# Observation of the $\phi(1680)$ and the $Y(2175)$ in $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-}$ 

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#### Abstract

The cross sections for $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-}$and $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ are measured from threshold to $\sqrt{s}=3.0 \mathrm{GeV}$ using initial state radiation. The analysis is based on a data sample of $673 \mathrm{fb}^{-1}$ collected on and below the $\Upsilon(4 S)$ resonance with the Belle detector at the KEKB asymmetricenergy $e^{+} e^{-}$collider. First measurements are reported for the resonance parameters of the $\phi(1680)$ in the $\phi \pi^{+} \pi^{-}$mode: $m=(1689 \pm 7 \pm 10) \mathrm{MeV} / c^{2}$ and $\Gamma=(211 \pm 14 \pm 19) \mathrm{MeV} / c^{2}$. A structure at $\sqrt{s}=2.1 \mathrm{GeV} / c^{2}$, corresponding to the so called $Y(2175)$, is observed; its mass and width are determined to be $2079 \pm 13_{-28}^{+79} \mathrm{MeV} / c^{2}$ and $192 \pm 23_{-61}^{+25} \mathrm{MeV} / c^{2}$, respectively.


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Although vector mesons are produced copiously in $e^{+} e^{-}$annihilation, the resonance parameters of excited vector states are not well measured [1]. In the $s \bar{s}$ sector, the $\phi(1680)$ was first observed by the DM1 Collaboration in the reaction $e^{+} e^{-} \rightarrow K \bar{K} \pi[2]$ and was recently studied in the initial-state radiation (ISR) events of the $e^{+} e^{-} \rightarrow \phi \eta$ and $K^{*}(892) \bar{K}+$ c.c. modes by the BaBar Collaboration, which measured its mass and width to be $1723 \pm 20 \mathrm{MeV} / c^{2}$ and $371 \pm 75 \mathrm{MeV} / c^{2}$, respectively [3]. In a study of ISR events of the type, $e^{+} e^{-} \rightarrow \gamma_{\mathrm{ISR}} \phi \pi^{+} \pi^{-}$, the BaBar Collaboration observed two clear structures near $\sqrt{s}=1.68 \mathrm{GeV} / c^{2}$ and $2.175 \mathrm{GeV} / c^{2}$, the latter was produced dominantly via a $\phi f_{0}(980)$ intermediate state, and was dubbed the $Y(2175)$ [4]. The BES Collaboration confirmed the $Y(2175)$ in the $\phi f_{0}(980)$ invariant mass spectrum of $J / \psi \rightarrow \eta \phi f_{0}(980)\left(\eta \rightarrow \gamma \gamma, \phi \rightarrow K^{+} K^{-}\right.$ and $f_{0}(980) \rightarrow \pi^{+} \pi^{-}$) decays with a statistical significance of about $5 \sigma$ [5]. The Particle Data Group (PDG) assigns all these observations to a new state referred to as the $\phi(2170)$ [1].

Since the $Y(2175)$ resonance is produced via ISR in $e^{+} e^{-}$collisions, its $J^{P C}=1^{--}$. This observation stimulated the theoretical speculation that $Y(2175)$ may be an $s$-quark counterpart of the $Y(4260)$ [6, 7] since both are produced in $e^{+} e^{-}$annihilation and exhibit similar decay patterns. On the other hand, a number of different interpretations have been proposed for the $Y(2175)$ with predicted masses that are consistent, within errors, with the experimental measurements. These include: an $s \bar{s} g$ hybrid [8]; a $2^{3} D_{1} s \bar{s}$ state [9] with a width predicted to be in the range $120-210 \mathrm{MeV} / c^{2}$; a tetraquark state [10, 11] ; a $\Lambda \bar{\Lambda}$ bound state [12]; or an ordinary $\phi f_{0}(980)$ resonance produced by interactions between the final state particles [13]. The possibility that the $Y(2175)$ is a $3^{3} S_{1} s \bar{s}$ state is disfavored by the rather large predicted width $\left(\Gamma \sim 380 \mathrm{MeV} / c^{2}\right)$ [14], which disagrees with the experimental upper limit of $100 \mathrm{MeV} / c^{2}$ [4]. A recent review [15] discusses the basic problem of the large expected decay widths into two mesons, which contrasts with experimental observations.

In the analysis reported here, we use a data sample with an integrated luminosity of $673 \mathrm{fb}^{-1}$ collected with the Belle detector [16] operating at the KEKB asymmetric-energy $e^{+} e^{-}(3.5$ on 8 GeV$)$ collider [17] to investigate the $\phi \pi^{+} \pi^{-}$final state via ISR. About $90 \%$ of data were collected at the $\Upsilon(4 S)$ resonance $(\sqrt{s}=10.58 \mathrm{GeV})$, and the rest were taken at a center-of-mass (C.M.) energy that is 60 MeV below the $\Upsilon(4 S)$ peak.

For Monte Carlo (MC) simulation of the ISR process, we generate signal events with the PHOKHARA program [18]. In this program, after one or two photons are emitted, the lower energy $e^{+} e^{-}$pair forms a resonance $X$ that subsequently decays to $\phi \pi^{+} \pi^{-}$or $\phi f_{0}(980)$ with the $\phi$ decaying into $K^{+} K^{-}$and the $f_{0}(980)$ decaying into $\pi^{+} \pi^{-}$. In the $X \rightarrow \phi \pi^{+} \pi^{-}$ generation, we assume that the $\pi \pi$ system is pure $S$-wave and that it and the $\phi$ are also in a relative $S$-wave. The $\pi^{+} \pi^{-}$invariant mass distributions are generated according to phase space.

To select the $\phi \pi^{+} \pi^{-}$final states, we use the following event selection criteria. We require four good charged tracks with net charge equal to zero. Good tracks should have impact parameters perpendicular to and along the beam direction with respect to the interaction point of less than 0.3 and 2 cm , respectively, and transverse momentum is restricted to be higher than $0.1 \mathrm{GeV} / c$. For each charged track, the particle identification information from different detector subsystems is combined to form a likelihood for each particle species $(i)$, $L_{i}$ [19]. Tracks with $\mathcal{R}_{K}=\frac{L_{K}}{L_{K}+L_{\pi}}<0.9$ are identified as pions, tracks with $\mathcal{R}_{K}>0.6$ are identified as kaons. We require that two tracks be identified as pions, and assign kaon masses to the other two tracks, of which only one is required to be identified as a kaon. The sum of the charged track energies and that of reconstructed photons (neutral clusters in the electromagnetic calorimeter with energies greater than 100 MeV and not associated with
charged tracks in the central drift chamber) must be greater than 9 GeV ; this ensures that an ISR photon is detected. The efficiency of this requirement is nearly $100 \%$ for $\phi \pi^{+} \pi^{-}$invariant masses below $3.0 \mathrm{GeV} / c^{2}$. We require $-1.0\left(\mathrm{GeV} / c^{2}\right)^{2}<M_{\text {rec }}^{2}<2.0\left(\mathrm{GeV} / c^{2}\right)^{2}$ to remove multi-photon backgrounds, where $M_{\text {rec }}^{2}$ is the square of the mass that is recoiling against the four charged tracks. The MC simulation indicates that the application of these requirements reduces the contamination from the background ISR process $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-} \pi^{0}(\gamma)$ [20] to the $1 \%$ level; the effect of the residual background is included as a systematic error.

In events with $K^{+} K^{-} \pi^{+} \pi^{-}$mass below $3.0 \mathrm{GeV} / c^{2}$, a $\phi$ meson signal is evident in the $K^{+} K^{-}$invariant mass distribution, as shown in Fig. 1. This distribution is fitted with a Breit-Wigner (BW) (the $\phi$ meson mass and width fixed to PDG values) convolved with a Gaussian resolution function as the $\phi$ signal and a second-order polynomial as the background shape; here the width of the Gaussian resolution function is fixed at $\sigma=0.95 \mathrm{MeV} / c^{2}$, its MC-determined value. The fit yields $4832 \pm 132 \phi \pi^{+} \pi^{-}$events.


FIG. 1: The $K^{+} K^{-}$invariant mass distribution for selected $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{+} \pi^{-}$candidates with $K^{+} K^{-} \pi^{+} \pi^{-}$invariant mass less than $3.0 \mathrm{GeV} / c^{2}$. The curves show the fit projections for the full fit and for the background component.

We obtain the number of $\phi \pi^{+} \pi^{-}$events in each $K^{+} K^{-} \pi^{+} \pi^{-}$invariant mass bin by fitting the $K^{+} K^{-}$invariant mass distribution for each interval, using the same fit model as in the fit for the overall sample. In order to reliably control the background shape, we restrict the coefficients of the background polynomials in nearby bins to vary smoothly along parabolas. The parameters of these parabolas are determined from fits to the coefficients obtained from fits to the $K^{+} K^{-}$invariant mass distribution in each $K^{+} K^{-} \pi^{+} \pi^{-}$invariant mass bin. The resulting $\phi \pi^{+} \pi^{-}$invariant mass distribution is shown in Fig. 2(a). Combinatorial background has a smooth distribution in the $K^{+} K^{-}$invariant mass distribution and does not affect the results since we fit the $K^{+} K^{-}$distribution to obtain the number of $\phi \pi^{+} \pi^{-}$events in each $K^{+} K^{-} \pi^{+} \pi^{-}$mass bin. In Fig. 2(a) there are two distinct peaks: one near $1.7 \mathrm{GeV} / c^{2}$ and another near $2.1 \mathrm{GeV} / c^{2}$, corresponding to the $\phi(1680)$ and $Y(2175)$ states. In addition, there is a cluster of events near $2.4 \mathrm{GeV} / c^{2}$.


FIG. 2: The $\phi \pi^{+} \pi^{-}$invariant mass distribution obtained from fitting to $m\left(K^{+} K^{-}\right)$in each $m\left(K^{+} K^{-} \pi^{+} \pi^{-}\right)$bin (a) as described in the text, and the measured $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-}$cross section (b). The errors are statistical only.

The $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-}$cross section for each $\phi \pi^{+} \pi^{-}$mass bin is computed using

$$
\sigma_{i}=\frac{n_{i}^{\mathrm{fit}}}{\varepsilon_{i} \mathcal{L}_{i} \mathcal{B}\left(\phi \rightarrow K^{+} K^{-}\right)}
$$

where $n_{i}^{\text {fit }}, \varepsilon_{i}$, and $\mathcal{L}_{i}$ are the number of $\phi \pi^{+} \pi^{-}$events fitted in data, the efficiency, and the effective luminosity [21] in the $i$-th $\phi \pi^{+} \pi^{-}$mass bin, respectively; $\mathcal{B}\left(\phi \rightarrow K^{+} K^{-}\right)=$ $49.2 \%$ [1]. According to the MC simulation, the efficiency increases smoothly from $1.72 \%$ to $2.67 \%$ for $\phi \pi^{+} \pi^{-}$masses ranging from threshold to $3.0 \mathrm{GeV} / c^{2}$. The resulting cross sections are shown in Fig. [2(b) [22].

We now consider the quasi-two-body intermediate state $\phi f_{0}(980)$. Figure 3(a) shows a scatter plot of $m\left(\phi \pi^{+} \pi^{-}\right)$versus $m\left(\pi^{+} \pi^{-}\right)$for events in the $\phi$ signal region ( $1.013 \mathrm{GeV} / c^{2}<$ $\left.m_{K^{+} K^{-}}<1.025 \mathrm{GeV} / c^{2}\right)$. Non- $\phi$ background is subtracted according to the normalized $\phi$ mass sidebands ( $m_{K^{+} K^{-}} \in[0.989,1.013] \mathrm{GeV} / c^{2}$ or $m_{K^{+} K^{-}} \in[1.025,1.049] \mathrm{GeV} / c^{2}$ ). It can be seen that within the $f_{0}(980)$ mass band there are clusters of events that correspond to $Y(2175) \rightarrow \phi f_{0}(980)$ and $J / \psi \rightarrow \phi f_{0}(980)$. Figure $3(\mathrm{~b})$ shows the projected $m\left(\pi^{+} \pi^{-}\right)$distribution, where a clear $f_{0}(980)$ signal is evident. For each $25 \mathrm{MeV} / c^{2}$ bin of $m\left(K^{+} K^{-} \pi^{+} \pi^{-}\right)$, we select events with $m\left(\pi^{+} \pi^{-}\right) \in[0.85,1.10] \mathrm{GeV} / c^{2}$ [23], and fit their $m\left(K^{+} K^{-}\right)$distribution to extract the number of $\phi f_{0}(980)$ events in a way similar to that used to extract the number of $\phi \pi^{+} \pi^{-}$events described above. The resulting $\phi f_{0}(980)$ invariant mass distribution is shown in Fig. [4(a). A very clear $Y(2175)$ signal is observed with an accumulation of events around $2.4 \mathrm{GeV} / c^{2}$. The $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ cross section for each $\phi f_{0}(980)$ mass bin is computed in the same way as that for the $\phi \pi^{+} \pi^{-}$mode using $\mathcal{B}\left(f_{0}(980) \rightarrow \pi^{+} \pi^{-}\right)=\frac{2}{3}$. The efficiency is nearly independent of $\phi f_{0}(980)$ mass, $2.3 \%$ from threshold to $3.0 \mathrm{GeV} / c^{2}$. The resulting cross sections are shown in Fig. 4(b).

The sources of the systematic errors for the cross section measurements are summarized in Table I. The uncertainty is $1.0 \%$ for pion identification, and is negligible for kaons since the identification of only one of the kaons is required; the uncertainty in the tracking efficiency is $1 \%$ per track, and is additive; the efficiency uncertainty associated with the $M_{\text {rec }}^{2}$ requirement is determined to be $1.0 \%$ [7]; the uncertainties in the yields of $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ events for each mass bin due to the $\phi$ signal fit are estimated to be $5.0 \%$ and $4.0 \%$ as determined by: changing the $\phi$ mass resolution, the orders of the background polynomial, and the


FIG. 3: Scatter plot of $m\left(\phi \pi^{+} \pi^{-}\right)$versus $m\left(\pi^{+} \pi^{-}\right)$, where non- $\phi$ background is subtracted according to the normalized $\phi$ mass sidebands (a), and the projection on $\pi^{+} \pi^{-}$mass, a clear $f_{0}(980)$ signal is visible (b).


FIG. 4: The $\phi f_{0}(980)$ invariant mass distribution (a) obtained from fitting to $m\left(K^{+} K^{-}\right)$in each $m\left(K^{+} K^{-} f_{0}(980)\right)$ bin as described in the text, and (b) the measured $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ cross sections. The errors are statistical only.
fitting range. The error due to the uncertainty of the background parameters changes with $m_{\phi \pi^{+} \pi^{-}}\left(\mathrm{GeV} / c^{2}\right)$ because of signal-to-noise ratio variations between threshold and $3 \mathrm{GeV} / c^{2}$. The error is approximated as $\left(5.92-5.44 m_{\phi \pi^{+} \pi^{-}}+1.28 m_{\phi \pi^{+} \pi^{-}}^{2}\right) /\left(0.02+\left(m_{\phi \pi^{+} \pi^{-}}-1.9\right)^{2}\right) \%$, $m_{\phi \pi^{+} \pi^{-}}$in $\mathrm{GeV} / c^{2}$, which is about $4 \%$ in the $\phi(1680)$ and the $Y(2175)$ mass regions, and increases continuously to about $10 \%$ in between and decreases to about $1 \%$ above $2.3 \mathrm{GeV} / c^{2}$. This is common to the $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ modes. The $\phi \pi^{+} \pi^{-} \pi^{0}(\gamma)$ background further contributes a $1 \%$ uncertainty to the $\phi \pi^{+} \pi^{-}$, and a negligible fraction to the $\phi f_{0}(980)$ mode. Belle measures luminosity with a precision of $1.4 \%$ using wide angle Bhabha events. The uncertainty of the ISR photon radiator is $0.1 \%$ [21]. The main uncertainty in the PHOKHARA [18] generator is due to the modelling of the $\pi^{+} \pi^{-}$mass spectrum. This is tested by generating events with different $\pi^{+} \pi^{-}$mass distributions. We take $5 \%$ and $3 \%$ as conservative uncertainties related to the MC generators for the $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ modes, respectively. According to the MC simulation, the trigger efficiency for these events is around $(97 \pm 1) \%$ with little dependence on the $\phi \pi^{+} \pi^{-}$invariant mass. The uncertainty of $\mathcal{B}\left(\phi \rightarrow K^{+} K^{-}\right)$is $1.2 \%$ [1]. Finally the error due to MC statistics is $0.8 \%$. We assume all the sources are independent and add them (excluding that from background parameters) in
quadrature, resulting in the total systematic errors on the cross sections of $8.6 \%$ and $6.9 \%$ for $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ modes, respectively.

TABLE I: Systematic errors (\%) in the $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ cross sections measurements. The systematic error from the uncertainty of background parameters [(5.92-5.44m $m_{\phi \pi^{+} \pi^{-}}+$ $\left.1.28 m_{\phi \pi^{+} \pi^{-}}^{2}\right) /\left(0.02+\left(m_{\phi \pi^{+} \pi^{-}}-1.9\right)^{2}\right) \%, m_{\phi \pi^{+} \pi^{-}}$in $\left.\mathrm{GeV} / c^{2}\right]$ is not included.

| Source | $\phi \pi^{+} \pi^{-}$ | $\phi f_{0}(980)$ |
| :---: | :---: | :---: |
| Particle ID | 1.0 | 1.0 |
| Tracking | 4.0 | 4.0 |
| $M_{\text {rec }}^{2}$ selection | 1.0 | 1.0 |
| $\phi$ signal fit | 5.0 | 4.0 |
| $\phi \pi^{+} \pi^{-} \pi^{0}(\gamma)$ background | 1.0 | 0.0 |
| Integrated luminosity | 1.4 | 1.4 |
| Generator | 5.0 | 3.0 |
| Trigger efficiency | 1.0 | 1.0 |
| Branching fractions | 1.2 | 1.2 |
| MC statistics | 0.8 | 0.8 |
| Sum in quadrature | 8.6 | 6.9 |

There are strong $J / \psi$ signals in both $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ samples (see Fig. 5). $254 \pm 23$ $J / \psi \rightarrow \phi \pi^{+} \pi^{-}$and $60 \pm 11 J / \psi \rightarrow \phi f_{0}(980)$ events are obtained from fitting the $J / \psi$ signals and subtracting the peaking backgrounds by fitting the normalized $\phi$ sidebands events. We determine $\mathcal{B}\left(J / \psi \rightarrow \phi \pi^{+} \pi^{-}\right) \times \Gamma_{e^{+} e^{-}}=(4.50 \pm 0.41 \pm 0.26) \mathrm{eV} / c^{2}$ and $\mathcal{B}(J / \psi \rightarrow$ $\left.\phi f_{0}(980)\right) \times \Gamma_{e^{+} e^{-}}=(2.22 \pm 0.41 \pm 0.13) \mathrm{eV} / c^{2}$, where the first errors are statistical, and the second systematic. The sources of the systematic errors are almost the same as those listed in Table [ except that the uncertainty of the $\phi$ signal fit is replaced by that of the $J / \psi$ signal fit. Our results are consistent with the PDG values [1] and BaBar's measurements [4], but are more precise.


FIG. 5: The $J / \psi \rightarrow \phi \pi^{+} \pi^{-}$(a) and $J / \psi \rightarrow \phi f_{0}(980)$ (b) signals in the $\phi$ signals regions (open histograms) and in the $\phi$ sidebands (shaded histograms, normalized). The curves are the best fits to the signal and sidebands distributions, respectively.

In order to obtain the resonance parameters of the $\phi(1680)$ and $Y(2175)$, a least squares fit is applied to the $\phi \pi^{+} \pi^{-}$cross section distribution. Since the $\phi(1680)$ decays into $\phi \pi^{+} \pi^{-}$ while the $Y(2175)$ decays dominantly into $\phi f_{0}(980)$, we use two incoherent BW functions in the fit, one for the $\phi(1680)$ and the other for the $Y(2175)$. The fit result is shown in Fig. [6(a), with a goodness-of-the-fit of $\chi^{2} / n d f=68 / 55$, corresponding to a C.L. of $12 \%$. The statistical significance of each resonance is greater than $10 \sigma$. From the fit we obtain the following resonance parameters of the $\phi(1680)$ : $m=(1689 \pm 7) \mathrm{MeV} / c^{2}, \Gamma=$ $(211 \pm 14) \mathrm{MeV} / c^{2}$ and $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(1.24 \pm 0.09) \times 10^{-7}$, while those of the $Y(2175)$ are $m=(2079 \pm 13) \mathrm{MeV} / c^{2}, \Gamma=(192 \pm 23) \mathrm{MeV} / c^{2}$ and $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=$ $(1.10 \pm 0.10) \times 10^{-7}$, where the errors are statistical only. We also perform a fit with an additional incoherent BW function centered near $2.4 \mathrm{GeV} / c^{2}$. The fitted parameters of this structure are $m=(2406 \pm 32) \mathrm{MeV} / c^{2}$ and $\Gamma=(57 \pm 58) \mathrm{MeV} / c^{2}$ with a goodness-of-the-fit of $\chi^{2} / n d f=62 / 52$, corresponding to a C.L. of $15 \%$, where the errors are statistical only. The statistical significance of the structure at $2.4 \mathrm{GeV} / c^{2}$ is $1.5 \sigma$ as determined from the change in the $\chi^{2}$ value. A fit with an additional non-resonant component does not improve the fit quality, and the contribution of the non-resonant term is negligibly small.

We fit the $\phi f_{0}(980)$ cross section distribution with a single BW function that interferes with a non-resonant component which is partly from non- $\phi f_{0}(980)$ contribution, and partly from the possible $\phi(1680) \rightarrow \phi f_{0}(980)$ at the high mass tail of the $\phi(1680)$, as in BaBar's analysis [4]. There are two solutions with very similar resonance parameters. The interference is constructive for one solution and destructive for the other. Figure 6(b) shows the result. The fit yields $m=(2163 \pm 32) \mathrm{MeV} / c^{2}$ and $\Gamma=(125 \pm 40) \mathrm{MeV} / c^{2}$, with a goodness-of-the-fit of $\chi^{2} / n d f=49 / 36$, corresponding to a C.L. of $8.0 \%$. Here we quote simple averages of the two solutions, and enlarge the errors to cover the full uncertainties. We also perform a fit with an additional coherent BW function near $2.4 \mathrm{GeV} / c^{2}$. The fitted mass and width of the $Y(2175)$ are consistent with the results listed above, and the statistical significance of the structure at $2.4 \mathrm{GeV} / c^{2}$ is estimated to be $2.3 \sigma$.


FIG. 6: Fit to (a) $e^{+} e^{-} \rightarrow \phi \pi^{+} \pi^{-}$cross section with two incoherent BW functions, one for the $\phi(1680)$ and the other for the $Y(2175)$ and (b) $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ cross section with a single BW function that interferes with a non-resonant component. The curves show the projections from the best fit and the contribution from each component. In (b), the dashed curves are for the destructive interference solution and the dot-dashed curves for the constructive interference solution

Since the differences in the $Y(2175)$ resonance parameters from fits to $\phi \pi^{+} \pi^{-}$and
$\phi f_{0}(980)$ are due to the assumptions on the background shape and the existence of additional resonances, and $\phi f_{0}(980)$ events are a subsample of $\phi \pi^{+} \pi^{-}$events, we take the values of the $Y(2175)$ resonance parameters from the fit to the $\phi \pi^{+} \pi^{-}$cross section with two incoherent resonances as the central values and the differences are taken as one source of systematic errors, which is the dominant one for the $Y(2175)$ resonance parameters. For the $\phi(1680)$ and $Y(2175)$ resonance parameters, we have also considered the uncertainties in the absolute mass scale, the mass resolution, the parameterization of the resonances and background shape, fit range and possible existence of additional resonances as systematic errors. Finally, we obtain $m(\phi(1680))=(1689 \pm 7 \pm 10) \mathrm{MeV} / c^{2}, \Gamma(\phi(1680))=$ $(211 \pm 14 \pm 19) \mathrm{MeV} / c^{2}$, and $\mathcal{B}\left(\phi(1680) \rightarrow \phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(1.24 \pm 0.09 \pm 0.14) \times 10^{-7}$, and $m(Y(2175))=\left(2079 \pm 13_{-28}^{+79}\right) \mathrm{MeV} / c^{2}, \Gamma(Y(2175))=\left(192 \pm 23_{-61}^{+25}\right) \mathrm{MeV} / c^{2}$, and $\mathcal{B}\left(Y(2175) \rightarrow \phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(1.10 \pm 0.10 \pm 0.12) \times 10^{-7}$ [24], where the first errors are statistical, the second systematic.

In summary, we present the most precise measurements of the cross sections for $e^{+} e^{-} \rightarrow$ $\phi \pi^{+} \pi^{-}$and $e^{+} e^{-} \rightarrow \phi f_{0}(980)$ from threshold to $\sqrt{s}=3.0 \mathrm{GeV}$. The masses and widths of the $\phi(1680)$ and $Y(2175)$ are determined and are in agreement with the previous measurements [1, 2, 3, 4, 5]. The width of the $Y(2175)$ tends to be larger than in previous measurements [4, 5] although the error is large. We find that the widths of the $\phi(1680)$ and $Y(2175)$ are quite similar and both are at the $200 \mathrm{MeV} / c^{2}$ level. This may suggest that the $Y(2175)$ is an excited $1^{--} s \bar{s}$ state. Since the $f_{0}(980)$ is thought to have a large $s \bar{s}$ component, $Y(2175) \rightarrow \phi f_{0}(980)$ can be viewed as an open-flavor decay as opposed to the case of $Y(4260) \rightarrow J / \psi \pi^{+} \pi^{-}$, which is a hadronic transition. The study of the $Y(2175)$ in other decay modes would be useful for distinguishing between different possibilities. The branching fractions of $J / \psi$ decays into $\phi \pi^{+} \pi^{-}$and $\phi f_{0}(980)$ are measured with improved precision; the results are in good agreement with the existing results.

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[22] The cross section values obtained in this paper will be submitted to the Durham HEP database website: http://durpdg.dur.ac.uk.
[23] This neglects the interference between $\phi f_{0}(980)$ and other $\phi \pi^{+} \pi^{-}$intermediate states, and allows a fraction of non- $\phi f_{0}(980)$ events counted as $\phi f_{0}(980)$ signal. The fraction is at about the $10 \%$ level according to the fit to the $\pi^{+} \pi^{-}$invariant mass distribution, which is consistent with BaBar's estimate [4].
[24] We also fit with two coherent BW functions. There are two solutions with equally good fit quality (the goodness-of-the-fit is $\chi^{2} / n d f=64 / 54$ ). The masses and widths of the resonances are the same for both solutions. The partial widths to $e^{+} e^{-}$and the relative phases between them are different. The resonance parameters of the $\phi(1680)$ are $m=(1682 \pm 8 \pm 10) \mathrm{MeV} / c^{2}$, $\Gamma=(226 \pm 19 \pm 19) \mathrm{MeV} / c^{2}$, and $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(1.02 \pm 0.08 \pm 0.12) \times 10^{-7}$ for the constructive solution, $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(1.64 \pm 0.08 \pm 0.18) \times 10^{-7}$ for the destructive solution, and those of the $Y(2175)$ are $m=\left(2097 \pm 17_{-45}^{+72}\right) \mathrm{MeV} / c^{2}, \Gamma=\left(212 \pm 31_{-79}^{+22}\right) \mathrm{MeV} / c^{2}$, and $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times \mathcal{B}_{e^{+} e^{-}}=(0.36 \pm 0.07 \pm 0.03) \times 10^{-7}$ for the constructive solution, $\mathcal{B}\left(\phi \pi^{+} \pi^{-}\right) \times$ $\mathcal{B}_{e^{+} e^{-}}=(3.82 \pm 0.39 \pm 0.38) \times 10^{-7}$ for the destructive solution, where the first errors are statistical and the second systematic.

