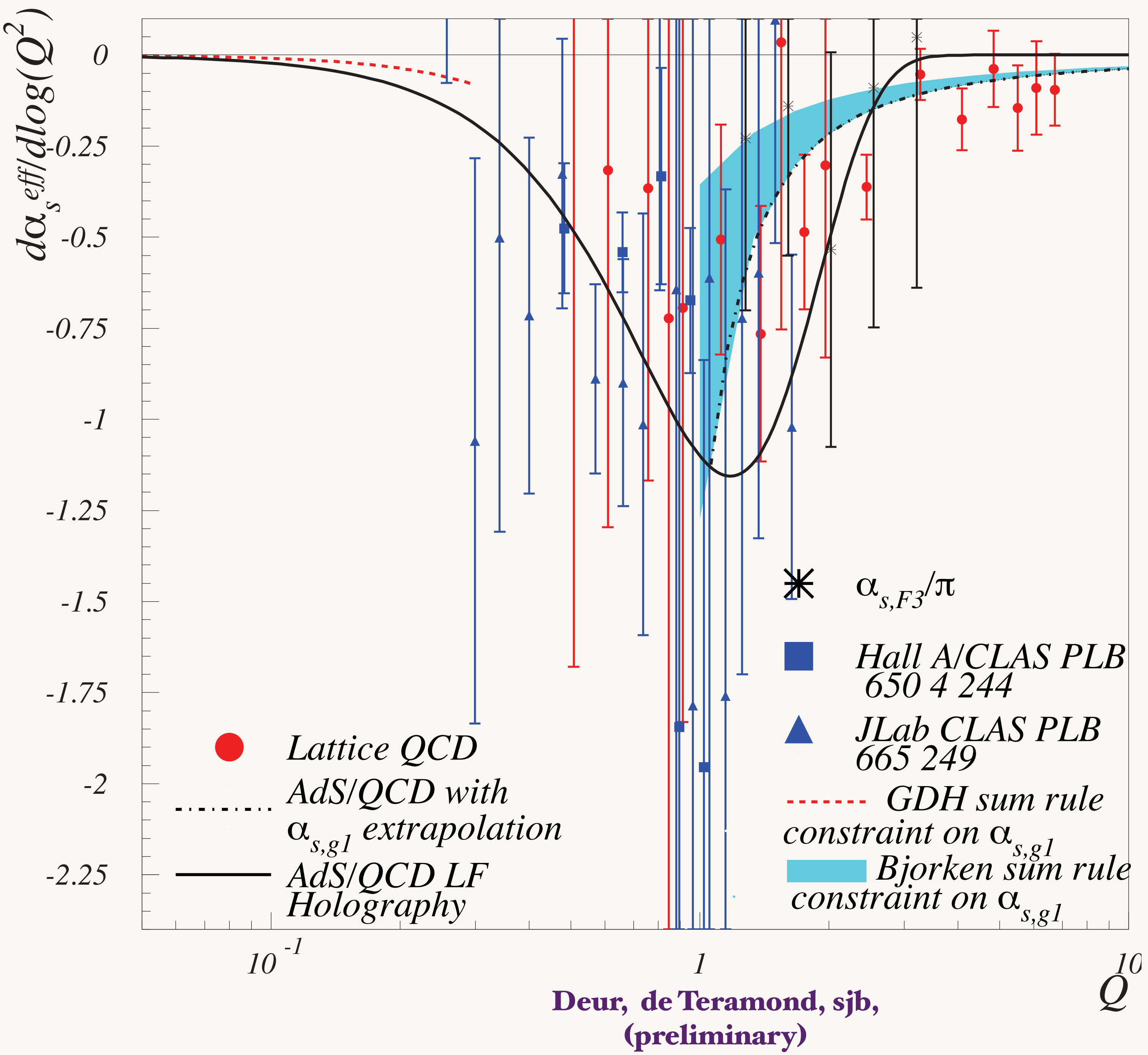
$$\alpha_s(q^2) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \qquad \text{where } \alpha_s(z) = e^{-\kappa^2 z^2} \alpha_s(0)$$

Running Coupling from AdS/QCD

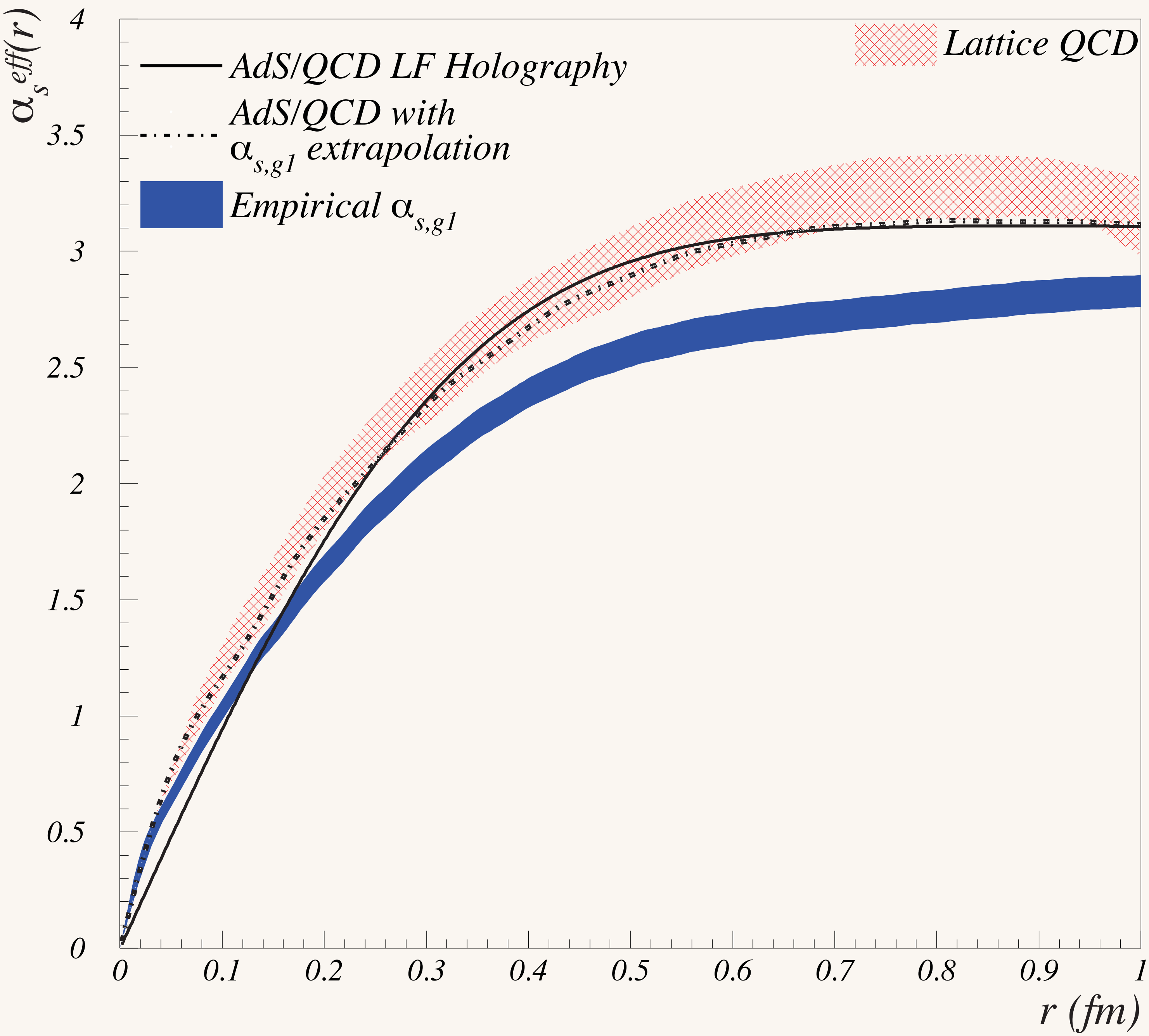
normalization



Deur, de Teramond, sjb,
(preliminary)



$d\alpha_s^{eff}/dlog(Q^2)$
0
-0.25
-0.5
-0.75
-1
-1.25
-1.5
-1.75
-2
-2.25
10^{-1}
1
10
Q
$\alpha_{s,F3}/\pi$
Hall A/CLAS PLB 650 4 244
JLab CLAS PLB 665 249
GDH sum rule constraint on $\alpha_{s,g1}$
Bjorken sum rule constraint on $\alpha_{s,g1}$
Lattice QCD
AdS/QCD with $\alpha_{s,g1}$ extrapolation
AdS/QCD LF Holography
Deur, de Teramond, sjb,
(preliminary)



$\alpha_s^{eff}(r)$
AdS/QCD LF Holography
AdS/QCD with
$\alpha_{s,g1}$ extrapolation
Empirical $\alpha_{s,g1}$
Lattice QCD
0
0.5
1
1.5
2
2.5
3
3.5
4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r (fm)

Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion (m_q =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S. Spectrum is independent of S
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize H_{LF} on AdS basis

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering

Regge Trajectories

QCD at the Amplitude Level

↓

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

Holography

↓

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

↓

Hadron Spectra, Wavefunctions, Dynamics

Light-Front QCD

Heisenberg Matrix Formulation

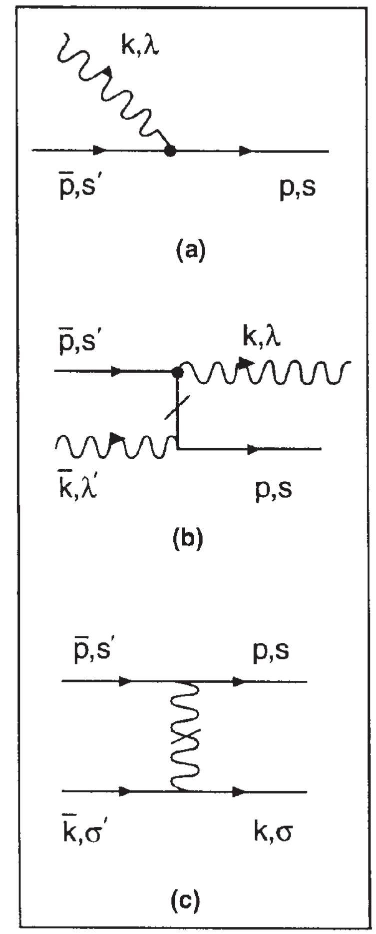
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

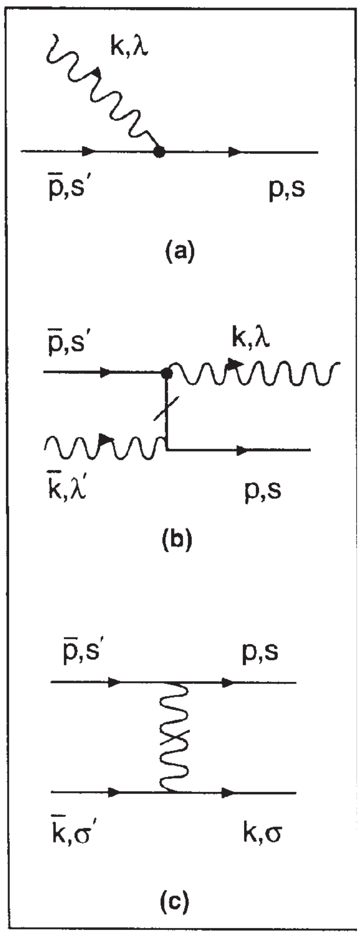
$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

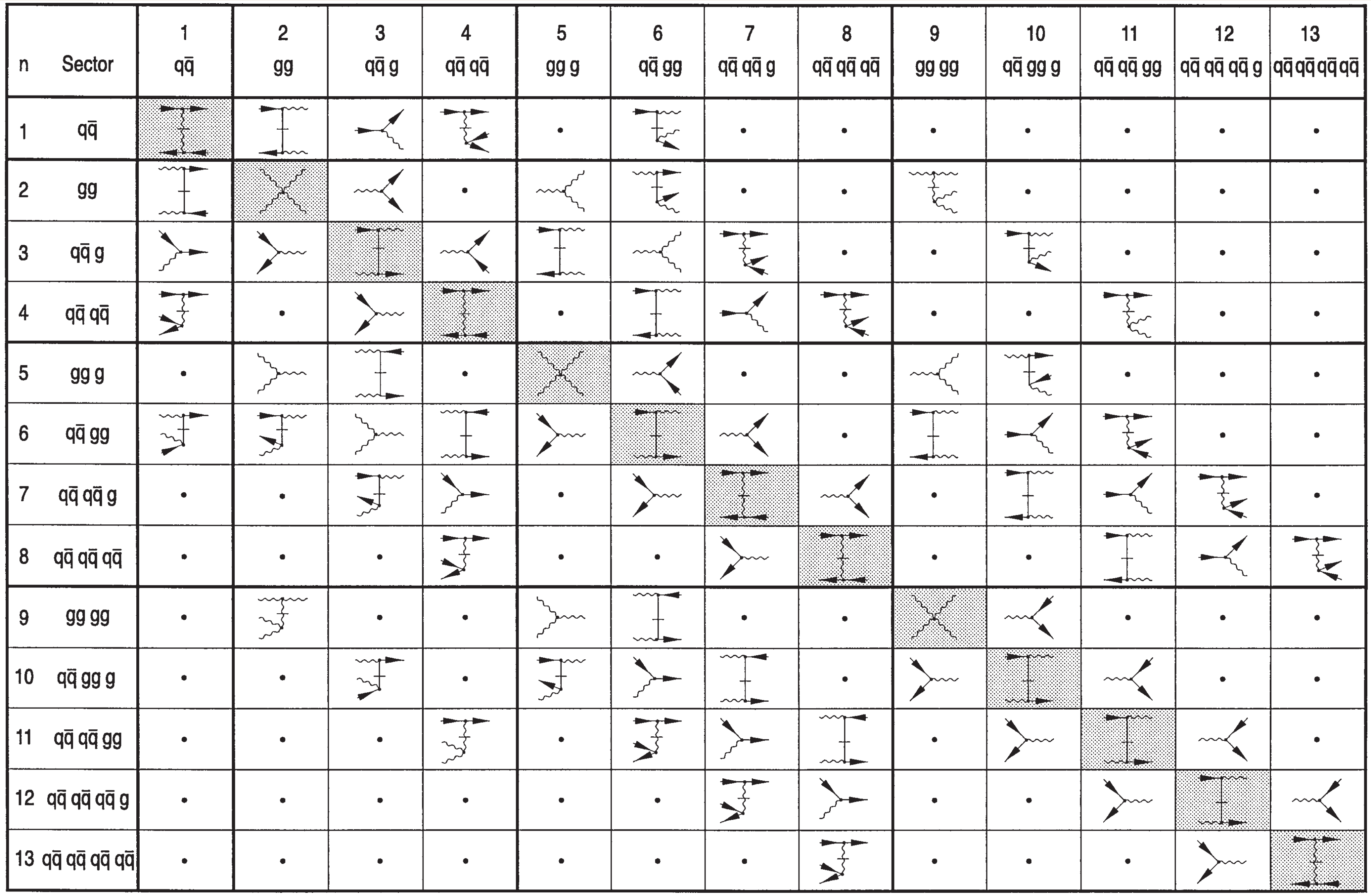


Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

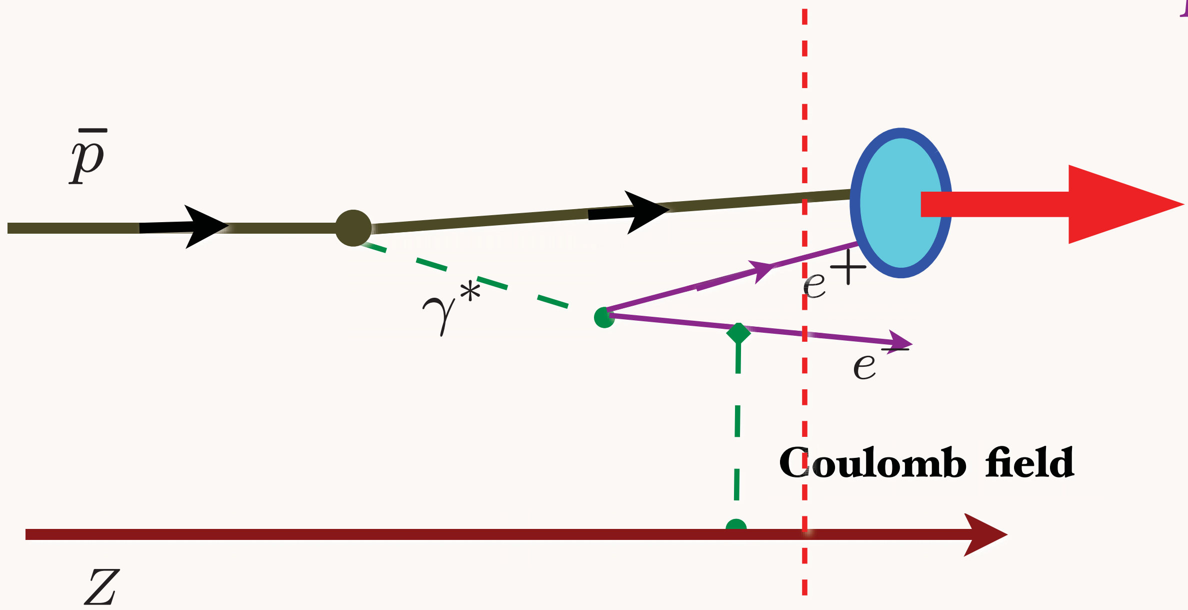
- Good initial approximant
- Better than plane wave basis Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

  Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

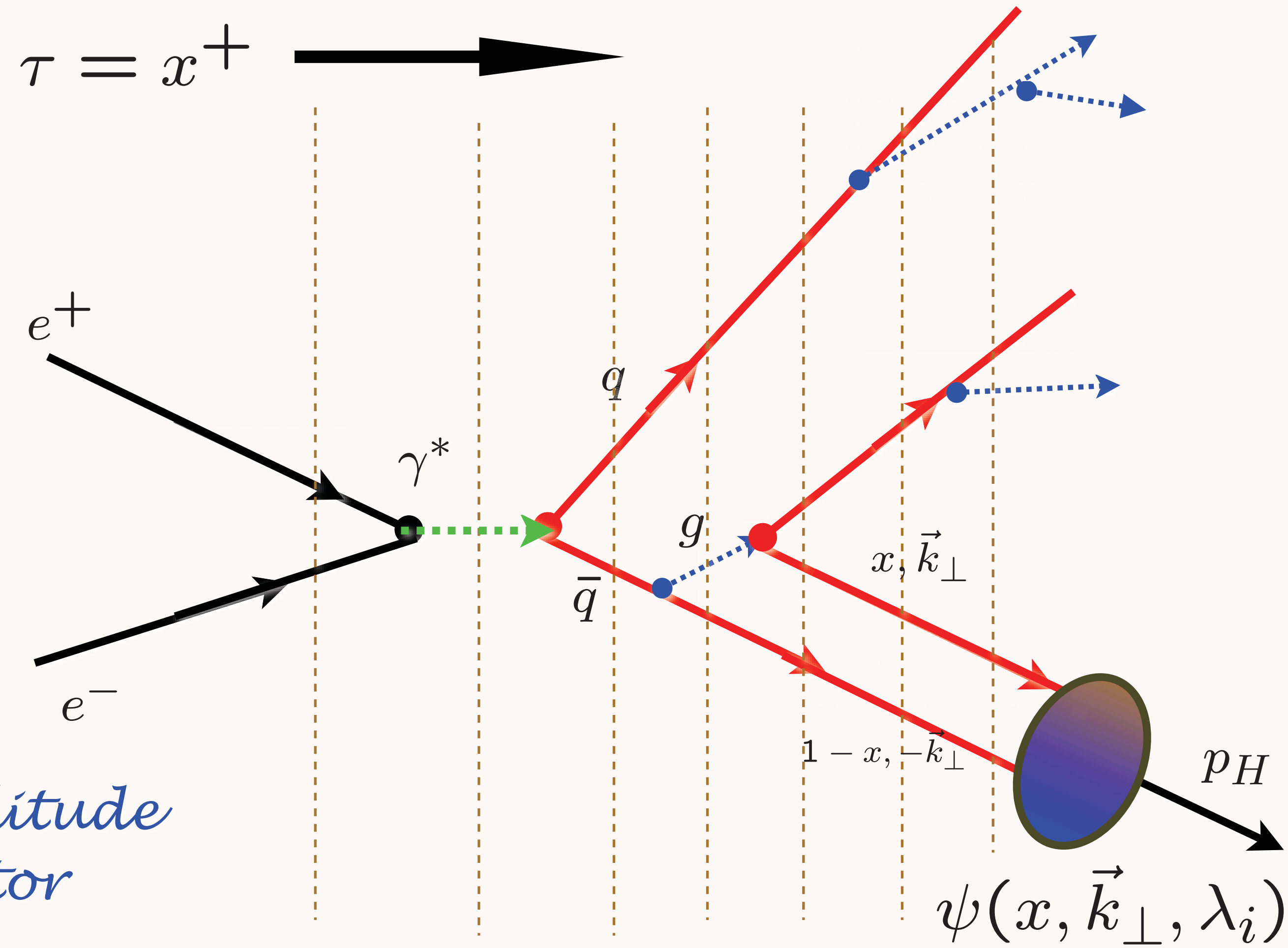
$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell *co-moving positron and antiproton*

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

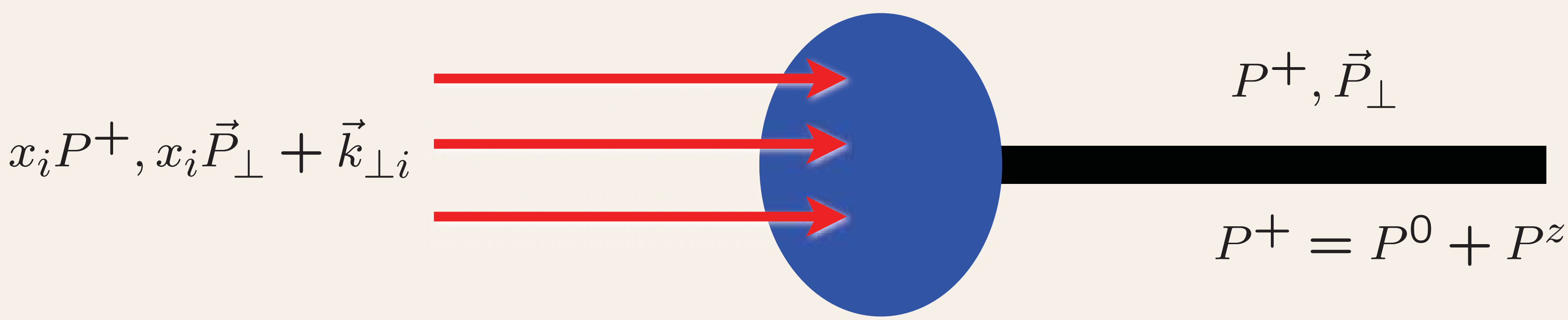
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda^2_{QCD}$$

- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement

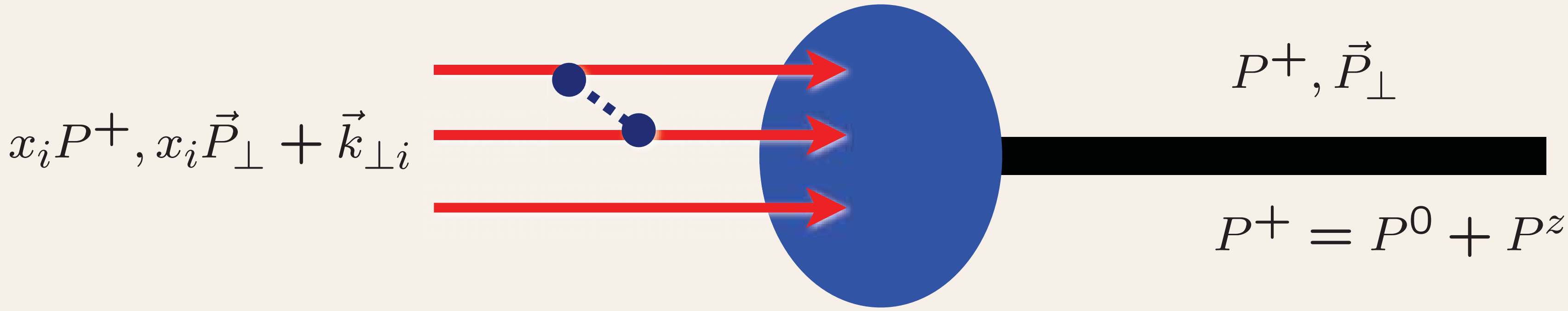


Features of LF T-Matrix Formalism
"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- Generates PQCD Hard Tail of LFWF at high x and high transverse momentum
- Dimensional Counting rules and Color Transparency for Hard Exclusive Channels
- Counting rules for structure functions and fragmentation functions at large x and z:

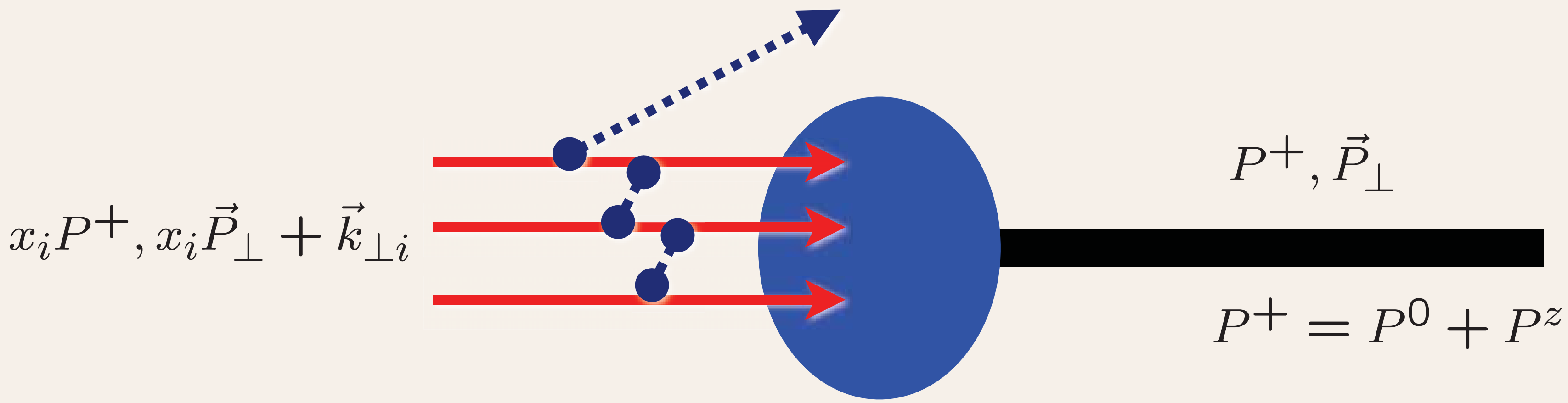
$$(1-x)^{2n_{spect}-1}, (1-z)^{2n_{spect}-1}$$



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange
- Factorization Scale for structure functions and fragmentation functions set: $\mu_{fact} = \Lambda_{QCD}$



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

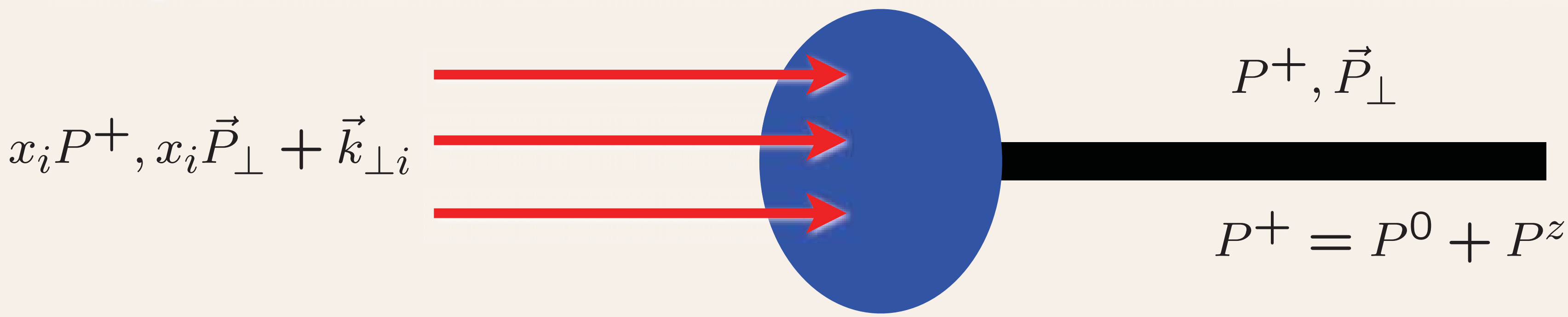
- **Same principle as antihydrogen production: off-shell coalescence**

- **coalescence to hadron favored at equal rapidity, small transverse momenta**

- **leading heavy hadron production: D and B mesons produced at large z**

- **hadron helicity conservation if hadron LFWF has $L^z = 0$**

- **Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin**

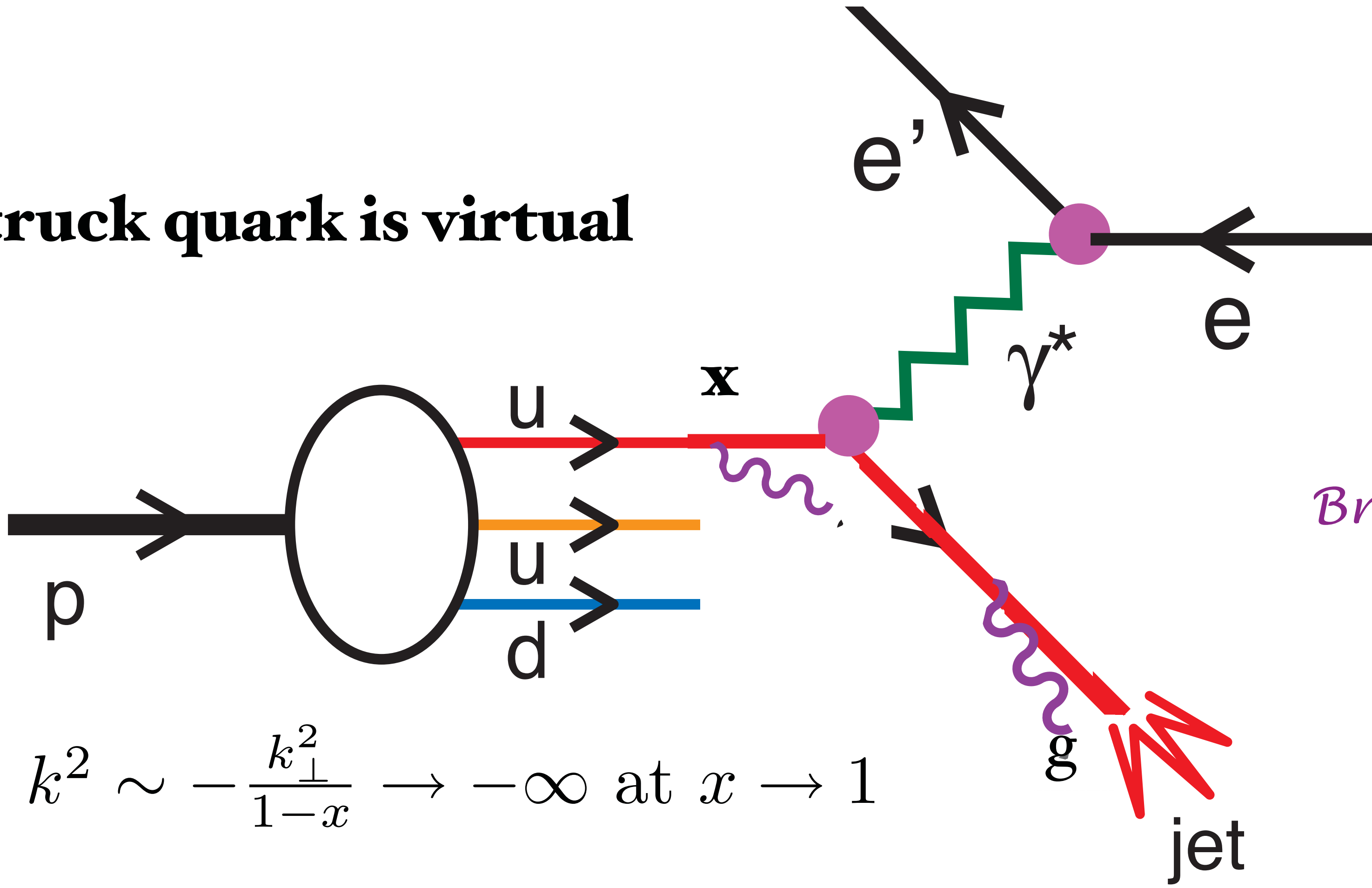


Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of P^+ and P^z
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using "alternating denominator" method
- Multiple renormalization scales (BLM)
- Cluster Decomposition; Unitary Cuts; `History'

Deep Inelastic Electron-Proton Scattering

Struck quark is virtual



Gluonic Bremmstrahlung

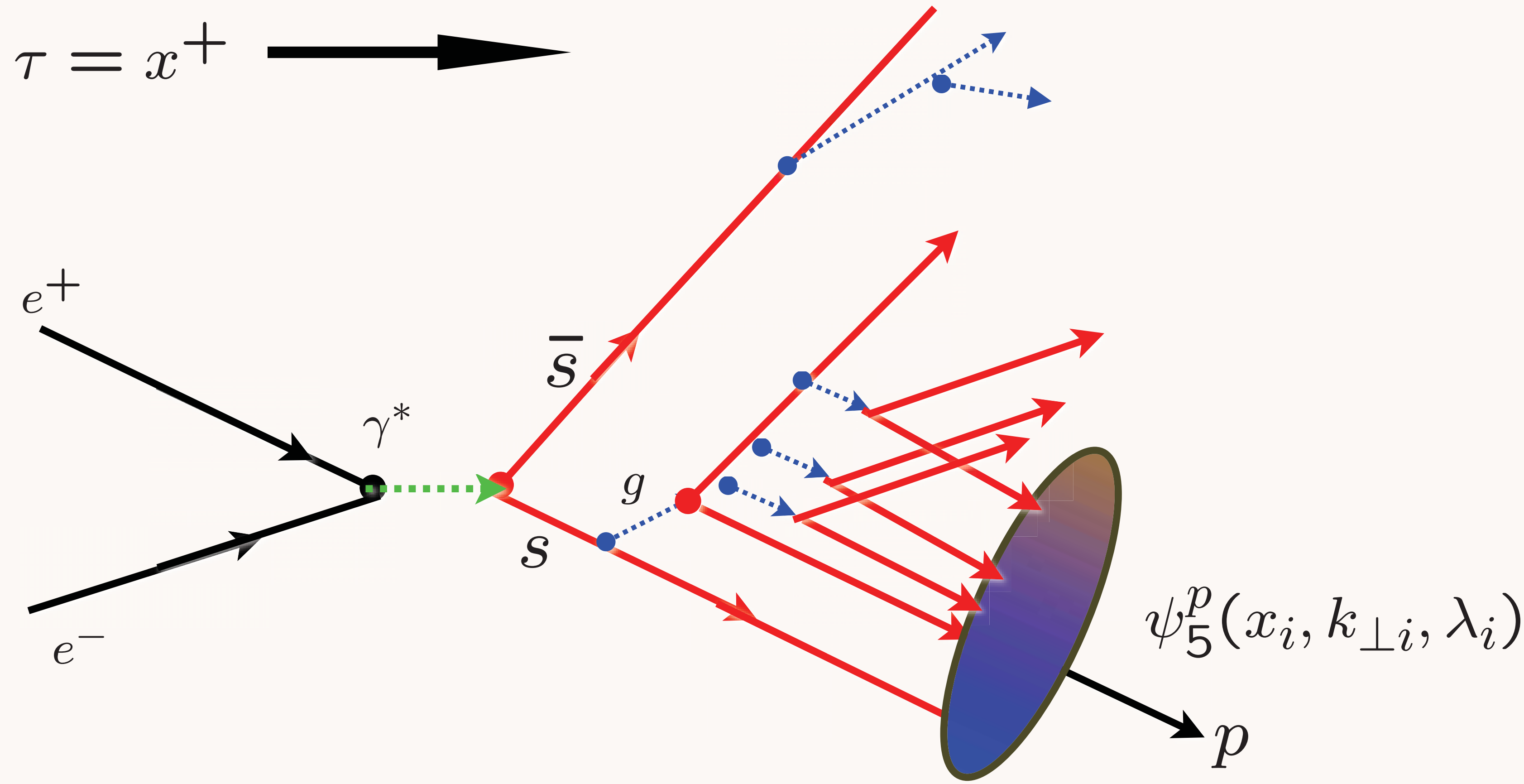
DGLAP Evolution

$$k^2 \sim -\frac{k_\perp^2}{1-x} \to -\infty \text{ at } x \to 1$$

Off-shell Effect: Breakdown of DGLAP at $x \sim 1$!

Off-shell Effect: Breakdown of DGLAP at $z \sim 1$!

Hadronization at the Amplitude Level



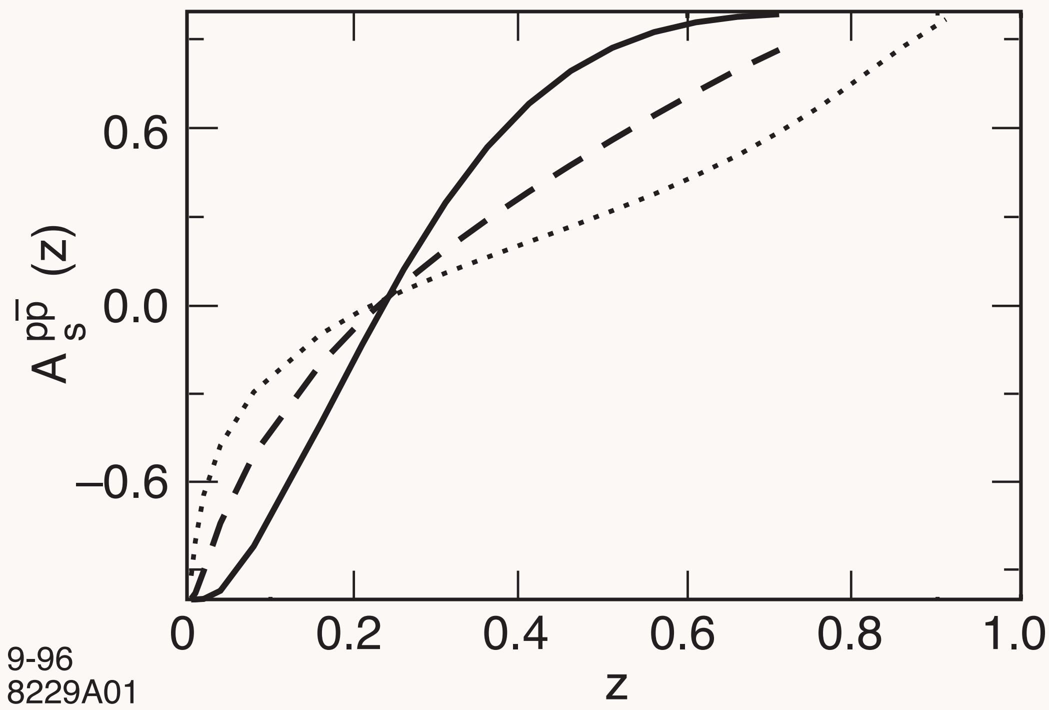
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s \to p}(z) \neq D_{s \to \bar{p}}(z)$

B-Q Ma, sjb

B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to\bar{p}}(z)}{D_{s\to p}(z) + D_{s\to\bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s}> \simeq |K^+\Lambda>$

Chiral Symmetry Breaking in AdS/QCD

Erlich et al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted z^2 distribution, not constant condensate**

$$\delta M^2 = -2m_q < \bar{\psi}\psi > \times \int dz \ \phi^2(z) z^2$$

- **z^2 weighting consistent with higher Fock states at periphery of hadron wavefunction**

- **AdS/QCD: confined condensate**

- **Suggests "In-Hadron" Condensates**

de Teramond, Shrock, sjb

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

Light-Front
(Front Form)
Formalism

Determinations of the vacuum Gluon Condensate

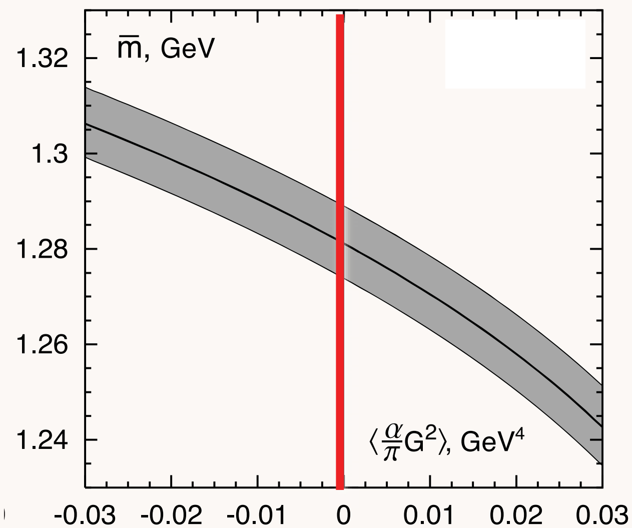
$$< 0|\frac{\alpha_s}{\pi} G^2|0 > [\mathrm{GeV}^4]$$

-0.005 ± 0.003 from τ decay. Davier et al.

$+0.006 \pm 0.012$ from τ decay. Geshkenbein, Ioffe, Zyablyuk

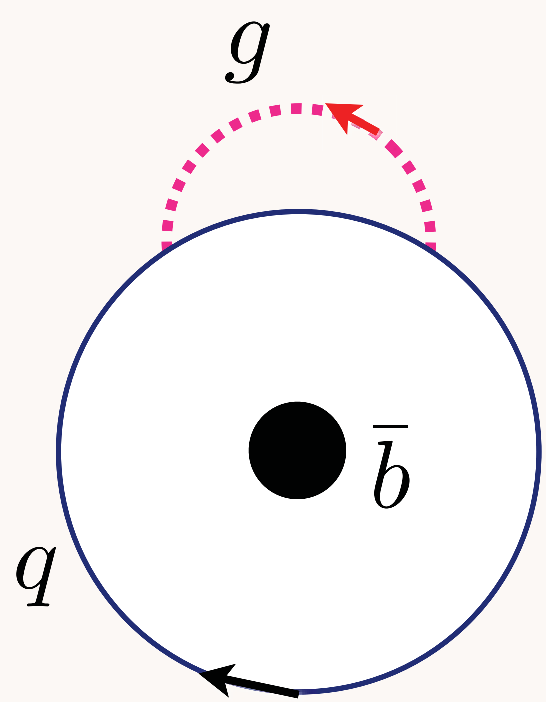
$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk



Consistent with zero vacuum condensate

Use Dyson-Schwinger Equation for bound-state quark propagator:



B-Meson

Shrock, sjb

Roberts, Tandy Maris

Alkofer

$$< \bar{b}|\bar{q}q|\bar{b} > \text{ not } < 0|\bar{q}q|0 >$$

Pion mass **and decay constant.**
Pieter Maris, Craig D. Roberts (Argonne, PHY) , Peter C. Tandy (Kent State U.) . ANL-PHY-8753-TH-97, KSUCNR-103-97, Jul 1997. 12pp.
Published in **Phys.Lett.B420:267-273,1998**.
e-Print: **nucl-th/9707003**

Pi- and K meson Bethe-Salpeter amplitudes.
Pieter Maris, Craig D. Roberts (Argonne, PHY) . ANL-PHY-8788-TH-97, Aug 1997. 34pp.
Published in **Phys.Rev.C56:3369-3383,1997**.
e-Print: **nucl-th/9708029**

Concerning the quark condensate.
K. Langfeld (Tubingen U.) , H. Markum (Vienna, Tech. U.) , R. Pullirsch (Regensburg U.) , C.D. Roberts (Argonne, PHY & Rostock U.) , S.M. Schmidt (Tubingen U. & HGF, Bonn) . ANL-PHY-10460-TH-2002, MPG-VT-UR-239-02, Jan 2003. 7pp.
Published in **Phys.Rev.C67:065206,2003**.
e-Print: **nucl-th/0301024**

"In-Meson Condensate"

$$- \langle \bar{q} q \rangle_{\zeta}^{\pi} = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle \,.$$

Valid even for $m_q \to 0$

f_π nonzero

Quark and Gluon condensates reside within hadrons, not LF vacuum

- **Bound-State Dyson-Schwinger Equations**

Maris, Roberts, Tandy

- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**

Casher Susskind

- **Finite size phase transition - infinite # Fock constituents**
- **AdS/QCD Description -- CSB is in-hadron Effect**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant -- reduction by 45 orders of magnitude!**

Shrock, sjb

"Confined QCD Condensates"

- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Conformal Template (BLM, CSR, ...)
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model

Shrock and sjb