Multiple Scattering of 600-Mev Electrons in Thin Foils*

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The beam of the Stanford Mark III linear accelerator has been used to measure the width of the angular distribution of electrons multiply scattered in thin Be, Al, and Au foils. The results are in agreement with the predictions of the Molière theory.

The very small angular divergence of the electron beam from the Stanford linear accelerator makes it particularly suitable for measuring multiple scattering in thin foils. Measurements were made using 600-Mev electrons in the undeflected beam (Figs. 1 and 2). The foils were placed behind a 1-in. thick copper collimator with a $\frac{1}{16}$-in. diameter aperture. To ensure a small angular divergence, $\frac{1}{4}$-in. aperture collimators were placed in the accelerator 85 ft and 125 ft before the foil. The angular divergence was found to be smaller than that defined by this geometry. After passing through the foil the scattered electrons continued for 40 ft in a vacuum pipe, and the angular distribution was measured just after they passed through a 2-mil aluminum vacuum window. The electrons were detected by measuring the darkening they caused in passing through a glass plate.

LINEARITY OF DARKENING

In order to establish the linearity of darkening of the glass plates, the approximate region of linearity was first determined by making a series of exposures varying the number of electrons in each exposure. The number of electrons was measured by a secondary-emission monitor.[1] By measuring the peak darkening and plotting it against integrated current, the linearity was found to be adequate up to approximately 10^{13} electrons/cm^2. The accuracy of comparison of the plates is limited by their fading and possibly by other variable glass characteristics. In obtaining multiple-scattering data at least two exposures of different intensity were made for each foil measured, and the darkening of the plates was plotted against angle (Figs. 3 and 4). Since saturation would cause a difference in the width of the curves from the same foil, the data used came from curves where no such effects were visible. It was found by checking curve shapes at different times that the shapes stayed constant although the absolute values of the densities changed, thus indicating that the fading was linear.

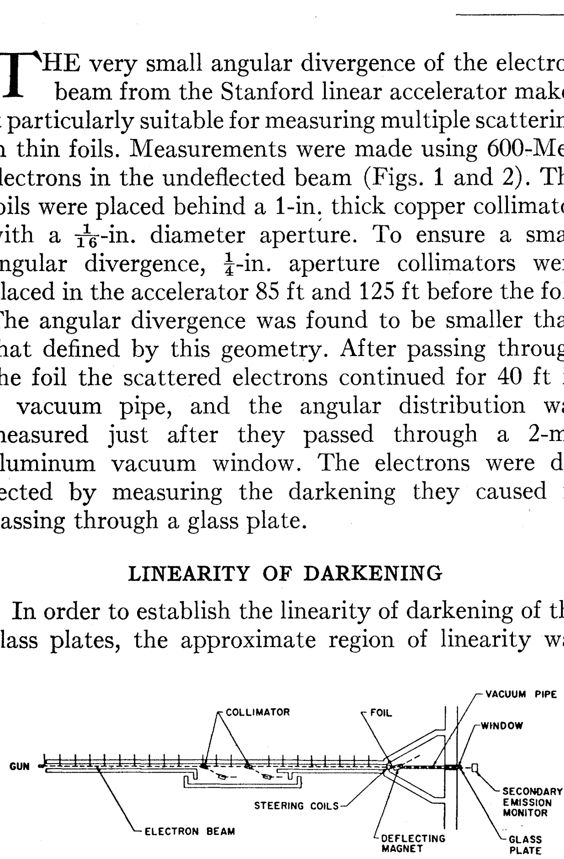

Fig. 1. Linear accelerator and equipment for multiple-scattering measurement.

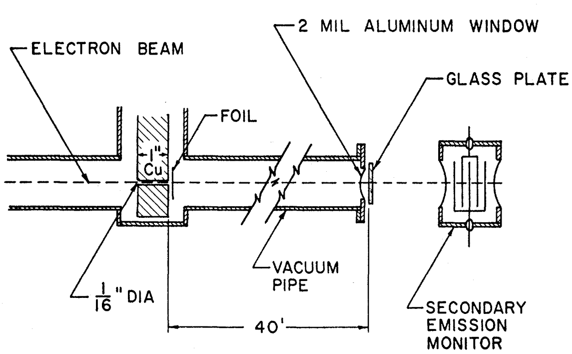

Fig. 2. Details of equipment for multiple-scattering measurement.

RESOLUTION

An exposure was made with no foil present in order to determine the effect of the $\frac{1}{16}$-in. diameter collimator aperture, the angular divergence of the electron beam, and the resolution of the densitometer. The resulting curve (Fig. 5) had a noticeable effect only on the width of scattering from the thinnest ($\frac{1}{3}$-mil Al) foil. All of the

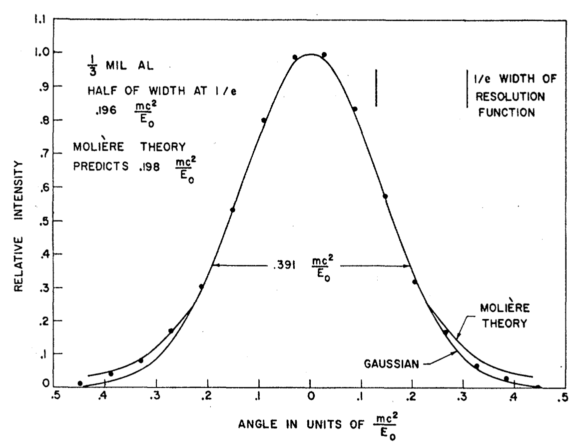

Fig. 3. Densitometer plot of glass darkening *vs* angle for $\frac{1}{3}$-mil aluminum scattering foil.

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[1] G. W. Tautfest and H. R. Fechter, Revs. Sci. Instr. **26**, 229 (1955).

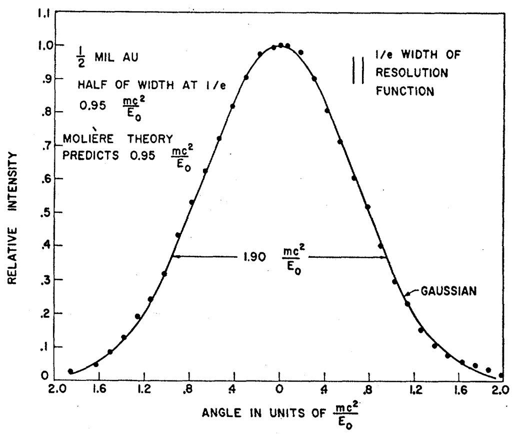

FIG. 4. Densitometer plot of glass darkening *vs* angle for $\frac{1}{2}$-mil gold scattering foil.

multiple-scattering curves were indistinguishable from Gaussian, and the aperture function resembled a Gaussian so closely that no appreciable error results from unfolding the two curves assuming that both are Gaussian. In a background exposure made with the collimator hole blocked, no measurable darkening was obtained.

ENERGY DETERMINATION

The unanalyzed beam of the accelerator was used during the actual multiple-scattering runs and the beam energy was measured before and after the runs

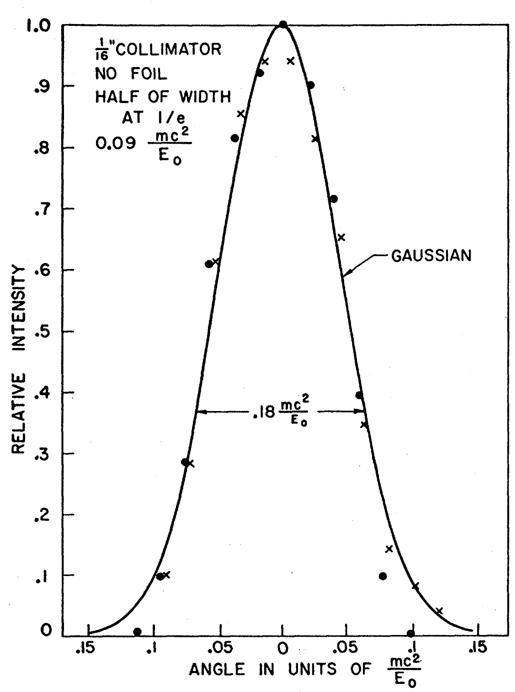

FIG. 5. Densitometer plot of glass darkening *vs* angle for $\frac{1}{16}$-in. collimator with no scattering foil.

on each foil. The drift in energy was less than $\frac{1}{2}\%$ while the accuracy of the energy calibration of the magnets is estimated at $1\frac{1}{2}\%$. In addition, the beam had an energy width of approximately $1\frac{1}{2}\%$ which would make a negligible contribution to the energy error.

FOIL THICKNESS

Except for beryllium, where a 1-cm^2 piece was used, the foil thicknesses were determined by weighing and measuring the area of a large (25–100 cm^2) section of foil. The foil used was then cut from the measured foil. After the foils had been used their nonuniformity was checked by measuring the actual foil sample after it had been cut into two sections of approximately $\frac{1}{2}$ cm^2. The area measurement in this case was made by placing the foils on an accurately ruled graph paper and photographing this to give a first negative of approximately fifty times the area. The areas of the negatives were measured and compared with a measurement of the graph paper on the same negative. The accuracy of the measurement of areas and of the small weights involved was approximately 2% for the thick and 3% for the thin foils. The foils showed no nonuniformity larger than this error. The thickness used in the data is that taken from the large area foil. An error of 2–3% in foil thickness has been assumed due primarily to the uncertainty in uniformity of the foil.

ERRORS

A very important source of error was in recording and reading the curves. The densitometer readings were not completely reproducible. The widths were determined by making a visual fit of a Gaussian to the densitometer curve and measuring the width of the Gaussian at the $1/e$ point. Since the curve is quite steep in this region, the percentage error contributed was about $1\frac{1}{2}\%$ for the thick and 2% for the thin foils. The errors taken into account were: curve plotting and reading, $1\frac{1}{2}$ to 2%; energy determination, $1\frac{1}{2}\%$; and foil thickness 1 to $1\frac{1}{2}\%$. (Since the multiple scattering varies as the square root of the foil thickness, a 2% error in foil thickness contributes only 1% to the angular error.) Nonlinearity of darkening was not observed, and errors due to this would be included in the estimate of error due to curve reading. The errors in reading and in foil thickness determination were larger for the thinner foils. The errors are treated as random and the rms errors derived from the estimates above are about 2.5% for the thick and 3% for the thin foils.

RESULTS AND COMPARISON WITH THEORY

Both the width and shape of the curves were compared with Molière's predictions.[2] The difference between the Molière prediction and Gaussian would be most easily seen in the large-angle data from the

[2] G. Molière, Z. Naturforsch. **3A**, 78 (1948); **2A**, 133 (1947).

thinnest foil ($\frac{1}{3}$-mil Al), Fig. 3. A Gaussian and the Molière prediction are fitted to the data, and it can be seen that the accuracy does not allow any conclusion to be reached regarding a possible divergence from a Gaussian.

The width of the curves were compared with the values predicted by the Molière theory using a relation derived by Hanson *et al.*[3]:

$$\theta_w=\theta_1(B-1.2)^{\frac{1}{2}},$$

$$\theta_1^2=0.157\frac{Z(Z+1)}{A}\frac{t}{E_0^2},$$

$$B-\ln B=\ln[(\theta_1/\theta_a)^2]-0.154,$$

$$\left(\frac{\theta_1}{\theta_a}\right)^2=7800\frac{(Z+1)Z^{\frac{1}{3}}t}{A[1+3.35Z^2(e^2/\hbar c)^2]},$$

where θ_w is the scattering angle at the $1/e$ point, θ_1 the maximum scattering angle, θ_a the screening angle, A the atomic weight, t the foil thickness in g/cm^2, and E the electron energy in Mev.

A comparison of the widths of the curves with the predictions of Molière theory is given in Table I. It can be seen that they are in excellent agreement. In this there is a slight and possibly not significant disagreement with the lower energy data of Hanson *et al.*[3] They found excellent agreement in the case of gold, but their value for beryllium indicated scattering about 3–7% less in width than the Molière predictions. It was suggested that this disagreement with theory might be caused by a breakdown of the Fermi-Thomas atomic model used in the Molière calculations since this calculation would not necessarily be valid for such a light element as beryllium. However, later work by Mohr and Tassie[4] using a Hartree model gives the same result as the Fermi-Thomas calculation.

[3] Hanson, Lanzl, Lyman, and Scott, Phys. Rev. **84**, 634 (1951).

TABLE I. Comparison of the measured multiple scattering with the predictions of the Molière theory.

Foil material	Thickness in g/cm^2	Half of the width at $1/e$ (in radians $\times E_0/mc^2$) Molière theory	Measured (aperture function unfolded)
Be	0.0123 (5 mil)	0.282	0.281±0.009
Al	0.00244 ($\frac{1}{3}$ mil)	0.176	0.174±0.005
Al	0.00706 (1 mil)	0.359	0.350±0.009
Al	0.0205 (3 mil)	0.699	0.685±0.017
Au	0.0103 ($\frac{1}{2}$ mil)	0.952	0.950±0.024

ACKNOWLEDGMENTS

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[4] C. B. O. Mohr and L. J. Tassie, Australian J. Phys. **7**, 217 (1954).

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Time-Reversal Invariance and Radiative Muon Decay*

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The radiative muon decay $\mu\rightarrow e+\nu+\bar{\nu}+\gamma$ is investigated as a source of information on the time-reversal properties of the muon decay interaction. It is shown that for any muon decay process, which may include electromagnetic interactions, terms in the transition probability which violate time-reversal invariance must be pseudoscalars. Further, only ten combinations of the coupling constants of the four-fermion interaction can occur in the transition probability. These may be classified in accordance with the types of terms in which they occur; a term which violates time-reversal invariance must contain as a factor either the electron mass or the transverse electron polarization. In the radiative muon decay, if such a term is proportional to the electron mass, it will also contain as a factor the longitudinal electron polarization.

If the two-component neutrino theory is assumed, the four-fermion interaction representing muon decay is characterized by two coupling constants, g_V and g_A.[1] The transition probability for a decay process must be quadratic in these constants; the combinations which may occur are $|g_V|^2+|g_A|^2$, $|g_V|^2-|g_A|^2$, $\mathrm{Re}(g_A^*g_V)$, and $\mathrm{Im}(g_A^*g_V)$.[2] A normalized decay spectrum will be

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[1] T. Lee and C. Yang, Phys. Rev. **105**, 1671 (1957).

[2] D. Candlin, Nuovo cimento **6**, 390 (1957). A more complete expression for the transition probability has been given by R. Sharp and G. Bach, Can. J. Phys. **35**, 1199 (1957). Similar remarks hold for the general four-component theory with its ten coupling constants; this has been treated by T. Kinoshita and A. Sirlin, Phys. Rev. **108**, 844 (1957).