Triplet Production by Linearly Polarized Photons

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Abstract

The process of electron-positron pair production by linearly polarized photons is used as a polarimeter to perform mobile measurement of linear photon polarization. In the limit of high photon energies, ω, the distributions of the recoil-electron momentum and azimuthal angle do not depend on the photon energy in the laboratory frame. We calculate the power corrections of order m/ω to the above distributions and estimate the deviation from the asymptotic result for various values of ω.

1 Introduction

The differential cross-section for electron-positron pair production by linearly polarized photons was derived in a series of papers during the period 1970–1972 [1] (see also [2, 3] and references therein). Expressed as a function of $s = 2m\omega$, where m is the electron mass (which we shall set equal to unity) and ω is the photon energy in the laboratory reference frame, the differential cross-section with respect to the azimuthal angle, ϕ, between the photon

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polarization vector e and the plane containing the initial-photon and recoil-electron momenta, is given by

$$\frac{\mathrm{d}\sigma^{\mathrm{asym}}}{\mathrm{d}\phi} = \frac{\alpha^3}{m^2}\left[\frac{28}{9}L - \frac{218}{27} - P\left(\frac{4}{9}L - \frac{20}{27}\right)\right], \tag{1}$$

with

$$P = \xi_1 \sin(2\phi) + \xi_3 \cos(2\phi), \qquad L = \ln\frac{s}{m^2}.$$

Here ξ_1 and ξ_3 are the Stokes parameters describing the photon polarization, introduced through its spin-density matrix:

$$\rho_{ij} = \overline{\mathrm{e}_i \mathrm{e}_j} = \frac{1}{2}(1 + \boldsymbol{\sigma\xi})_{ij}.$$

In the derivation of (1) terms of order m^2/s were systematically neglected. The main contribution, $\sim \mathcal{O}(L)$, arises from configurations with small recoil momentum, $q = m\frac{2\cos\theta}{\sin^2\theta} \ll m$, where θ is the polar angle of the recoil electron (i.e., the angle between the initial photon and recoil-electron directions). However, the corresponding events presumably cannot be measured experimentally. For the region $q \sim m$ ($\theta \sim 50°$), the doubly differential cross-section was obtained in [2]:

$$2\pi\frac{\mathrm{d}^2\sigma^{\mathrm{asym}}}{\mathrm{d}q\,\mathrm{d}\phi} = \frac{2\alpha r_0^2}{3}\frac{q}{\varepsilon(\varepsilon-1)^2}\left[a_0 - b_0 P\right], \tag{2}$$

with

$$r_0 = \frac{\alpha}{m}, \qquad a_0 = 1 + \frac{2\varepsilon - 3}{q}\ln(q+\varepsilon), \qquad b_0 = 1 - \frac{1}{q}\ln(q+\varepsilon),$$

where $\varepsilon = \sqrt{q^2+1}$. The comparatively large magnitude of the azimuthal asymmetry

$$\mathcal{A} = \frac{b_0}{a_0} = \frac{1}{7} - \frac{1}{245}q^2 + \frac{51}{34300}q^4 + O(q^6) \sim 14\% \tag{3}$$

is, in fact, the reason this process is used for the polarimetry of linearly polarized photons [3, 4].

The aim of the present paper is to calculate the power corrections of order $1/s$ to the asymptotic expression for the asymmetry. The calculation of radiative corrections to the asymptotic expression for the asymmetry is a rather difficult problem, which we shall not touch here. A rough estimate gives $\Delta\mathcal{A}^{\mathrm{rad}} \sim \frac{\alpha}{\pi}L \sim 2 - 3\%$.

The differential cross-section of electron-positron pair photoproduction off a free electron in the Born approximation is described by eight Feynman diagrams. It was calculated numerically in particular by K. Mork [5]. The closed expression for the unpolarized case is very cumbersome and was first obtained in a complete form by E. Haug during the period 1975–1985 [6]. To the best of our knowledge, the exact analytical expression for the differential cross-section in the case of a polarized photon has not yet been published. Special attention has been paid to the so-called Bethe-Heitler (BH) subset of Feynman diagrams, whose contribution does not vanish in the high-energy limit, $s \to \infty$ [7]. The power corrections to this contribution to the total cross-section, behaving as L^3/s [8], indicate the need for the exact expression.

A detailed analysis of the expressions of Haug's work reveals that the interference terms of the BH matrix elements with the other three gauge-invariant subsets (which take into account the bremsstrahlung mechanism of pair creation and Fermi statistics for fermions) turn out to be of the order of some percent for $s > 50 - 60m^2$. On the other hand, the difference between the asymptotic and the exact expression is still found to be of the order of several percent for $s > 3000m^2$, i.e., very far above threshold, rendering the asymptotic expression useless in the energy range of interest. Arguments of positivity of the cross-section provide the relevant upper bound for the polarized part of the differential cross-section.

In Ref. [9] a Monte Carlo simulation of the process under consideration was performed using the HELAS code, in which all eight lowest order diagrams can be numerically treated without approximation. There it was shown that one might consider only the two leading graphs in a wide range of photon energies from 50 to 550 MeV. Note that this observation was made earlier for the unpolarized case by Haug [6] (who presented his results in explicit analytical form).

Our paper is organized as follows. After introducing the problem, in section 2 we analyze the kinematics of the process and give a general expression for the differential cross-section taking into account only leading and non-leading ($\sim 1/s$) contributions. Section 3 is devoted to the derivation of the differential cross-section with respect to the azimuthal angle and the recoil momentum of the electron. In the concluding section we present the correction to the cross-section and asymmetry, together with some numerical estimates. Some details of the calculation may be found in the Appendix.

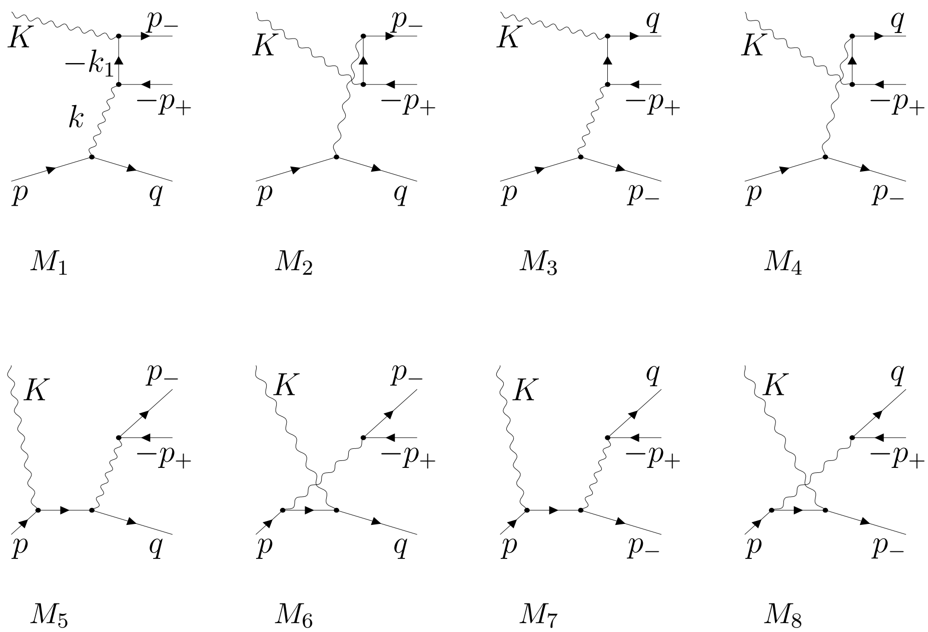

Figure 1: The Feynman diagrams contributing to triplet production

2 Kinematics and differential cross-section

In the Born approximation, the cross-section for the process of pair production off an electron,

$$\gamma(K,\mathrm{e}) + e(p) \to e(q) + e(p_-) + e(p_+), \tag{4}$$

with

$$q = K - k_1, \qquad p_- = p - k, \qquad p_+ = k_1 + k,$$

is described by eight Feynman diagrams (Fig. 1), which can be combined into four gauge-invariant subsets. Bearing in mind the desired application to the case of high photon energies, $\omega \gg m$, we shall present the total differential cross-section with leading terms (non-vanishing in the limit $s = 2m\omega \to \infty$) and terms of order $1/s$ (non-leading contributions). The first arise from the BH subset, denoted by the indices (12), whereas the non-leading terms come from interference of the BH amplitude with the sets denoted (34), (56) and (78), as well as from the BH amplitude itself.

We use the following Sudakov decomposition of the momenta in our problem:

$$k = \alpha_k p' + \beta_k K + k_\perp, \qquad k_1 = \alpha_1 p' + xK + k_{1\perp},$$

$$p' = p - K\frac{m^2}{s}, \qquad s = 2pK, \tag{5}$$

with the properties that

$$p'^2 = K^2 = 0, \qquad 2p'p = 1, \qquad k_\perp p = k_\perp K = 0.$$

The energy-momentum fractions of the created pair are x and $1-x$ while $-\vec{k}_1$ and $\vec{k}-\vec{k}_1$ are their momentum components transverse to the photon beam axis. The three mass-shell conditions,

$$\begin{aligned}
(K-k_1)^2 - 1 &= -s\alpha_1(1-x) - A = 0, \qquad A = \vec{k}_1^2 + 1, \\
(k_1+k)^2 - 1 &= s(\alpha_k+\alpha_1)(\beta_k+x) - B = 0, \qquad B = (\vec{k}+\vec{k}_1)^2 + 1, \\
(p-k)^2 - 1 &= -s\beta_k(1-\alpha_k) - \vec{k}^2 - \alpha_k = 0,
\end{aligned} \tag{6}$$

permit elimination of the following three Sudakov parameters:

$$\beta_k = -\frac{\vec{k}^2}{s}, \qquad \alpha_k = \frac{1}{s}(s_1 + \vec{k}^2), \qquad \alpha_1 = -\frac{A}{s(1-x)}. \tag{7}$$

Henceforth, we neglect terms contributing to the cross-section at order $\sim 1/s^2$. Keeping in mind the application to experiment, we shall assume

$$\vec{k}^2 \sim 1. \tag{8}$$

In this context, it should be noted that the above expressions for α_k and β_k are also valid only to order $\mathcal{O}(s^{-2})$. Here $s_1 = (k+K)^2$ denotes the invariant mass squared of the created pair and $\vec{k}^2$, the recoil-electron four-momentum transfer squared,

$$s_1 = \frac{1}{(x+\beta_k)(1-x)}\left[\left(\vec{k}_1 + (1-x)\vec{k}\right)^2 + 1\right], \qquad k^2 = -\frac{\vec{k}^2}{1-\alpha_k}. \tag{9}$$

The recoil-electron 3-momentum, q, is related to its component transverse to the photon beam axis, $|\vec{k}|$, as follows:

$$q^2 = \vec{k}^2 + \frac{1}{4}\vec{k}^4, \quad \vec{k}^2 = q^2\sin^2\theta = 2(\varepsilon - 1). \tag{10}$$

The final-state phase volume may be expressed as

$$\begin{aligned}
\mathrm{d}\Gamma &= \mathrm{d}^4k\,\mathrm{d}^4k_1\,\delta((K-k_1)^2-1)\,\delta((k+k_1)^2-1)\,\delta((p-k)^2-1) \\
&= \frac{\mathrm{d}^2k\,\mathrm{d}^2k_1\,\mathrm{d}x}{4s(1-\alpha_k)(x+\beta_k)(1-x)} \\
&= \frac{\mathrm{d}^2k\,\mathrm{d}^2k_1\,\mathrm{d}x}{4sx(1-x)}\left(1+\alpha_k-\frac{\beta_k}{x}\right).
\end{aligned} \tag{11}$$

In terms of these variables, the total differential cross-section may be written in the form:

$$
\begin{aligned}
\mathrm{d}\sigma &= \frac{\alpha^3}{\pi^2(\vec{k}^2)^2}\Big\{a^0_{1212} + \frac{1}{s}\Big[a^0_{1212}\Big(-s\alpha_k - s\frac{\beta_k}{x}\Big) + a^1_{1212} \\
&\qquad - \frac{2\vec{k}^2}{1-x}a_{1234} + \frac{2\vec{k}^2}{x+\beta_k}a_{1278} - \frac{2\vec{k}^2}{s_1}a_{1256}\Big]\Big\}\,\mathrm{d}^2k_1\,\mathrm{d}x\,\mathrm{d}^2k,
\end{aligned} \tag{12}
$$

with

$$
\begin{aligned}
a^0_{1212} &= \frac{\vec{k}^2}{AB} - 4x(1-x)\frac{R_{11}(B-A)^2}{A^2B^2} + 8x(1-x)\frac{R_1(B-A)}{AB^2} \\
&\quad - 4x(1-x)\frac{R}{B^2}, \\
a^1_{1212} &= \frac{\vec{k}^2}{(x+\beta_k)}\Big(\frac{B-A}{AB} - \frac{\vec{k}^2}{AB}\Big) \\
&\quad + 4\vec{k}^2\Big((3x-2)\frac{R_{11}(B-A)}{AB^2} + (1-x)\frac{R_{11}(B-A)}{A^2B}\Big) \\
&\quad - 4(3x-2)R\frac{\vec{k}^2}{B^2} - 4\vec{k}^2\Big((6x-4)\frac{R_1}{B^2} + (3-4x)\frac{R_1}{AB}\Big), \\
a_{1234} &= \frac{1}{4}\Big(\frac{B-A}{AB} - \frac{\vec{k}^2}{AB}\Big) - 2x(1-x)\frac{R_{11}(B-A)}{AB^2} + 2x(1-x)\frac{R}{B^2} \\
&\quad - 2x(1-x)\Big(\frac{R_1(B-A)}{AB^2} - \frac{R_1}{B^2}\Big), \\
a_{1256} &= \frac{1}{2(x+\beta_k)(1-x)}\Big(x\frac{A}{B} + (1-2x) - x\frac{\vec{k}^2}{B} - (1-x)\Big(\frac{B}{A} - \frac{\vec{k}^2}{A}\Big)\Big) \\
&\quad + 4\frac{R_{11}(B-A)}{AB} - 4(1-x)\frac{R}{B} + 4(1-x)\frac{R_1}{A} - 4(2-x)\frac{R_1}{B}, \\
a_{1278} &= -\frac{1}{4}\Big(\frac{B-A}{AB} + \frac{\vec{k}^2}{AB}\Big) + 2x(1-x)\frac{R_{11}(B-A)}{A^2B} \\
&\quad - 2x(1-x)\frac{R_1}{AB}
\end{aligned} \tag{13}
$$

and

$$
R_{11} = \mathrm{e}k_1\,\mathrm{e}^*k_1, \quad R_1 = \frac{1}{2}(\mathrm{e}k_1\,\mathrm{e}^*k + \mathrm{e}k\,\mathrm{e}^*k_1), \quad R = \mathrm{e}k\,\mathrm{e}^*k = \frac{\vec{k}^2}{2}(1+P).
$$

In general, the limits of variation for the parameters of the created pair are imposed by experimental cuts together with the following relations

$$
\begin{aligned}
(K-k_1)_0 = w(1-x) > m, &\qquad (k_1+k)_0 = (x+\beta_k)\omega > m, \\
s_1 < s, &\qquad s = 2\omega
\end{aligned}
$$

or

$$\epsilon = \frac{2m^2}{s} < (x + \beta_k, 1 - x), \qquad 0 < \vec{k}_1^2 < s(x + \beta_k)(1 - x) = \Lambda.$$

3 The inclusive distribution of the recoil electron

The leading and non-leading contributions to the inclusive cross-section in recoil-electron momentum may be organized as

$$2\pi \frac{\mathrm{d}\sigma}{\mathrm{d}q\,\mathrm{d}\phi} = \frac{q}{\varepsilon(\varepsilon - 1)^2} \alpha r_0^2 (I^{\ell}_{1212} + I^n), \qquad r_0 = \frac{\alpha}{m},$$

where I^{ℓ}_{1212} comes from the first term in Eq. (12) and I^n corresponds to the second. The leading and associated non-leading contributions may be decomposed as

$$\begin{aligned} I^{\ell}_{1212} &= \int\limits_{\epsilon}^{1-\epsilon} \mathrm{d}x \int\limits_{\vec{k}_1^2 < \Lambda} \frac{\mathrm{d}^2 k_1}{\pi} a^0_{1212} \qquad (14) \\ &= \int\limits_{\epsilon}^{1-\epsilon} \mathrm{d}x \Big\{ \int\limits_{0 < \vec{k}_1^2 < \infty} - \int\limits_{\Lambda < \vec{k}_1^2 < \infty} \Big\} \frac{\mathrm{d}^2 k_1}{\pi} a^0_{1212} = I^{\ell_1}_{1212} + I^{\ell_2}_{1212}. \end{aligned}$$

Using the table of integrals provided in the Appendix, for the first term in brackets we obtain

$$\begin{aligned} I^{\ell_1}_{1212} &= \int\limits_0^1 \mathrm{d}z \int\limits_{\epsilon - \beta_k}^{1-\epsilon} \mathrm{d}x \Big\{ \frac{\vec{k}^2}{\gamma} + 8x(1 - x) \Big[1 - \frac{4 + \vec{k}^2}{4\gamma} - R\frac{z(1 - z)}{\gamma} \Big] \Big\} \\ &= \int\limits_0^1 \mathrm{d}z \Big\{ \frac{\vec{k}^2}{\gamma}(1 - 2\epsilon + \beta_k) + \frac{4}{3} \Big[1 - \frac{4 + \vec{k}^2}{4\gamma} - R\frac{z(1 - z)}{\gamma} \Big] \Big\}, \qquad (15) \end{aligned}$$

where $\gamma = 1 + z(1 - z)\vec{k}^2$. For $\epsilon = 0$, we reproduce the result given in Eq. (2). In order to see this, one may use the expansion

$$\int\limits_0^1 \frac{\mathrm{d}z}{\gamma} = 1 - \frac{1}{6}\vec{k}^2 + \frac{1}{30}\vec{k}^4 + \dots$$

and the relation between $\vec{k}^2$ and q^2 given above in Eq. (10). The second term, $I^{\ell_2}_{1212}$, may be calculated using the expansion of a^0_{1212} for $\vec{k}_1^2 \gg 1$ (see

Eq. (30) in the Appendix)

$$I^{\ell_2}_{1212} = -\frac{2\vec{k}^2}{s}(L - \ln 2 - 1). \tag{16}$$

The quantity I^n may also be expressed as a sum: $I^n = I^c_{1212} + I^{\rm int}$. Consider now the contributions arising from corrections to the leading term (see Eq. (12)):

$$\begin{aligned}
I^c_{1212} &= \int\limits_\epsilon^{1-\epsilon} \mathrm{d}x \int\limits_{\vec{k}_1^2<\Lambda} \frac{\mathrm{d}^2 k_1}{\pi}\, a^0_{1212}\Big(-\alpha_k - \frac{\beta_k}{x}\Big) \\
&= \int\limits_\epsilon^{1-\epsilon} \mathrm{d}x \int \frac{\mathrm{d}^2 k_1}{\pi}\, a^0_{1212}\frac{(1-x)\vec{k}^2}{sx} \\
&\quad - \frac{1}{s}\int\limits_\epsilon^{1-\epsilon}\frac{\mathrm{d}x}{x(1-x)} \int\limits_{\vec{k}_1^2<\Lambda} \frac{\mathrm{d}^2 k_1}{\pi}\, a^0_{1212}\Big[1 + \Big(\vec{k}_1 + (1-x)\vec{k}\Big)^2\Big] \\
&= I^{c_1}_{1212} + I^{c_2}_{1212} + I^{c_3}_{1212}.
\end{aligned} \tag{17}$$

The first term on the RHS of Eq. (17) gives

$$I^{c_1}_{1212} = \frac{\vec{k}^2}{s}\int\limits_0^1 \mathrm{d}z\Big\{\frac{\vec{k}^2}{\gamma}(L - \ln 2 - 1) + \frac{8}{3}\Big[1 - \frac{4+\vec{k}^2}{4\gamma} - R\frac{z(1-z)}{\gamma}\Big]\Big\}.$$

It is convenient also to present the second term as a sum of two parts. The first, containing L^2, comes from the a^0_{1212} term, non-vanishing for both $x \to 0$ and $x \to 1$:

$$\begin{aligned}
I^{c_2}_{1212} &= -\frac{\vec{k}^2}{s}\int\limits_\epsilon^{1-\epsilon}\frac{\mathrm{d}x}{x(1-x)}\int\frac{\mathrm{d}^2 k_1}{\pi}\frac{xA + (1-x)B - x(1-x)\vec{k}^2}{AB} \\
&= \frac{\vec{k}^2}{s}\int\limits_0^1 \mathrm{d}z\frac{\vec{k}^2}{\gamma} - 2\frac{\vec{k}^2}{s}\Big[\frac{1}{2}(L^2 - \ln^2 2) - \frac{\pi^2}{6}\Big].
\end{aligned} \tag{18}$$

The remaining terms are

$$\begin{aligned}
I^{c_3}_{1212} &= -\frac{4}{s}\int\limits_\epsilon^{1-\epsilon}\mathrm{d}x \int_{\vec{k}_1^2<\Lambda}\frac{\mathrm{d}^2 k_1}{\pi}\Big[xA + (1-x)B - x(1-x)\vec{k}^2\Big] \\
&\qquad \Big[-R_{11}\Big(\frac{1}{A} - \frac{1}{B}\Big)^2 + 2R_1\Big(\frac{1}{AB} - \frac{1}{B^2}\Big) - \frac{R}{B^2}\Big] = r_1 + r_2.
\end{aligned}$$

The last term in the first brackets, which gives rise to r_2, is ultraviolet convergent upon integration over $\vec{k}_1$,

$$r_2 = -\frac{4\vec{k}^2}{3s}\int_0^1 \mathrm{d}z\Big[-1 + \frac{4+\vec{k}^2}{4\gamma} + \frac{z(1-z)}{\gamma}R\Big], \tag{19}$$

whereas the other is

$$r_1 = \frac{2}{s}\int_0^1 \mathrm{d}z\Big\{\Big(L - \frac{7}{2}\Big)\vec{k}^2 + R\Big\}. \tag{20}$$

The non-leading term, a^1_{1212}, along with the interference contribution (see Eq. (12)) to the inclusive cross-section yields

$$I^{\rm int} = \frac{\vec{k}^2}{s}\int_0^1 \mathrm{d}z\Big\{-\frac{\vec{k}^2}{\gamma}(L - \ln 2) - 4 + \frac{4+\vec{k}^2}{\gamma} + \frac{4z(1-z)}{\gamma}R\Big\}. \tag{21}$$

The contribution of the structure a_{1256} is exactly zero to order $1/s$. To make this clear, one can bring it into the following form,

$$\begin{aligned} a_{1256} \;=\; & \frac{1}{2x(1-x)A}\Big\{(1-x)(\vec{k}^2 + A - B) \\ & + 8x(1-x)(R_{11} + (1-x)R_1)\Big\} - \left(\begin{array}{c} x \leftrightarrow 1-x \\ \vec{k}_1 \to \vec{k}_1 + \vec{k} \end{array}\right), \end{aligned}$$

which is antisymmetric with respect to interchange of x and $1-x$.

4 Conclusions

Our result for the order $1/s$ correction to the inclusive cross-section is the sum of the expressions for I^{ℓ_1,ℓ_2}_{1212}, I^{c_1,c_2,c_3}_{1212} and I^n given in Eqs. (15–21). Organizing the inclusive cross-section in the form

$$2\pi\frac{\mathrm{d}\sigma}{\mathrm{d}q\,\mathrm{d}\phi} = \frac{2q}{3\varepsilon(\varepsilon-1)^2}\alpha r_0^2\Big[a_0 + \frac{2(\varepsilon-1)}{s}a_1 - P\Big(b_0 + \frac{2(\varepsilon-1)}{s}b_1\Big)\Big],$$

where a_0 and b_0 are given above in Eq. (2) and

$$a_1 = \frac{3}{2}\Big[-L^2 + C - 2L_q(\varepsilon+1)\Big], \qquad b_1 = -\frac{3}{2},$$
$$C = \ln^2 2 + 2\ln 2 + \frac{\pi^2}{3} - 4 \approx 1.1566, \qquad L_q = \frac{\ln(\varepsilon+q)}{q},$$

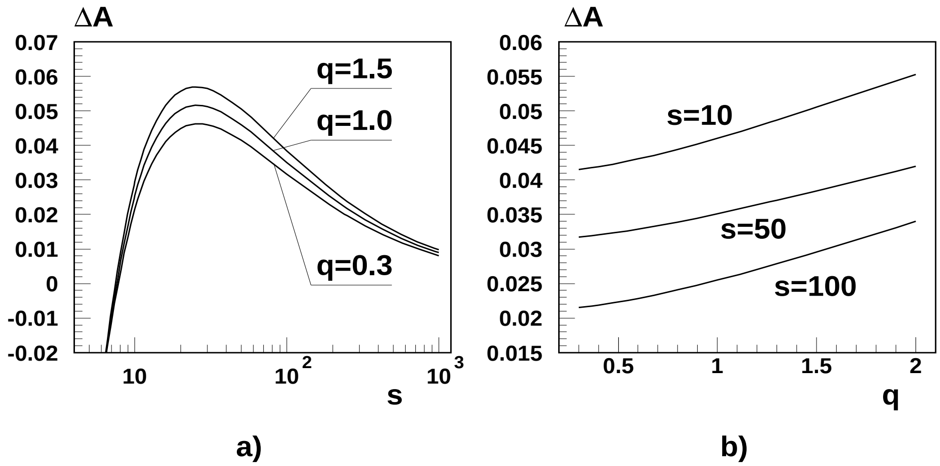

Figure 2: The dependence of the correction, $\Delta\mathcal{A}$, to the asymmetry as a function of a) s and b) q.

we extract the asymmetry,

$$\mathcal{A} = \mathcal{A} + \Delta\mathcal{A} = \frac{b_0}{a_0} + \frac{\vec{k}^2}{s}\frac{b_1 a_0 - a_1 b_0}{a_0^2}. \tag{22}$$

The expansion of $\Delta\mathcal{A}$ can be recast in the form (and this is our final result),

$$\Delta\mathcal{A} = \frac{3(\varepsilon - 1)}{s a_0^2}\Big[L^2 - C - 1 + L_q(5 - L^2 + C) - 2L_q^2(\varepsilon + 1)\Big]. \tag{23}$$

The the dependence of this quantity on q at fixed s and vice versa is shown in Figs. 2a and b (recall that we have set $m = 1$).

For small enough $s \lesssim 10$ the terms of order of $1/s^n$, for $n \geq 2$, become essential and the approach presented in this paper is not applicable.

We should like to point out that our results are in qualitative agreement with those obtained using the HELAS code [9] and reported in the talk delivered at the workshop [4].

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Appendix

Since the amplitudes of the gauge-invariant sets of diagrams (3,4), (5,6) and (7,8) are suppressed by at least one power of $\vec{k}^2/s$ as compared to amplitude (1,2), we need only consider interference terms. Thus, for the modulus of the matrix element, squared and summed over fermion spin states, we have within leading ($\sim s^2$) and non-leading ($\sim s$) accuracy, in order

$$\sum |M|^2 = \frac{(4\pi\alpha)^3}{\vec{k}^2}\left\{-\frac{2T_{1234}}{s(1-x)} - \frac{2T_{1256}}{s_1} + \frac{2T_{1278}}{sx} + \frac{T_{1212}(1-\alpha_k)^2}{\vec{k}^2}\right\}, \quad (24)$$

where

$$\begin{aligned}
T_{1212} &= \mathrm{Tr}\left\{(\not{q}+1)\gamma_\mu(\not{p}+1)\gamma_\nu\right\} \\
&\quad \times \mathrm{Tr}\left\{(\not{K}-\not{k}_1+1)O_{12}^{\mu\lambda}(\not{k}_1+\not{k}-1)\overline{O}_{34}^{\nu\sigma}\right\} e_\lambda e_\sigma^*, \\
T_{1234} &= \mathrm{Tr}\left\{(\not{q}+1)\gamma_\mu(\not{p}+1)\gamma_\nu(\not{K}-\not{k}_1+1)O_{12}^{\mu\lambda}(\not{k}_1+\not{k}-1)\overline{O}_{34}^{\nu\sigma}\right\} e_\lambda e_\sigma^*, \\
T_{1256} &= \mathrm{Tr}\left\{(\not{q}+1)\gamma_\mu(\not{p}+1)\overline{O}_{56}^{\nu\sigma}\right\} \qquad (25)\\
&\quad \times \mathrm{Tr}\left\{(\not{K}-\not{k}_1+1)O_{12}^{\mu\lambda}(\not{k}_1+\not{k}-1)\gamma_\nu\right\} e_\lambda e_\sigma^*, \\
T_{1278} &= \mathrm{Tr}\left\{(\not{q}+1)\gamma_\mu(\not{p}+1)\overline{O}_{78}^{\nu\sigma}(\not{K}-\not{k}_1+1)O_{12}^{\mu\lambda}(\not{k}_1+\not{k}-1)\gamma_\nu\right\} e_\lambda e_\sigma^*, \\
O_{12}^{\mu\lambda} &= -\frac{x+\beta_k}{B}\gamma_\mu(\not{K}-\not{k}_1-\not{k}+1)\gamma_\lambda - \frac{1-x}{A}\gamma_\lambda(-\not{k}_1+1)\gamma_\mu, \\
O_{34}^{\mu\lambda} &= -\frac{x}{B}\gamma_\mu(\not{K}-\not{k}_1-\not{k}+1)\gamma_\lambda - \frac{1}{s}\gamma_\lambda(\not{p}-\not{K}-\not{k}+1)\gamma_\mu, \\
O_{56}^{\mu\lambda} &= \frac{1}{s}\left[-\gamma_\lambda(\not{p}-\not{K}-\not{k}+1)\gamma_\mu + \gamma_\mu(\not{p}+\not{K}+1)\gamma_\lambda\right], \\
O_{78}^{\mu\lambda} &= -\frac{1-x}{A}\gamma_\lambda(-\not{k}_1+1)\gamma_\mu + \frac{1}{s}\gamma_\mu(\not{p}+\not{K}+1)\gamma_\lambda,
\end{aligned}$$

and $q = p - k$.

The quantities T_{ijkl} are related to the a_{ijkl} given in Eq. (13) by

$$\begin{aligned}
T_{1212} &= 16sx(1-x)\,[a^0_{1212}s + a^1_{1212}], \\
T_{1234,1278} &= 16s^2x(1-x)\,a_{1234,1278}, \\
T_{1256} &= 16sx(1-x)\,a_{1256}. \qquad (26)
\end{aligned}$$

To perform the integration over $\vec{k}_1$, we introduce an ultraviolet cut-off $\vec{k}_1^2 < \Lambda$, which may be omitted when calculating convergent integrals for the corrections. The integrals containing A or B are (hereinafter we omit the terms of order of $1/s$)

$$\int \frac{d^2\vec{k}_1}{\pi}\frac{1}{A^2} = \int \frac{d^2\vec{k}_1}{\pi}\frac{1}{B^2} = 1, \qquad \int \frac{d^2\vec{k}_1}{\pi}\frac{R_1}{B^2} = -R, \tag{27}$$
$$\int \frac{d^2\vec{k}_1}{\pi}\frac{R_{11}}{A^2} = \frac{1}{2}(\ln\Lambda - 1), \qquad \int \frac{d^2\vec{k}_1}{\pi}\frac{R_{11}}{B^2} = \frac{1}{2}(\ln\Lambda - 1) + R.$$

In order to avoid linearly divergent integrals, we combine denominators as follows:

$$\frac{1}{AB} = \int\limits_0^1 \frac{dz}{[(\vec{k}_1 + z\vec{k})^2 + \gamma]^2},$$

with $\gamma = 1 + \vec{k}^2 z(1-z)$. Integrating over $\vec{k}_1$ we obtain

$$\begin{aligned}
\int \frac{d^2\vec{k}_1}{\pi}\frac{1}{AB} &= \int\limits_0^1 \frac{dz}{\gamma} = \frac{1}{q}\ln(\varepsilon + q) = L_q, \\
\int \frac{d^2\vec{k}_1}{\pi}\frac{R_1}{AB} &= -\int\limits_0^1 \frac{z dz}{\gamma}R = -R\int\limits_0^1 \frac{dz}{2\gamma}, \\
\int \frac{d^2\vec{k}_1}{\pi}\frac{R_{11}}{AB} &= \int\limits_0^1 dz\Big[\frac{1}{2}(\ln\Lambda - 1) - \frac{1}{2}\ln\gamma + \frac{z^2}{\gamma}R\Big].
\end{aligned} \tag{28}$$

To evaluate the quantity r_1, we use the following set of integrals:

$$\begin{aligned}
\int \frac{d^2\vec{k}_1}{\pi}[xA + (1-x)B]\frac{R}{B^2} &= R\left(\ln\Lambda + x\vec{k}^2\right), \\
\int \frac{d^2\vec{k}_1}{\pi}[xA + (1-x)B]R_1\frac{A-B}{AB^2} &= -R\left(\ln\Lambda - 1 + x\vec{k}^2\right), \\
\int \frac{d^2\vec{k}_1}{\pi}[xA + (1-x)B]R_{11}\frac{A-B}{AB} &= \left[R + \frac{\vec{k}^2}{2}\right]\left[\ln\Lambda - \frac{3}{2}\right] + x\vec{k}^2 R.
\end{aligned} \tag{29}$$

In deriving (16), for a^0_{1212} averaged over angles with $\vec{k}_1^2 \gg 1$, we make use of

$$\overline{a^0_{1212}}|_{\vec{k}_1^2 \gg 1} \approx \frac{\vec{k}^2}{(\vec{k}_1^2)^2}\left(1 - 2x(1-x)\right) \tag{30}$$

and to perform the angular averaging in r_1 we use

$$\overline{R_{11}(\vec{k}\vec{k}_1)^2} \to \frac{1}{8}(\vec{k}_1^2)^2[\vec{k}^2 + 2R].$$

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