Prospects in Lattice QCD (for Gluex)

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Overview of the talk

I focus on lattice results for the exotic 1^{-+} meson from lattice QCD, because there are/used to be experimental signals at 1.4, 1.6, and 1.9 GeV.

Plan of the talk.

- What questions am I trying to answer
- Brief introduction to lattice QCD.
- Masses and widths of exotic mesons.
- Open decay channels.

I am not going to talk about radiative transitions, two photon decays or photo-production, because this is a strength of the research program JLAB theory group. (I will discuss the issue of the 1^{-+} versus 4^{-+} state on the lattice). To focus the talk to Gluex interests I looked at presentation to PAC30, "Mapping the Spectrum of Light Quark Mesons and gluonic excitations with linearly polarized photons."

Pehaps I have been too focussed and you would like to learn about searching for glueball and tetraquark degress of freedom in the $a_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, and $f_0(600)$ mesons (see my review arXiv:0710.0985 [hep-lat] from lattice 2007).

From models to solving QCD

The quark model (as described in the review section of the PDG) claims that light quark spectroscopy is “explained” by

- $\overline{\psi}\Gamma\psi$ Mesons
- $\psi\psi\psi$ Baryons

In QCD there are additional possible colour singlet operators that may form bound states:

- Glueballs: lumps of glue.
- Hybrids mesons: quark and anti-quark with excited glue.
- Multiquark states.

Is there any evidence for the above states in nature? Can theory accurately compute non-perturbative results?

Some background to lattice QCD

To solve QCD for bound state properties "all" that is required is to compute

$$c_{ij}(t) = \frac{1}{Z} \int du \int d\psi \overline{d\psi} O(t)_i O(0)^\dagger_j e^{-S_F - S_G}$$

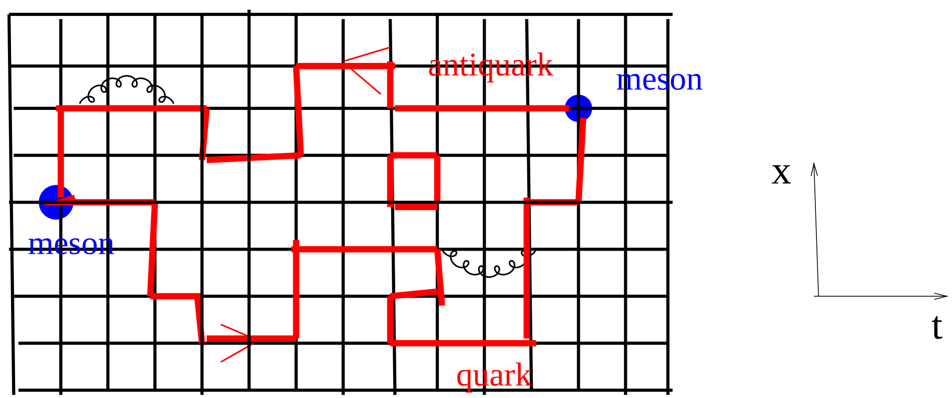
- The path integral is regulated by the introduction of a space-time lattice. Lattice calculations are now being done at several lattice spacings and volumes.
- The multi-dimensional integral is computed in Euclidean space using Monte Carlo techniques on the computer (similar $\int_{-\infty}^{-\infty} x^2 e^{-mx^2 - \lambda x^4} dx$).
- The dynamics of the gluon and quarks is in the Dirac action (S_F) and gauge action (S_G).

Pictures of lattice QCD (part of MC)

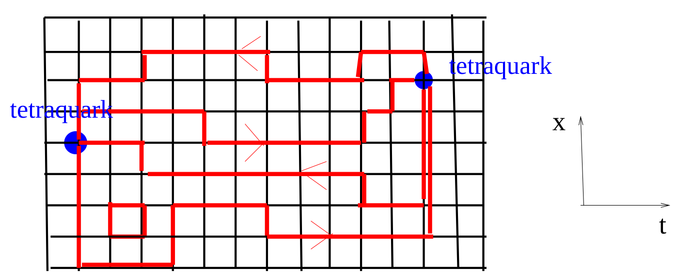
There is a physical picture (part of the Monte Carlo) of the process. For the meson interpolating operator

$$O(x,t) = \overline{\psi_1}(x,t)\psi_2(x,t)$$

where 1 and 2 are different flavours.



For tetraquark interpolating operator such as $\overline{q}\gamma_5 q\overline{q}\gamma_5 q$



A consumers view of lattice QCD

Now that quenched QCD is dead, we are solving full QCD, but need to control the parameters. For example, the European Twisted Mass Collaboration (ETMC) have reached the parameters below

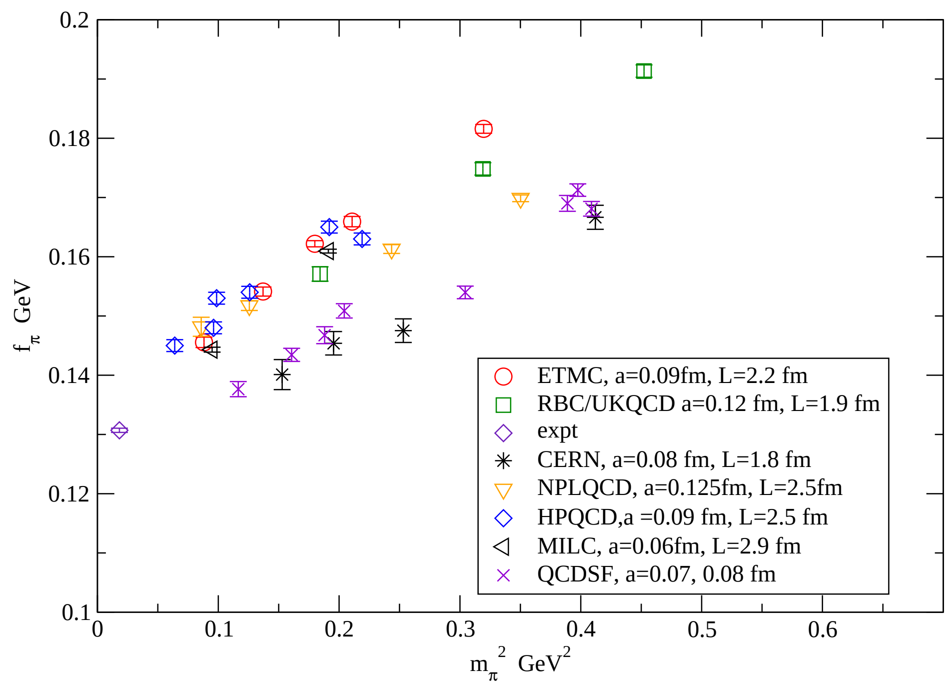
- Two flavours of sea quarks with range of pion masses 270 to 520 MeV. (2+1+1 flavours of quarks in the sea this year).
- Statistical errors under 1 % level for pseudo-scalar mesons.
- Two lattice spacings 0.07 and 0.09 fm.
- Finite volume effects are an issue. Volumes between 2.2 and 2.9 fm.

Other collaborations have reached similar parameters.

Validation, can you trust us?

Summary of the pion decay constant (f_π) as a function of square of the pion mass.

$$\Gamma(\pi^- \to e^- + \overline{\nu}_e) \propto \frac{G_F^2 \mid V_{ud} \mid^2 f_\pi^2}{m_\pi^3} m_l^2 (m_\pi^2 - m_l^2)^2$$



Exotic mesons

States with exotic quantum numbers J^{PC}= 1^{-+}, 0^{+-}, 2^{+-}, 0^{--} are definitely not described by the quark model (that predicts $C = (-1)^{l+s}$, $P = (-1)^{l+1}$).

- Hybrid exotic mesons with $\overline{q}q$ plus excited glue.
- Multiquark operators ($\overline{qq}qq$).

Interpolating operators used in lattice QCD calculations of hybrid meson spectroscopy.

1^{--}	$\overline{\psi}\gamma_j\psi$
1^{--}	$\epsilon_{ijk}\overline{\psi}\gamma_5 F_{ij}\psi$
1^{-+}	$\overline{\psi}\gamma_j F_{jk}\psi$
1^{-+}	$\overline{\psi}\gamma_j\gamma_5\psi$ $\overline{\psi}\gamma_5\psi$

where F is the QCD field strength tensor.

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Gregory (Scotland Yard detective): ‘‘Is there any other point to
   which you would wish to draw my attention?’’
Holmes: ‘‘To the curious incident of the dog in the night-time.’’
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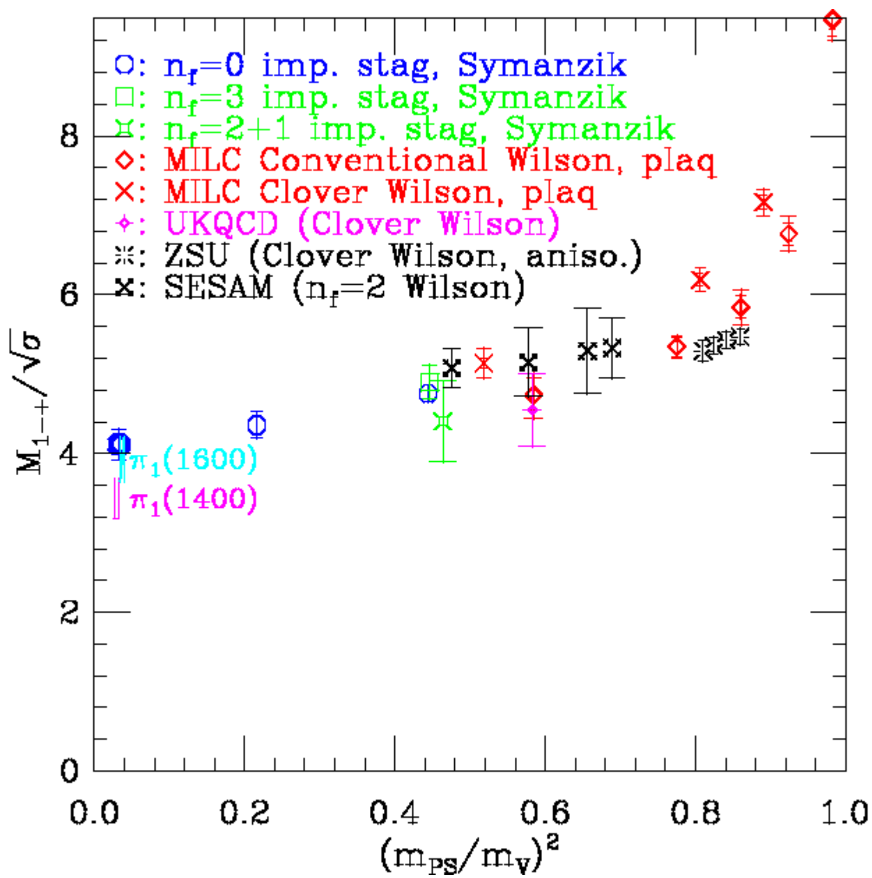
- No mention of potentials or wave functions for Hybrid mesons
- No mention of flux tubes

1^{-+} masses.

Summary of lattice results from the MILC collaboration, (hep-lat/0209097). Lattice mostly still predicts lightest 1^{-+} state to be at 1.9(2) GeV.



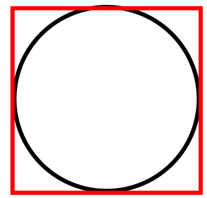
Two recent quenched calculations

Some recent lattice QCD calculations quote results close to 1600 MeV.

- The Adelaide group (hep-lat/0509106) computed $m_{1^{-+}}$ = 1.74(24) GeV from a quenched lattice QCD calculation. The lattice parameters were $a \sim 0.13$ fm and $L \sim$ 2.6 fm. Lightest pseudoscalar mass 320 MeV.
- Cook and Fiebig extracted the masses of the 1^{-+} mesons from a quenched lattice QCD calculation using the maximum entropy method. Lattice parameters $a_t \sim$ 0.07 fm and $L \sim 1.7$ fm. They claim to see masses of 1^{-+} at 1400, 1600, and 1900 MeV.
- The extrapolations of the 1^{-+} mass to the physical quark mass need physics of decays included.

Is the lattice 1^{-+} a non-exotic 4^{-+} ?

Issue recently raised by Dudek et al. (arXiv:0707.4162 [hep-lat]) in charmonium.



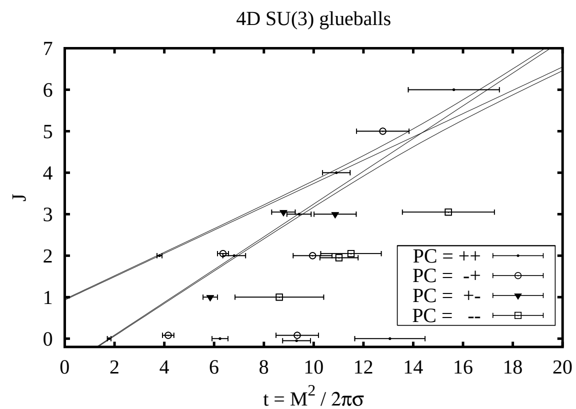
Symmetries of lattice are subgroup of continumm group, so care is required in construction of operators with J^{PC}.

J	Lattice Rep
0	A_1
1	T_1
2	$E\ T_2$
3	$A_2\ T_1\ T_2$
4	$A_1\ E\ T_1\ T_2$

The $\overline{\psi}\gamma_i\psi$ operator lives in T_1, so can couple to 1^{--} and 3^{--} (UKQCD hep-lat/9605025).

Lattice and spin identification

Meyer and Teper studied the J=4,6 glueballs (hep-ph/0409183) in quenched QCD to study Regge trajectories. Used continuum like interpolating operators and range of lattice spacings.



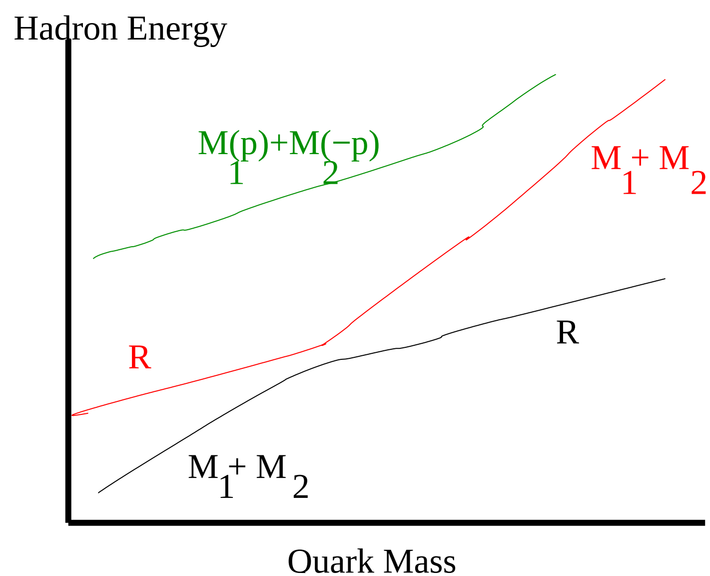
Current results for the masses of 1^{-+} states are lower bounds (although not a strict), but

- For light quarks the pressing issue is that the lattice result at 1.9(2) GeV is not in great agreement with experiment 1.6 GeV.
- For charmed hybrids the issue is whether the 1^{-+} is above or below the $D^{\star}D^{\star\star}$ threshold.

Cartoon of meson decay

Resonance decay $R \rightarrow M_1 + M_2$

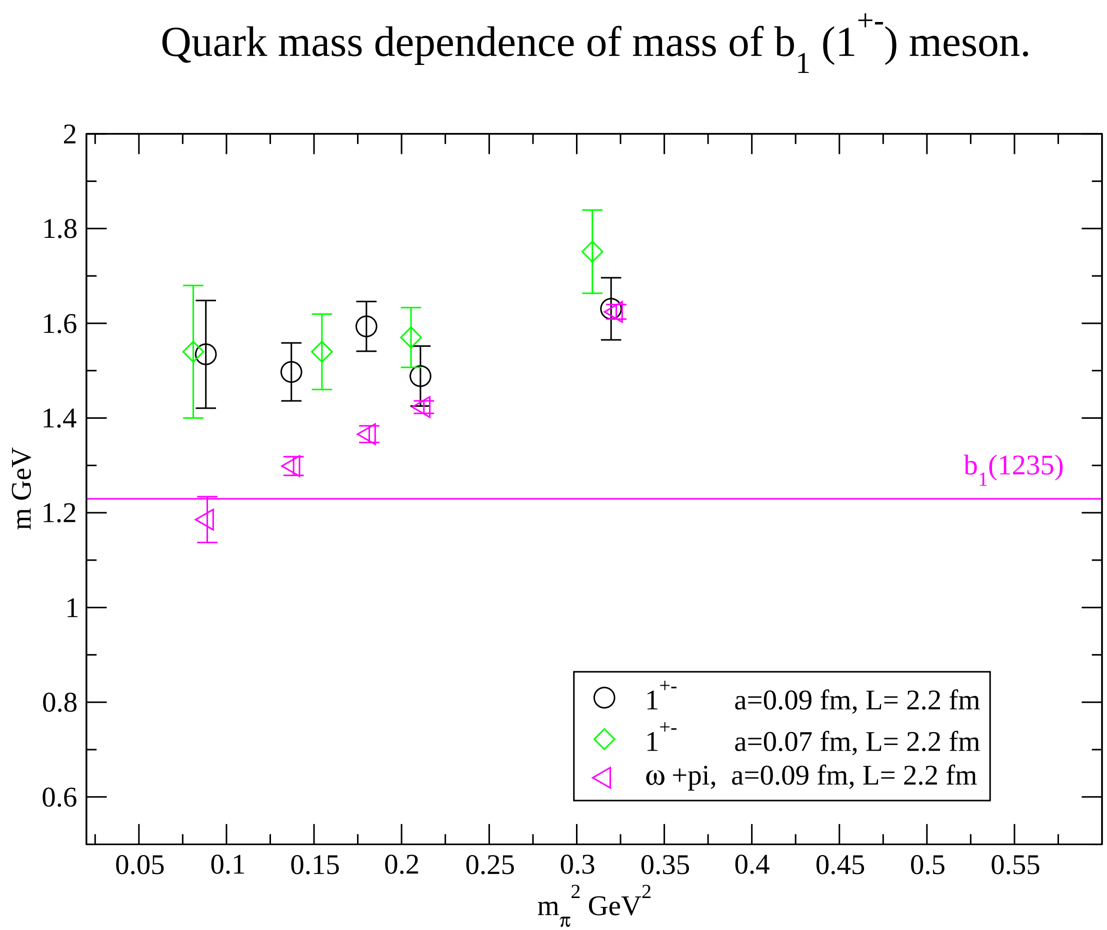
- S-wave decay, decay threshold $M_R = M_1 + M_2$
- P-wave decay, decay threshold $M_R = \sqrt{M_1^2 + p^2} + \sqrt{M_2^2 + p^2}$, p = $\frac{2\pi}{L}$ (~ 500 MeV)



(Elegant method proposed by Lüscher, Nucl. Phys. B364, 237, 1991. First application for ρ decay , ariv:0708.3705, by CP-PACS.) MILC (hep-lat/0104002) originally claimed evidence for a_0 decay to $\pi\eta$ from LGT.

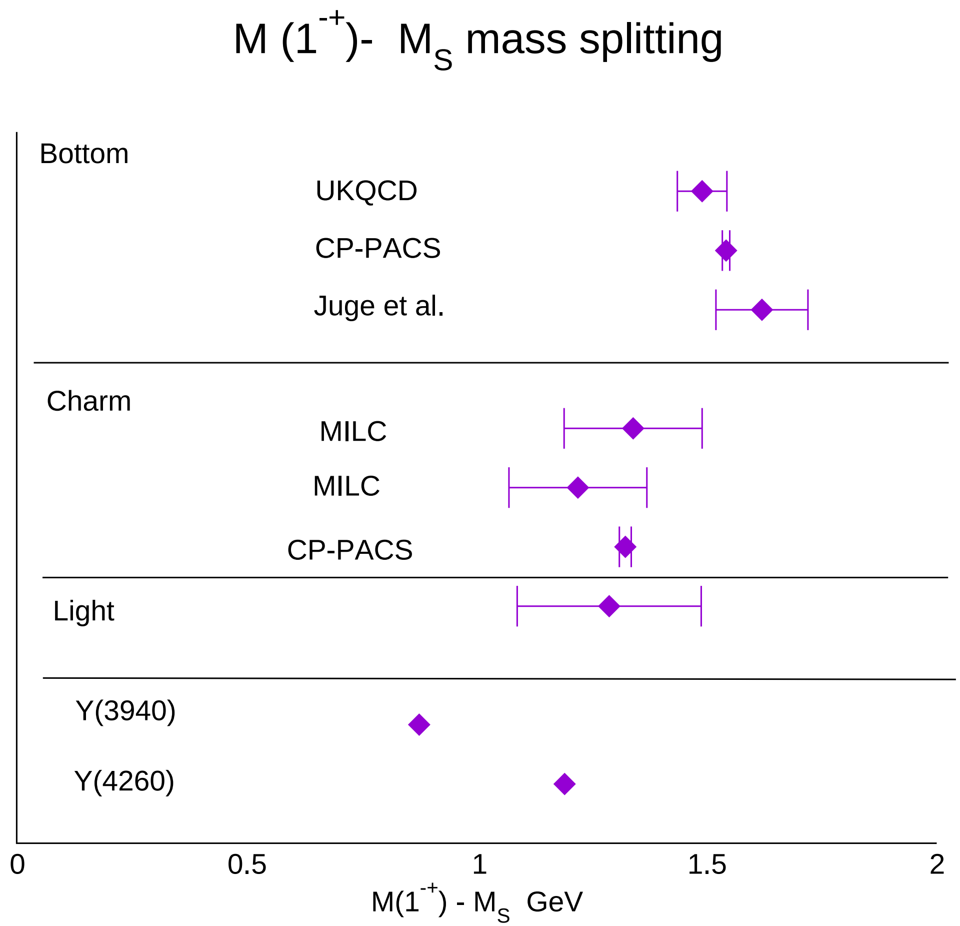
Preliminary results from ETMC

The b_1 meson has the dominant decay $\omega\pi$, that we should see on the lattice. (The difference between the ρ and ω is disconnected and small.) This is an updated analysis to my lattice 2007 presentation.

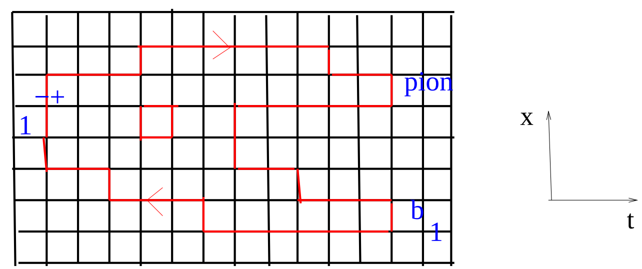


Heavy ($\bar{c}cg$ and $\bar{b}bg$) 1^{-+} states

From recent review by McNeile (hep-ph/0610139)



$1^{-+} \to \pi b_1$ (hep-lat/0603007)



$$H(t) = \frac{(1^{-+} \mid b_1 \pi)}{\sqrt{(1^{-+} \mid 1^{-+})(b_1 \mid b_1)(\pi \mid \pi)}} \sim xt$$

With transition matrix element xa plug into Fermi Golden rule to get width Γ.

$$\Gamma / k = \frac{1}{\pi} (xa)^2 (L/a)^3 \frac{E(b_1) a E(\pi)}{E(b_1) + E(\pi)}$$

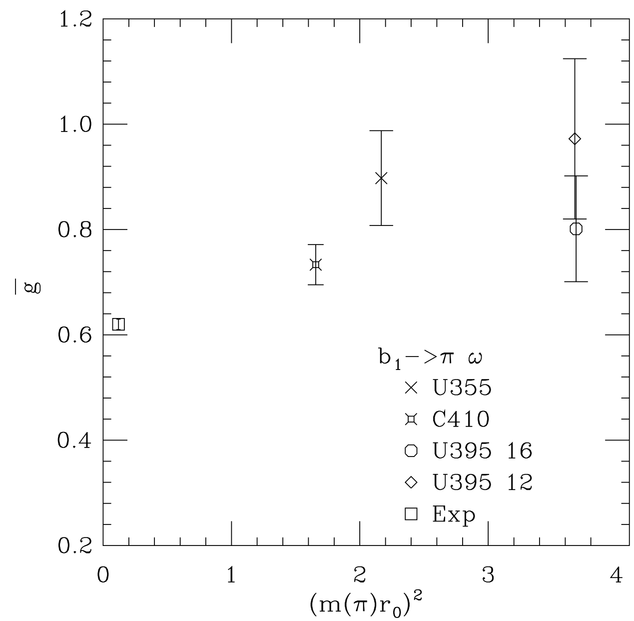
Decay widths from the lattice

Group	Decay	Result	Expt
CP-PACS	$\rho \to \pi\pi$	140(27) MeV	150 MeV
McNeile & Michael, hep-lat/0603007	$1^{-+} \to \pi b_1$	400(120) MeV	-
McNeile & Michael, hep-lat/0603007	$1^{-+} \to \pi f_1$	90(60) MeV	-
Cook and Fiebig, hep-lat/0606005	$1^{-+} \to \pi a_1$	~ 60 MeV	-

CMN and CM didn't look at $1^{-+} \to \rho\pi$, because this is not a S-wave decay, so requires momentum for the ρ and π. Many models claim that $1^{-+} \to S + P$ is small, but this caused confusion with E852 signal at 1600 MeV with decay $\rho\pi$.

Some systematics

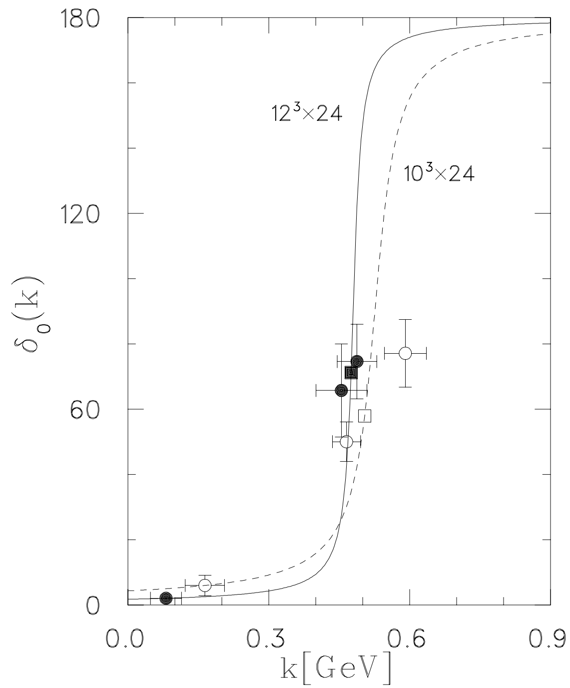
From McNeile and Michael, hep-lat/0603007. For the decay $b_1 \to \omega\pi$ compute the coupling $\bar{g}$ defined by: $\bar{g}^2 = \Gamma/k$



- Statistical errors large
- Large extrapolation

Phase shifts

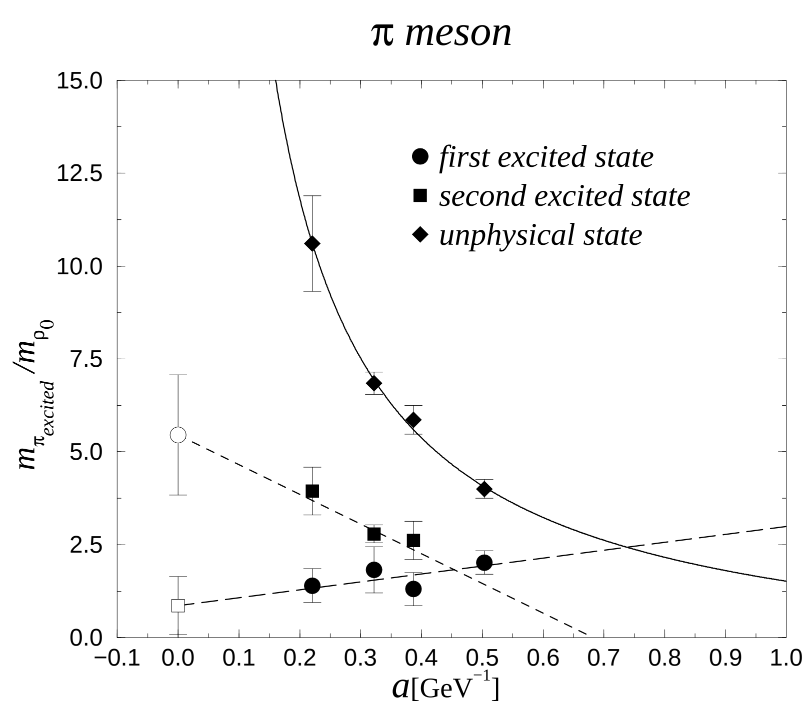
From Cook and Fiebig, hep-lat/0606005 an attempt to compute scattering phase shifts.



Extract width from fit to Breit-Wigner function. CP-PACS applied similar technique to the ρ decay (arXiv:0708.3705 [hep-lat]).

Another cross-check

CP-PACS collaboration used quenched QCD to study the excited state masses of light mesons using the maximum entropy method (hep-lat/0105030). They claim evidence for bound states of doublers (unphysical particles) that diverge in the continuum limit.



In place of conclusions: homework

A triumph, and yet some excuses.

- Many lattice collaborations now have access to full lattice QCD calculations with 300 MeV pions, a range of latice spacings and volumes. This will help the study of exotic mesons on the lattice.

Things to do before GlueX starts taking data in 2015.

- In lattice QCD we are starting to deal with resonances, such as the b_1 meson that have decayed in our lattice calculations.
- Need the value of mass of 1^{-+} from unquenched QCD, that includes the effects of the decay thresholds.
- Need spectroscopy of multi-quark 1^{-+} from unquenched QCD. Something like the calculation by the Kentucky lattice group of 0^{++} hep-ph/0607110, but with full QCD.
- Need to check whether decay of 1^{-+} meson to two S-wave mesons is suppressed, as is found in many models.

Some excuses. Our funding agency in the UK has now decided that “the Jefferson lab” is on the list of highest priority projects, but that the operation of high performance computers is on the low priority list.