Lattice QCD update - isoscalars ?

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Toward the excited meson spectrum of dynamical QCD
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Comments: 26 pages, 26 figures
Subjects: **High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Lattice (hep-lat)

work under the auspices of the
Hadron Spectrum Collaboration

two-point functions & the energy spectrum

a two-point correlator :

$$C_{\mathbf{\Gamma},\mathbf{\Gamma}'}(\vec{p};t,0) =$$

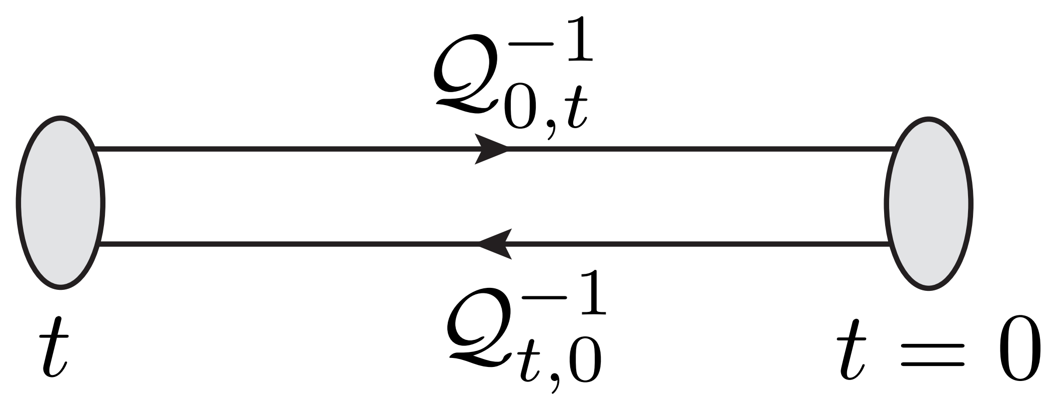
$$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}U \cdot \int d^3y\, e^{-i\vec{p}\cdot\vec{y}}\, \bar{\psi}_{y,t}\mathbf{\Gamma}\psi_{y,t} \int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \bar{\psi}_{x,0}\mathbf{\Gamma}'\psi_{x,0} \cdot e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

meson operator at mom $\vec{p}$

$$\mathcal{Q}^{-1}_{0,t}$$

$$\bar{\psi}_{y,t}\mathbf{\Gamma}\psi_{y,t}\ \bar{\psi}_{x,0}\mathbf{\Gamma}'\psi_{x,0}$$

$$\mathcal{Q}^{-1}_{t,0}$$



isovector

two-point functions & the energy spectrum

relation to the spectrum :

complete set of ***QCD*** eigenstates*

$$1 = \sum_n |n\rangle\langle n|$$

$$\langle 0|\bar{\psi}_t \mathbf{\Gamma} \psi_t \; \bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n \langle 0|\bar{\psi}_t \mathbf{\Gamma} \psi_t|n\rangle\langle n|\bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n e^{-E_n t}\langle 0|\bar{\psi}_0 \mathbf{\Gamma} \psi_0|n\rangle\langle n|\bar{\psi}_0 \mathbf{\Gamma}' \psi_0|0\rangle$$

$$= \sum_n Z_n^{\mathbf{\Gamma}} Z_n^{\mathbf{\Gamma}'} e^{-E_n t}$$

in principle - contribution from all states with the right q.n.'s

fitting a sum of exponentials is unstable
- noisy data
- possibly degenerate states

** in a finite volume*

operator basis & variational solution

better - build a basis of operators, calculate a matrix of correlators

$$\begin{bmatrix} \langle 0|\mathcal{O}_1\mathcal{O}_1|0\rangle & \langle 0|\mathcal{O}_1\mathcal{O}_2|0\rangle & \cdots \\ \langle 0|\mathcal{O}_2\mathcal{O}_1|0\rangle & \langle 0|\mathcal{O}_2\mathcal{O}_2|0\rangle & \\ \vdots & & \ddots \end{bmatrix}$$

diagonalising this matrix gives the optimal linear combination of operators for each state

eigenvalues → energies
eigenvectors → Z

optimal combinations are ***orthogonal*** - deals with degenerate states

our operators - built out of covariant derivatives

$$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\psi \qquad D = \partial - A$$

use Clebsch-Gordans to build the desired q.n.'s, e.g. 1^{--}

$$\bar{\psi}\vec{\gamma}\psi$$

$$D_{\pm,0} \sim \left\{ \begin{matrix} \frac{1}{\sqrt{2}}(D_x \pm i D_y) \\ D_z \end{matrix} \right\}$$

$$\bar{\psi}\overleftrightarrow{D}\psi$$

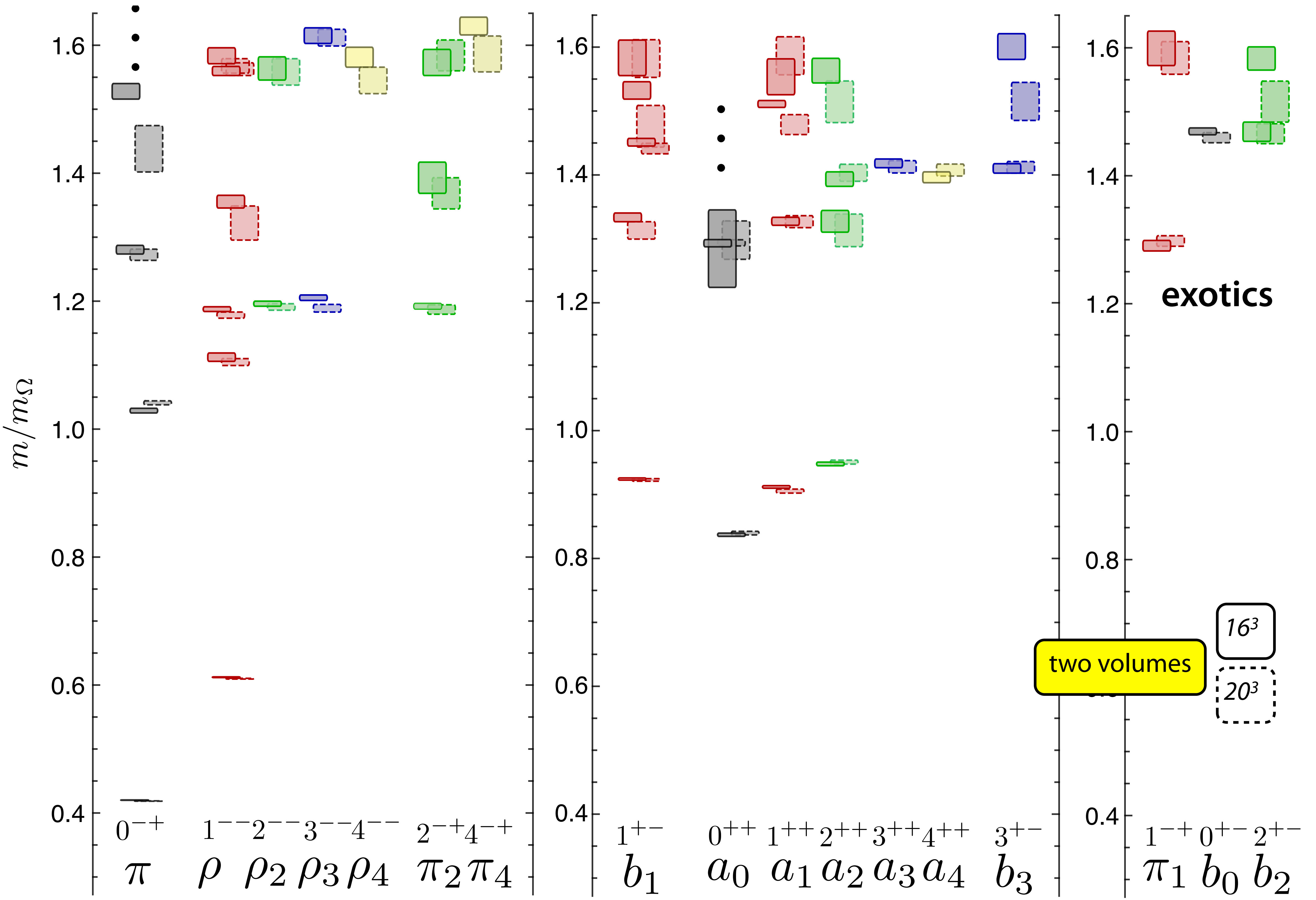
$$\langle 1, m_1; 1, m_2 | 1, m\rangle \, \bar{\psi}\gamma_5 \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}\psi$$

$$\begin{matrix}\langle 1, m_3; 2, m_D | 1, m\rangle \\ \langle 1, m_1; 1, m_2 | 2, m_D\rangle\end{matrix} \; \bar{\psi}\gamma_{m_3} \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}\psi$$

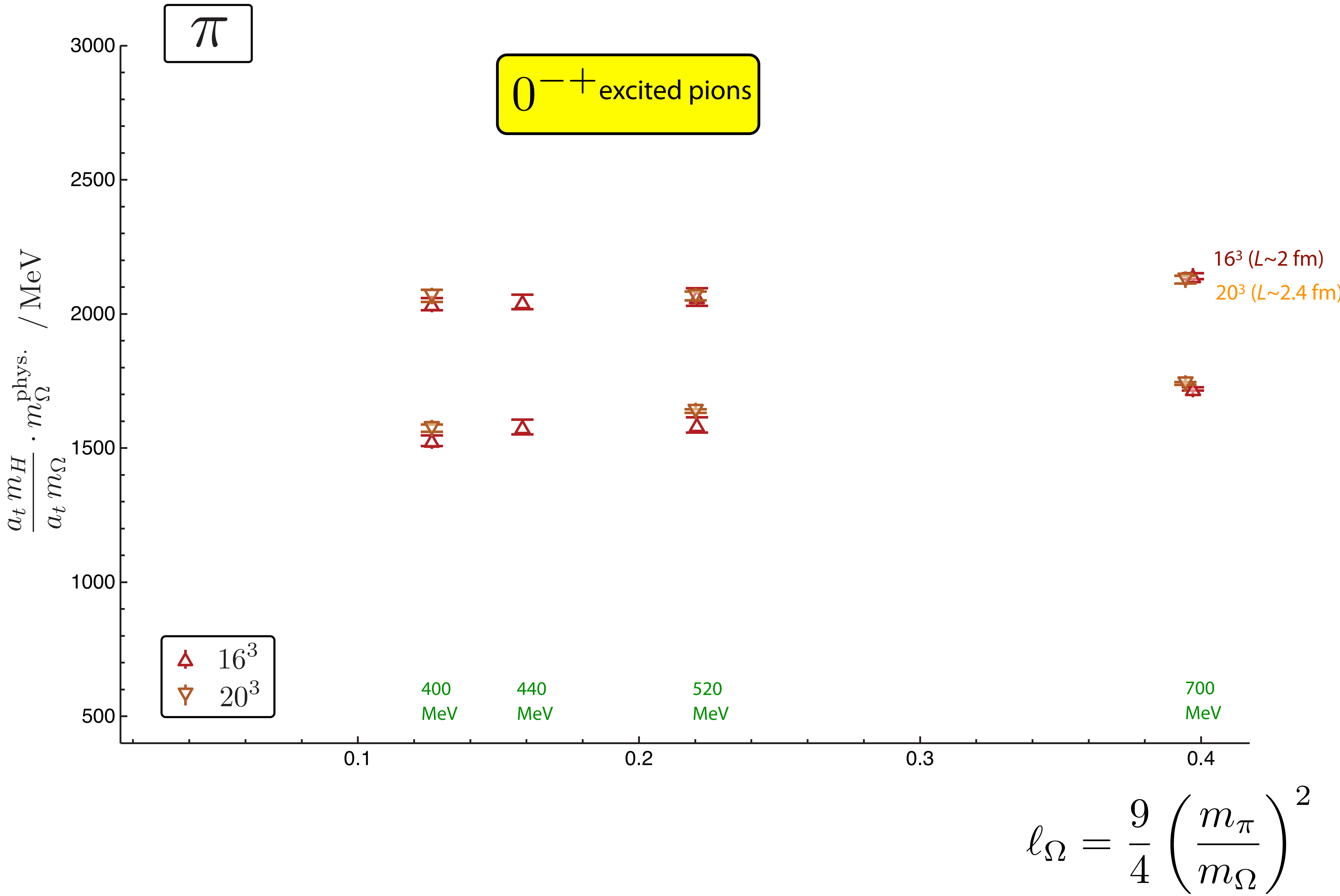
meson spectrum

$N_F = 3$ (s,s,s) $m_\pi \sim 700$ MeV

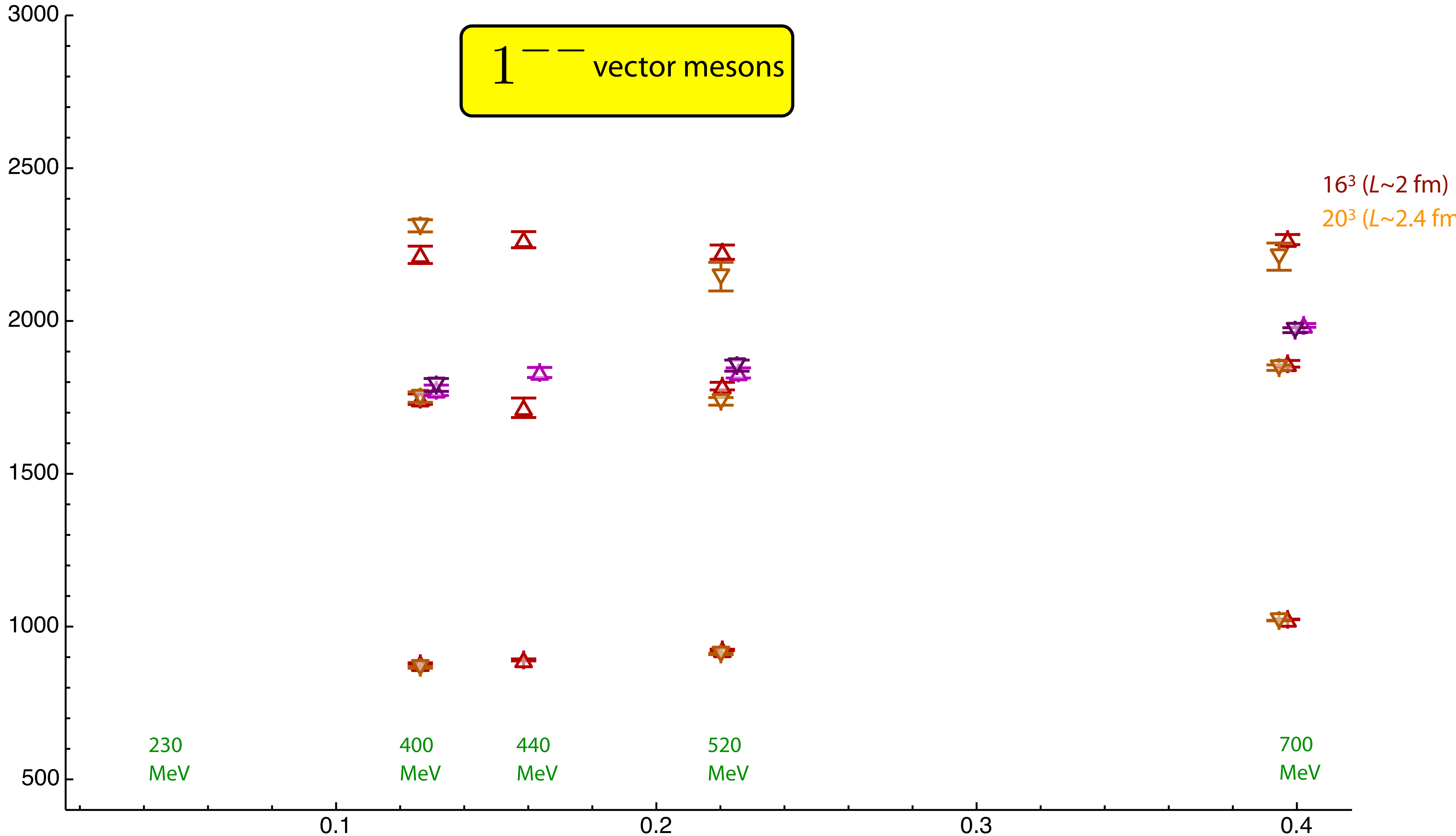
PRL 103:262001,2009



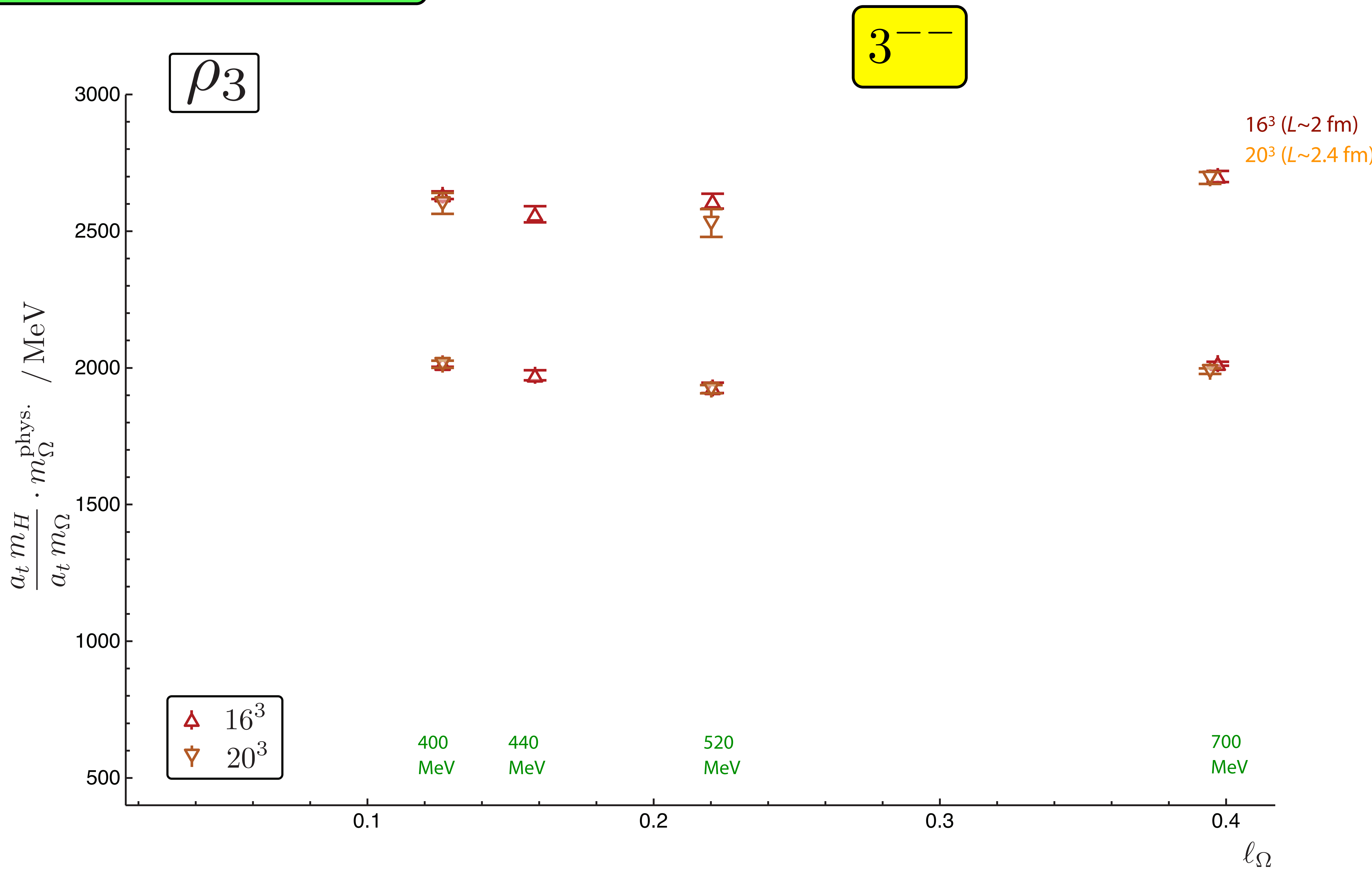
reducing the quark mass



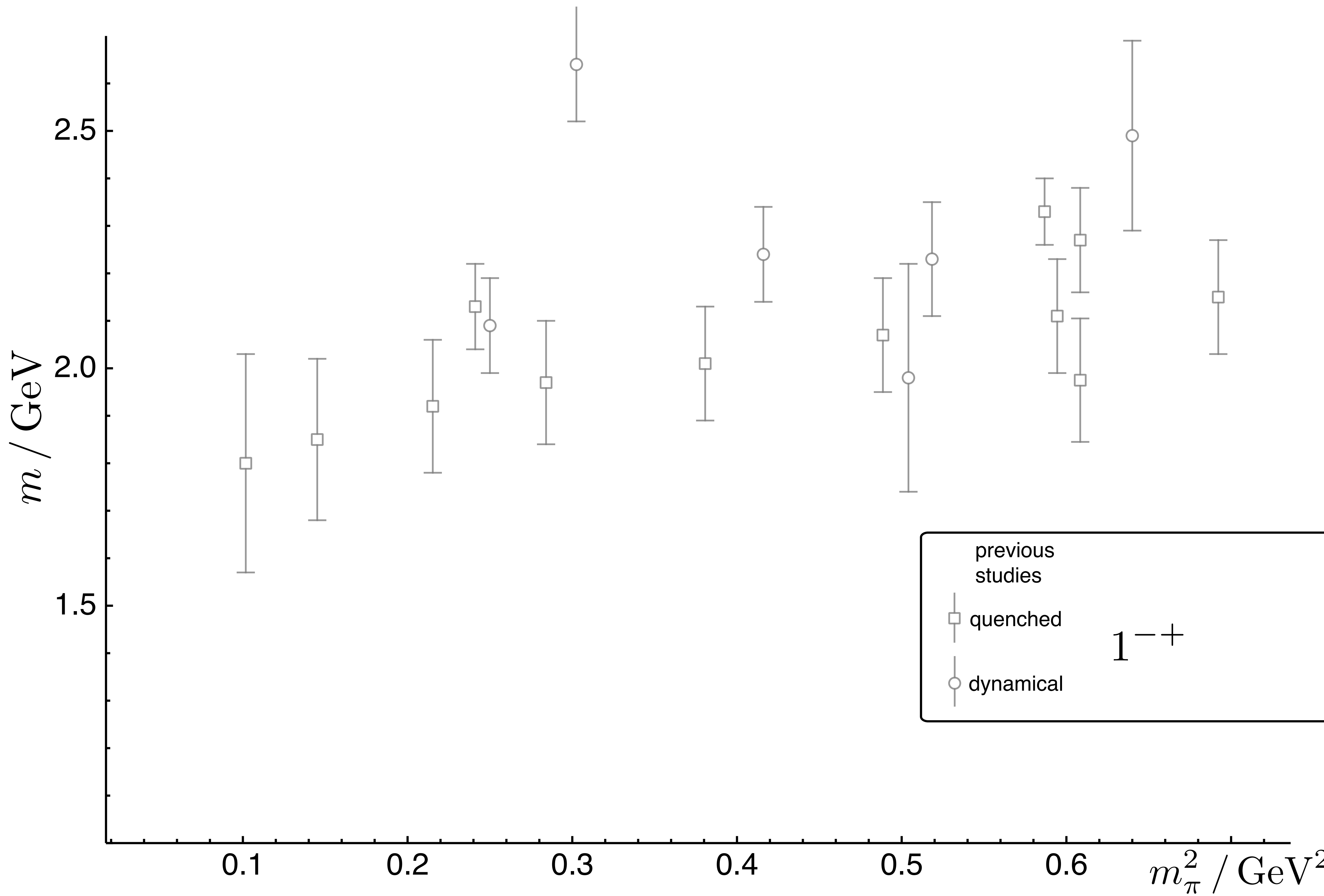
reducing the quark mass



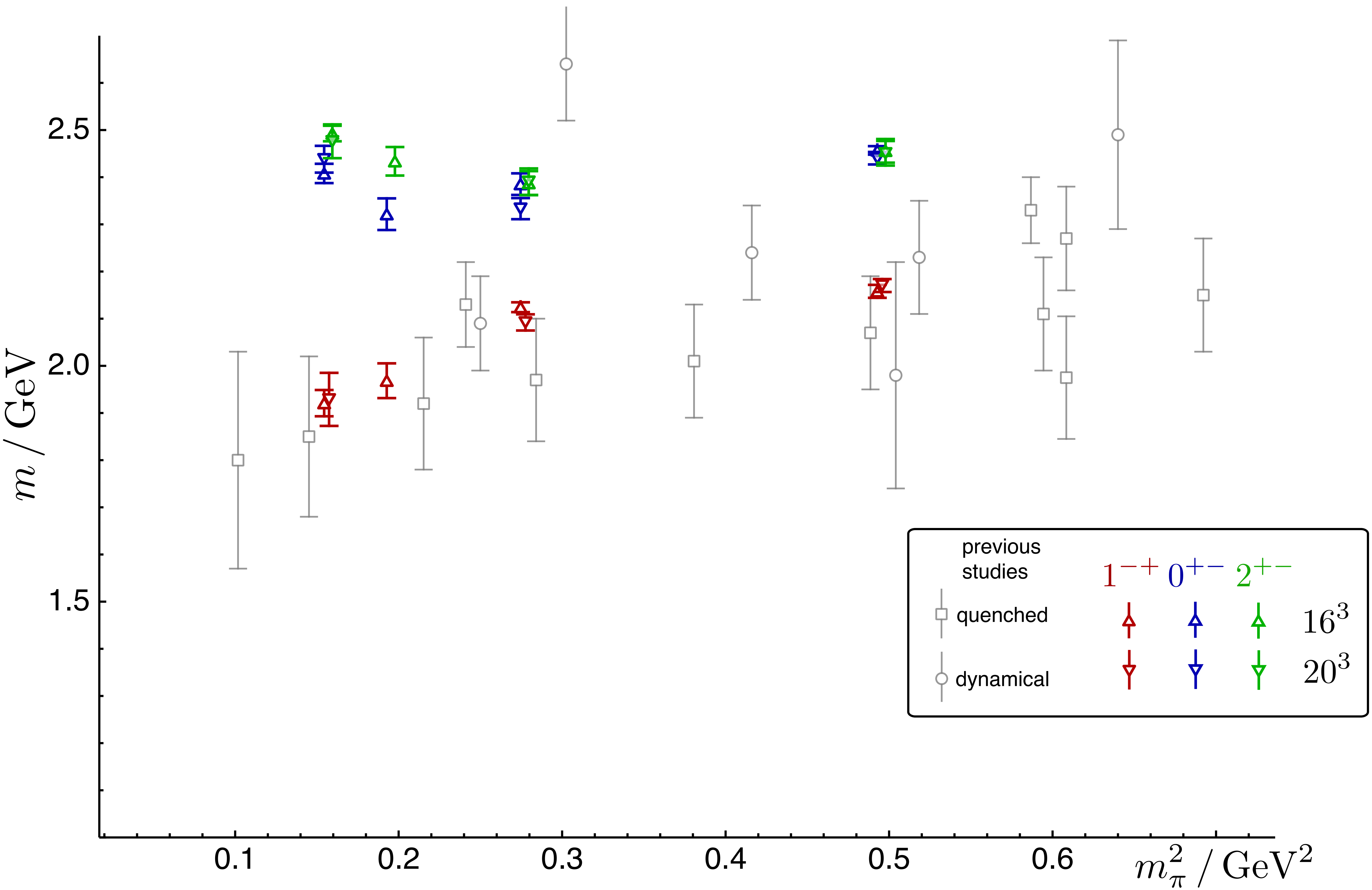
reducing the quark mass



exotics - world summary



exotics - world summary



isoscalar mesons

interesting observations in experiment :

η, ***η'*** mixed very close to octet ($\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$) , singlet ($\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$)

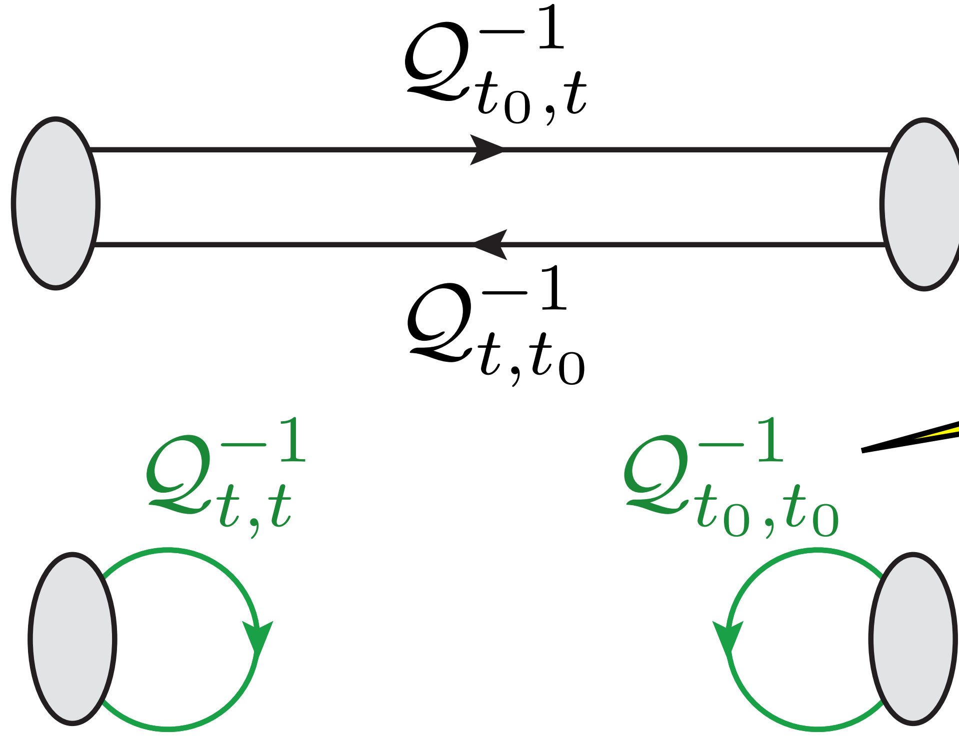
ω, **ϕ** mixed according to quark mass, $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $s\bar{s}$

f_0 sector overpopulated ? glueballs ?

GlueX relevant - how are the ***exotics*** flavour mixed ?

time for ***QCD*** to say something about this

isoscalars



these 'annihilation' propagators traditionally difficult to compute

light-strange (ℓs) basis

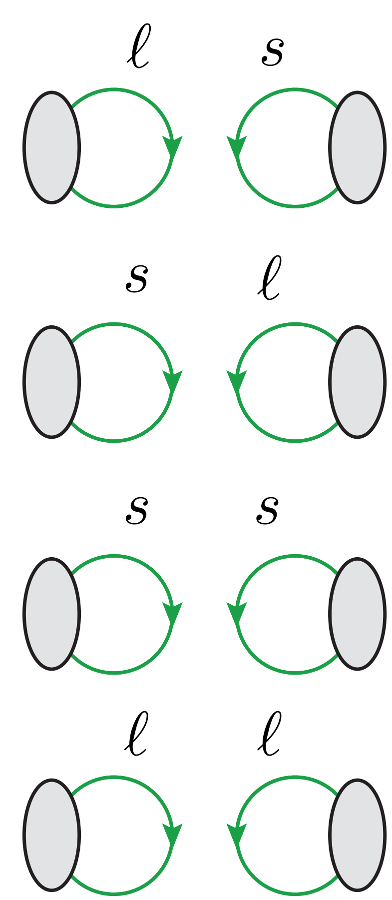
$$\mathcal{O}^{\mathbf{\Gamma}}_{\ell} = \frac{1}{\sqrt{2}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d\right)$$

$$\mathcal{O}^{\mathbf{\Gamma}}_{s} = \bar{s}\mathbf{\Gamma}s$$

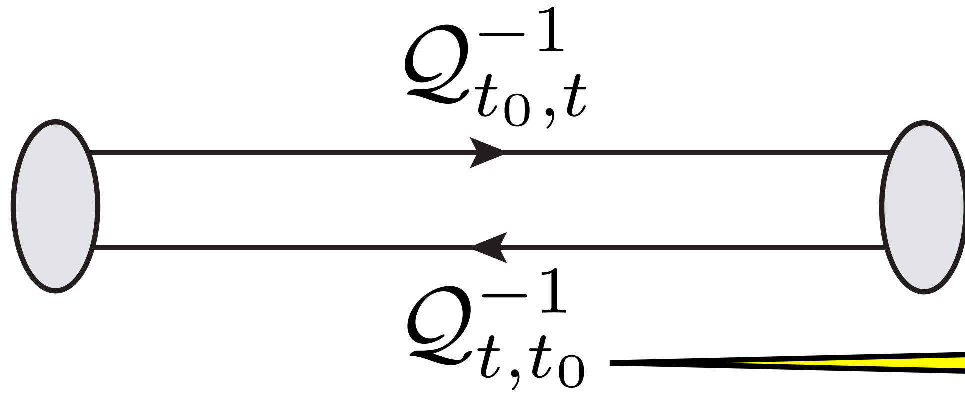
SU(3)$_F$ singlet-octet basis

$$\mathcal{O}^{\mathbf{\Gamma}}_{\mathbf{1}} = \frac{1}{\sqrt{3}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d + \bar{s}\mathbf{\Gamma}s\right)$$

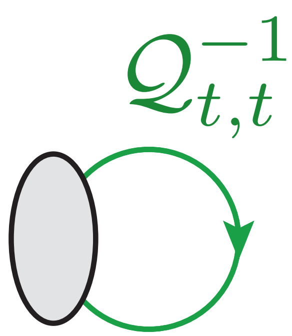
$$\mathcal{O}^{\mathbf{\Gamma}}_{\mathbf{8}} = \frac{1}{\sqrt{6}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d - 2\bar{s}\mathbf{\Gamma}s\right)$$

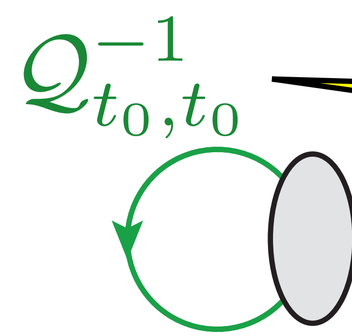


GPUs



typically <10 time-sources (t_0)





need ***all*** time-sources, i.e. for us 128 !

lots of matrix inversions - turns out ***GPUs*** are ideal for this

JLab group have computed all time-sources on a single lattice :

N_F = 2+1 (u,d,s)
$m_\pi \sim$ 400 MeV
16^3x128

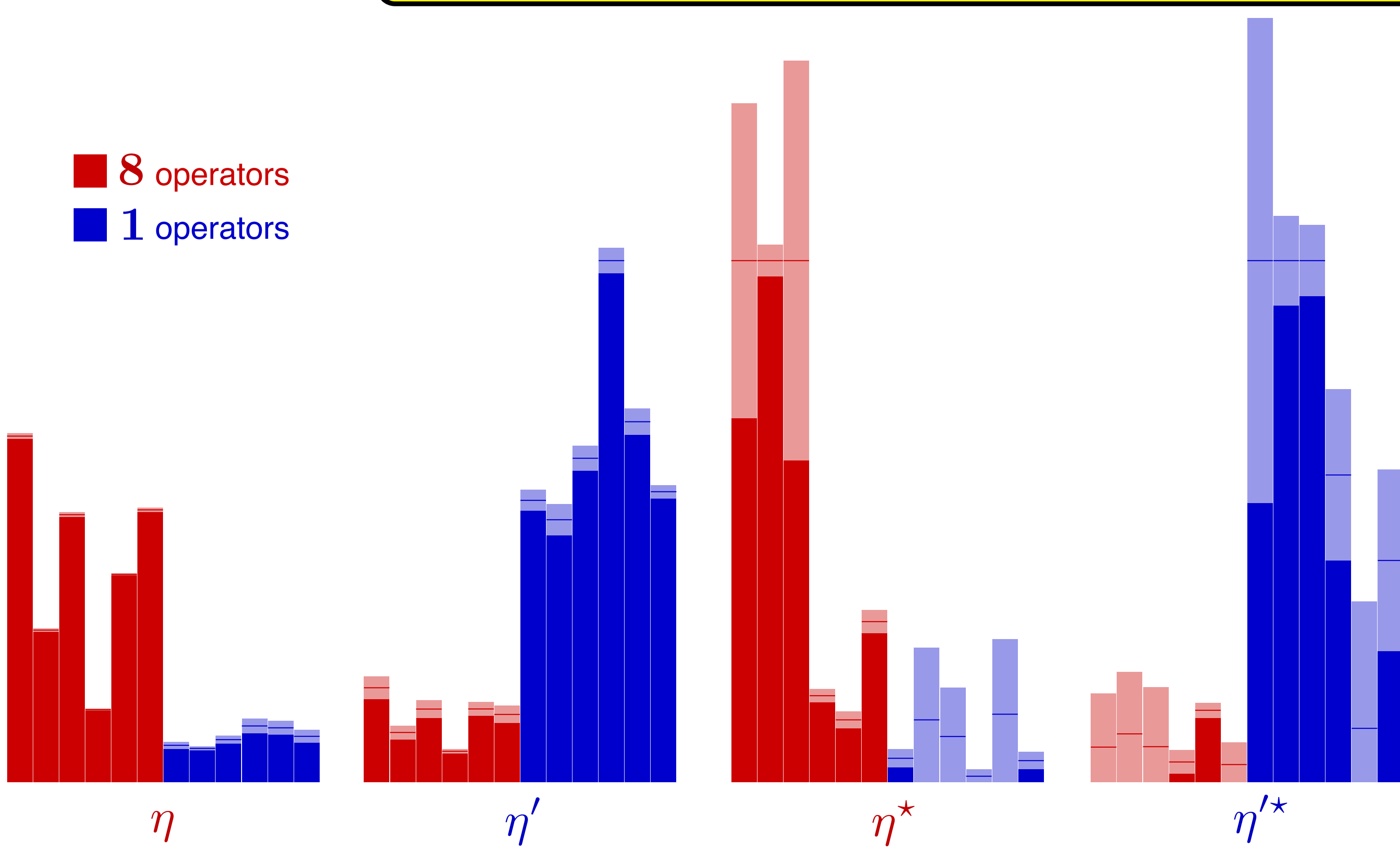
pseudoscalar isoscalars

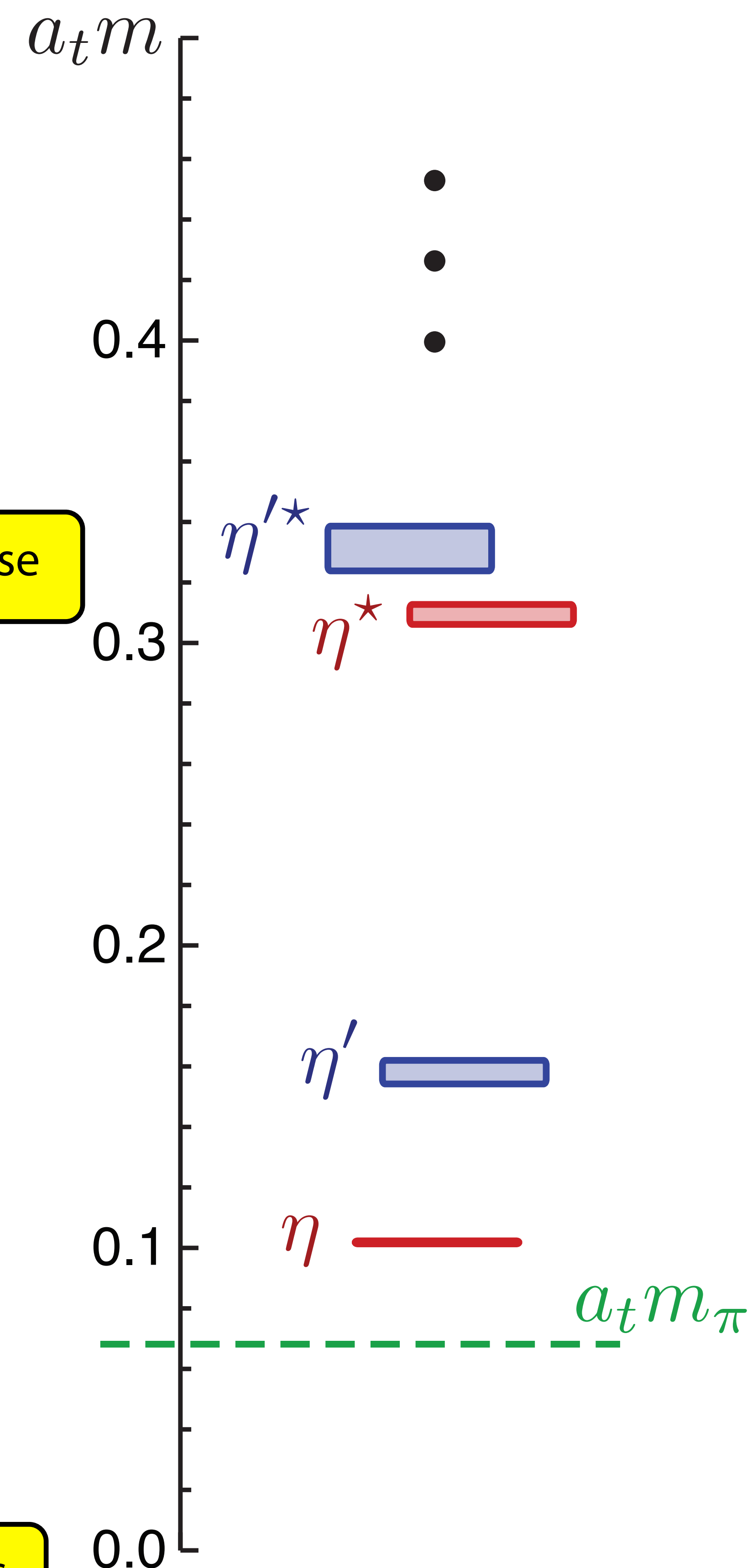
SU(3)$_F$ singlet-octet basis

$$\mathcal{O}_{\mathbf{8}}^{\mathbf{\Gamma}} = \frac{1}{\sqrt{6}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d - 2\bar{s}\mathbf{\Gamma}s\right)$$

$$\mathcal{O}_{\mathbf{1}}^{\mathbf{\Gamma}} = \frac{1}{\sqrt{3}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d + \bar{s}\mathbf{\Gamma}s\right)$$

no glueball operators as yet - straightforward to add these





almost diagonal in the ***SU(3)$_F$*** singlet-octet basis

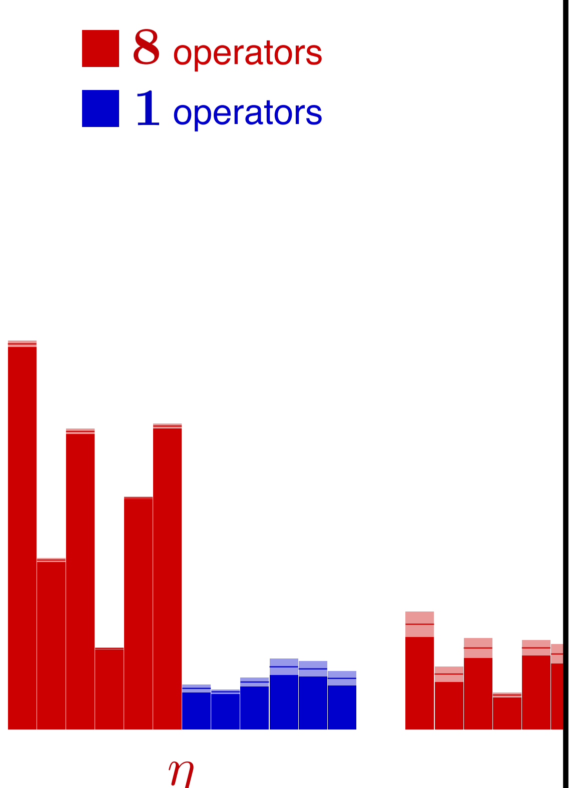
pseudoscalar isoscalars

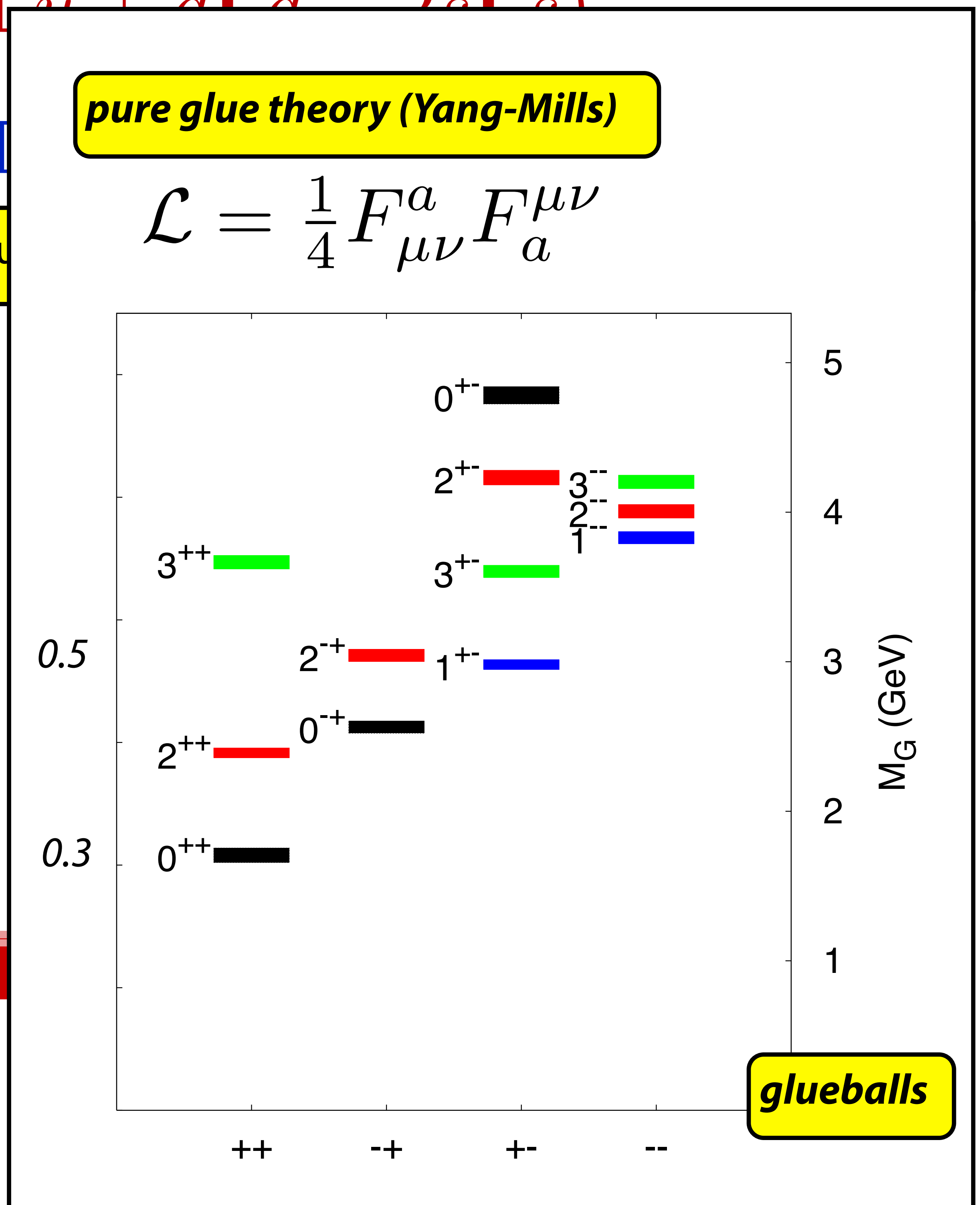
SU(3)$_F$ singlet-octet basis

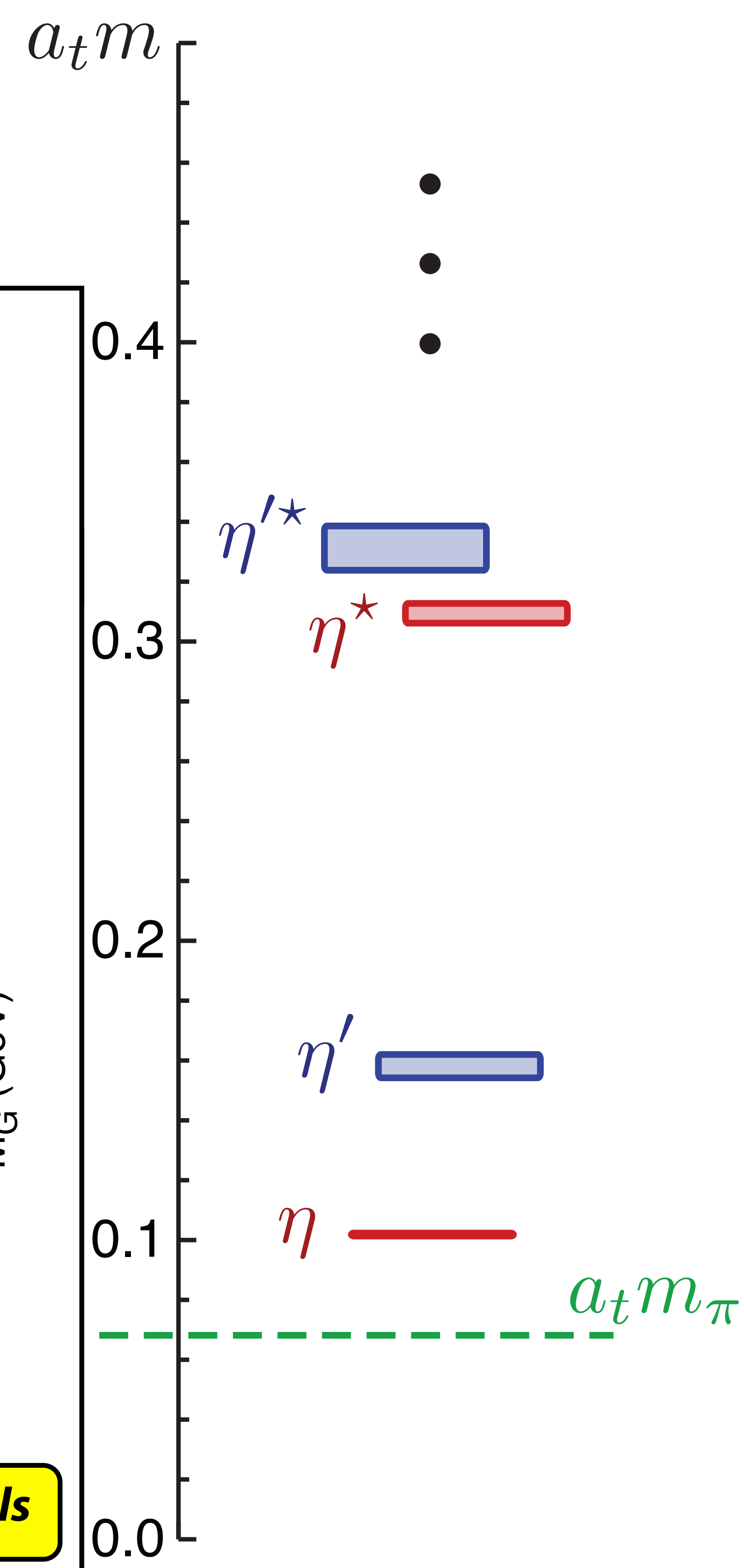
$$\mathcal{O}_{\mathbf{8}}^{\mathbf{\Gamma}} = \frac{1}{\sqrt{6}} \left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d - 2\bar{s}\mathbf{\Gamma}s \right)$$

$$\mathcal{O}_{\mathbf{1}}^{\mathbf{\Gamma}} = \frac{1}{\sqrt{3}} \left(\bar{u}\mathbf{\Gamma} \ldots \right.$$

no glu…







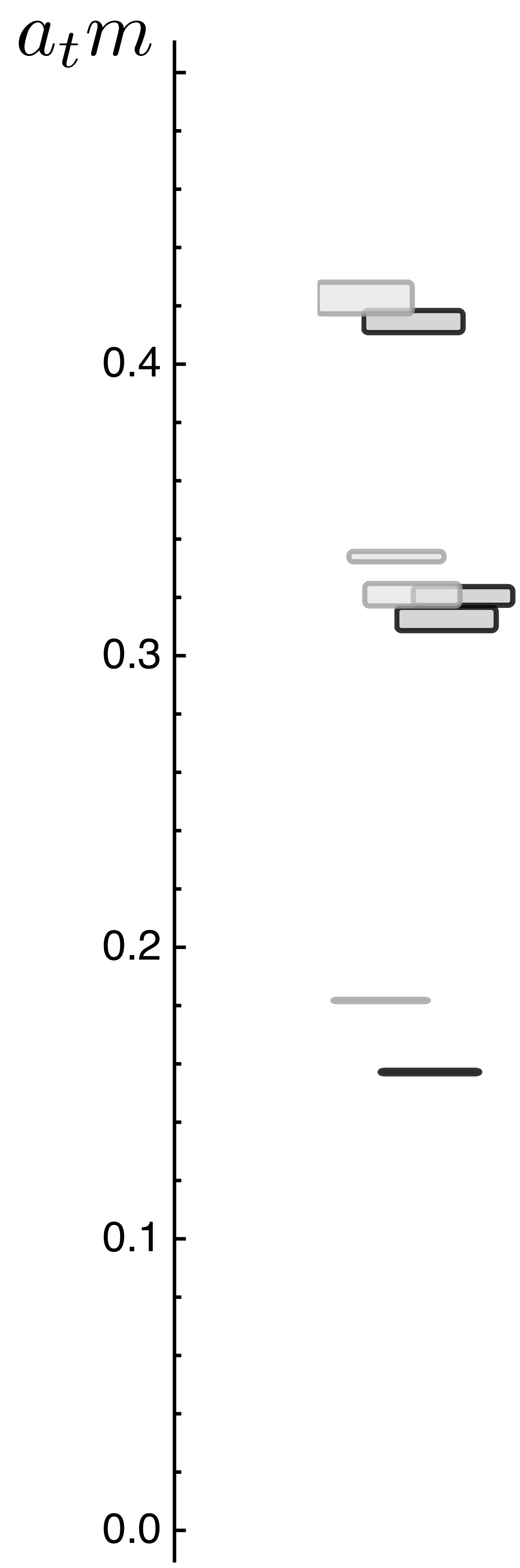
Monday, May 10, 2010

vector isoscalars

light-strange (ℓs) basis

$$\mathcal{O}_\ell^{\mathbf{\Gamma}} = \frac{1}{\sqrt{2}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d\right)$$

$$\mathcal{O}_s^{\mathbf{\Gamma}} = \bar{s}\mathbf{\Gamma}s$$



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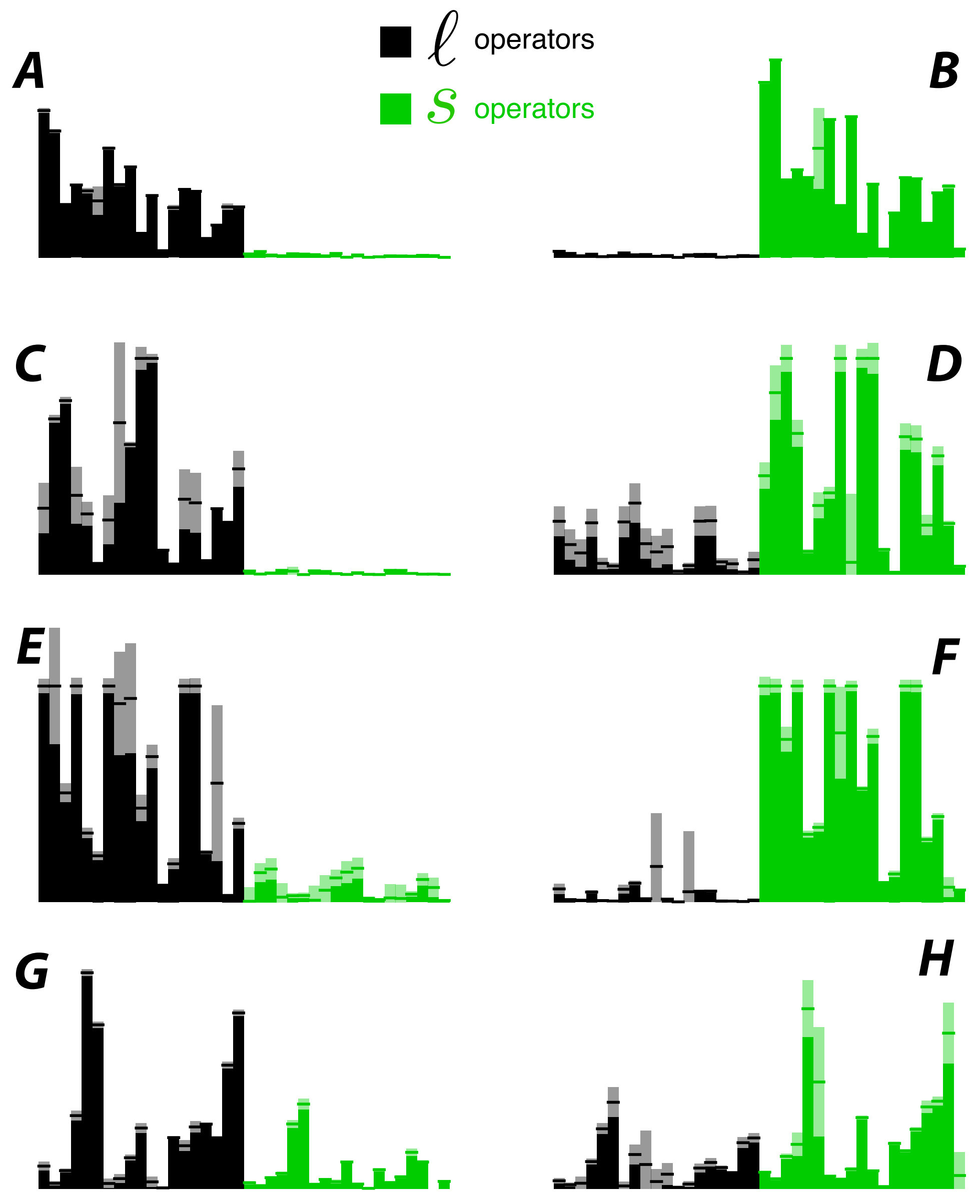

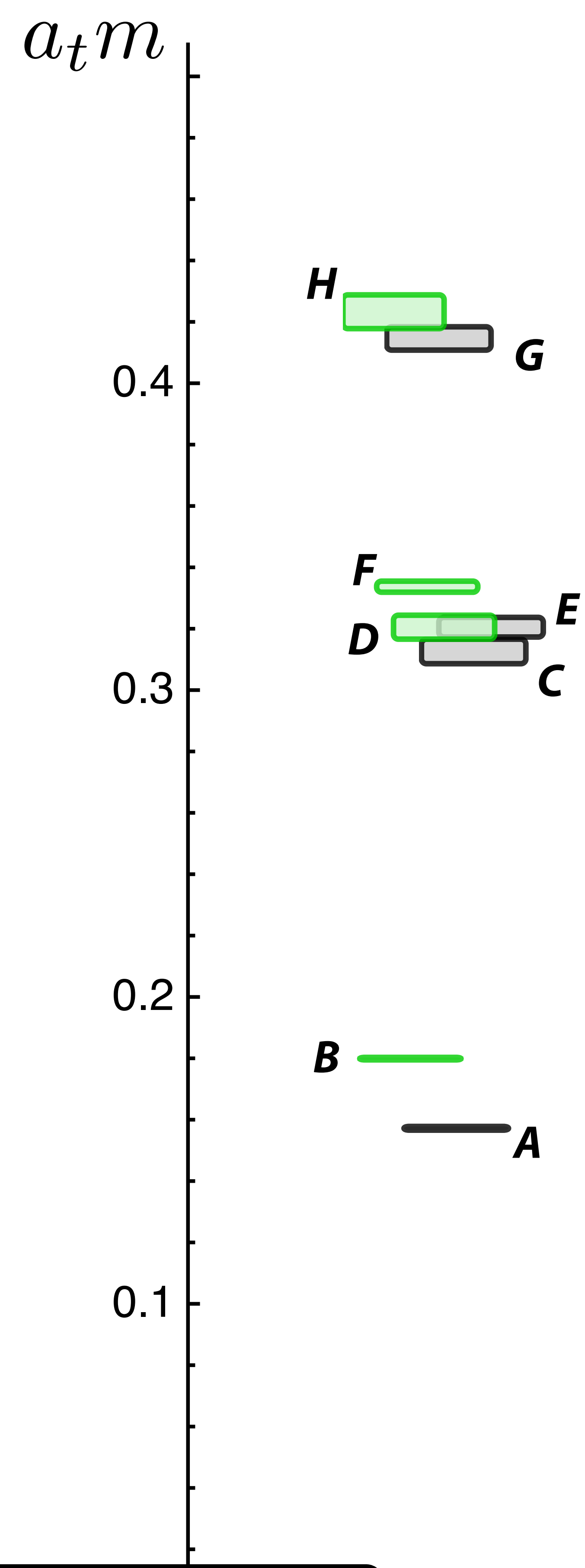

almost diagonal in the ℓs basis

Monday, May 10, 2010

vector isoscalars

light-strange (ℓs) basis

$$\mathcal{O}_\ell^{\mathbf{\Gamma}} = \frac{1}{\sqrt{2}}\left(\bar{u}\mathbf{\Gamma}u + \bar{d}\mathbf{\Gamma}d\right)$$

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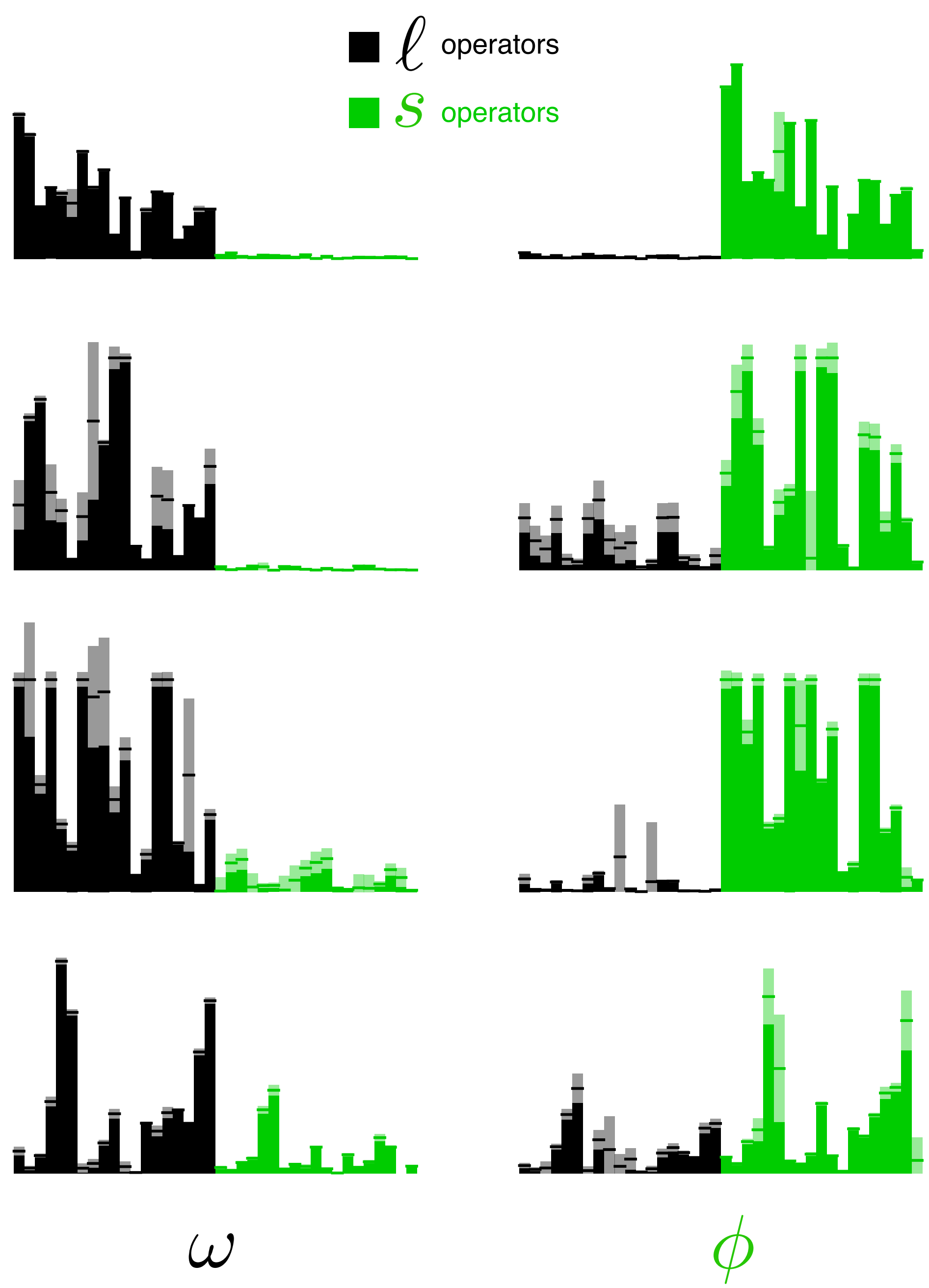
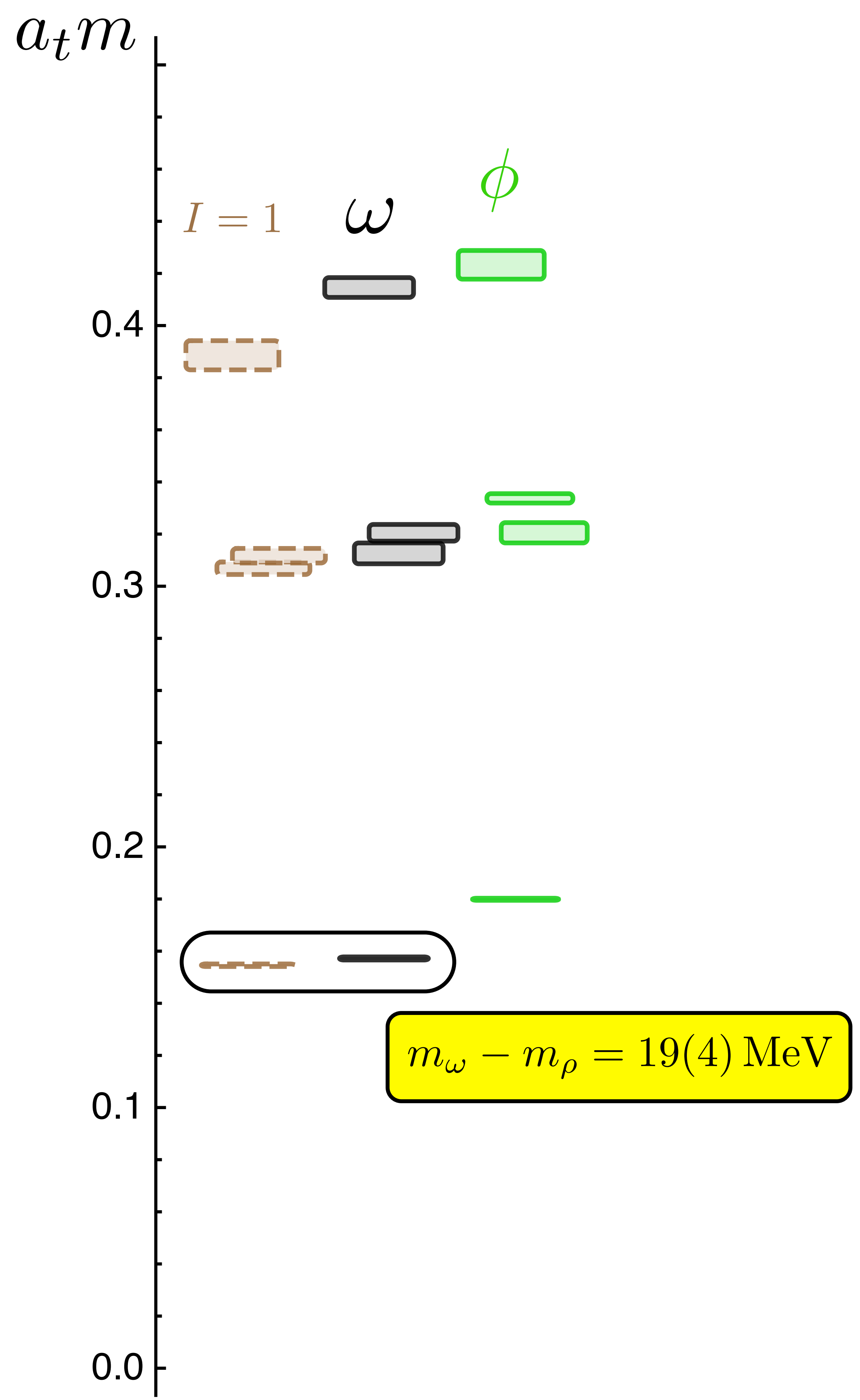
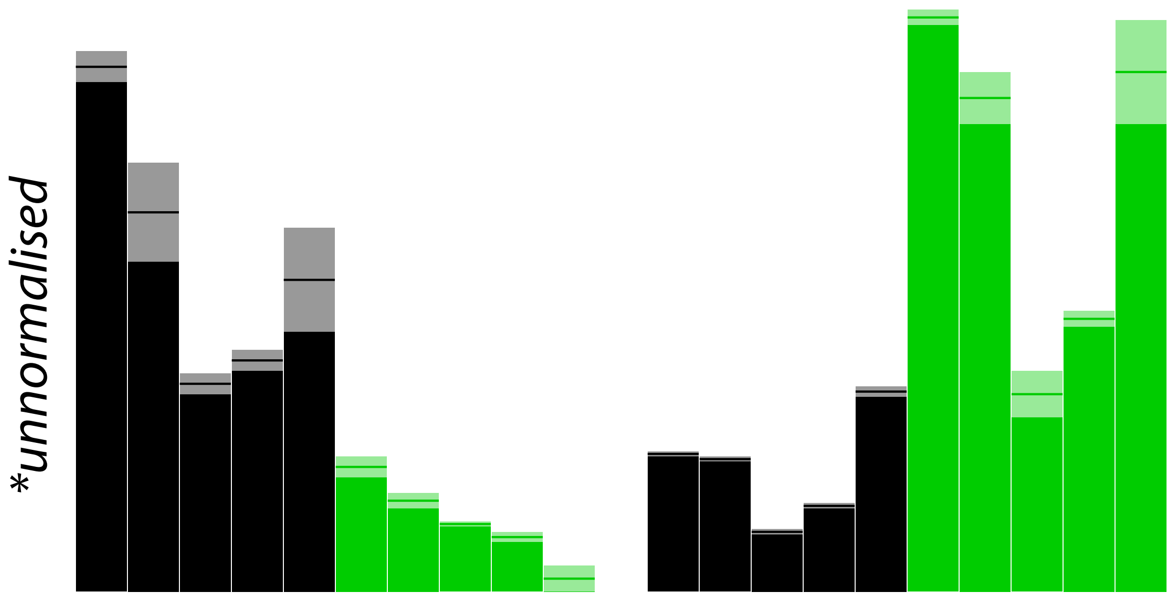




tensor isoscalars “f_2”

just look at the two lowest states initially

light-strange (ℓs) basis



f_2 — *dominantly light*

f_2' — *dominantly strange*

assume these two states are admixtures of light and strange (i.e. neglect any glueball component)

$$|f_2\rangle = \cos\alpha |\ell\bar{\ell}\rangle + \sin\alpha |s\bar{s}\rangle$$

$$|f_2'\rangle = -\sin\alpha |\ell\bar{\ell}\rangle + \cos\alpha |s\bar{s}\rangle$$

$$m(a_2) = .2614(13)a_t^{-1}$$

$$m(f_2) = .2596(31)a_t^{-1}$$

$$m(f_2') = .2807(9)a_t^{-1}$$

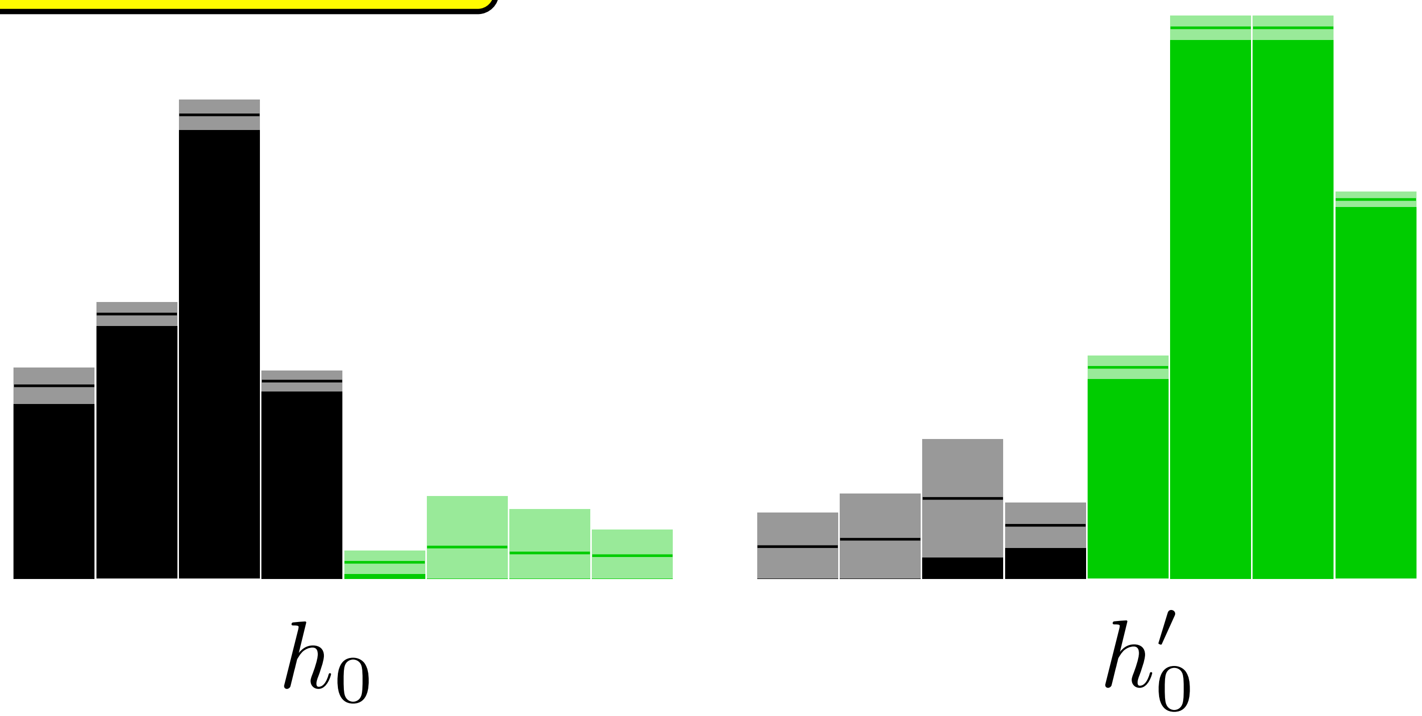
$$|\alpha| \sim 10^\circ$$

97% , 3%

exotics - h_0

0^{+-}

light-strange (ℓs) basis



dominantly light

dominantly strange

$$m(b_0) = .4237(36)\, a_t^{-1}$$
$$m(h_0) = .4347(39)\, a_t^{-1}$$
$$m(h_0') = .4552(43)\, a_t^{-1}$$

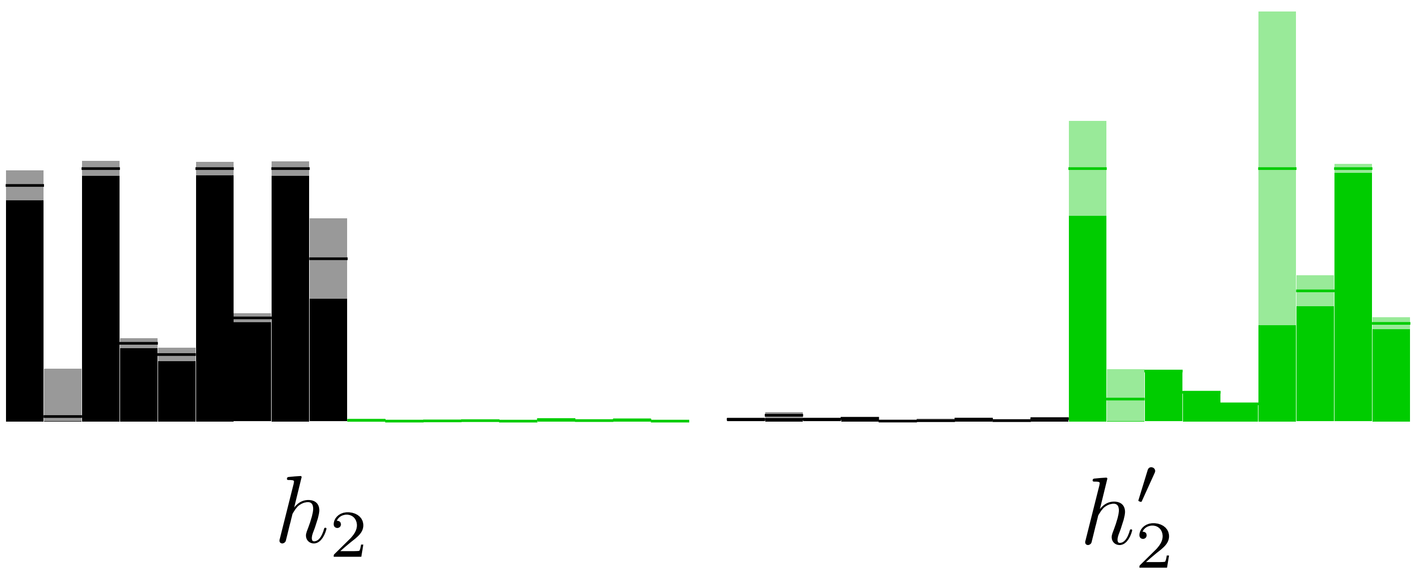
$|\alpha| \sim 14^\circ$

94% , 6%

exotics - h_2

2^{+-}

light-strange (ℓs) basis



dominantly light

dominantly strange

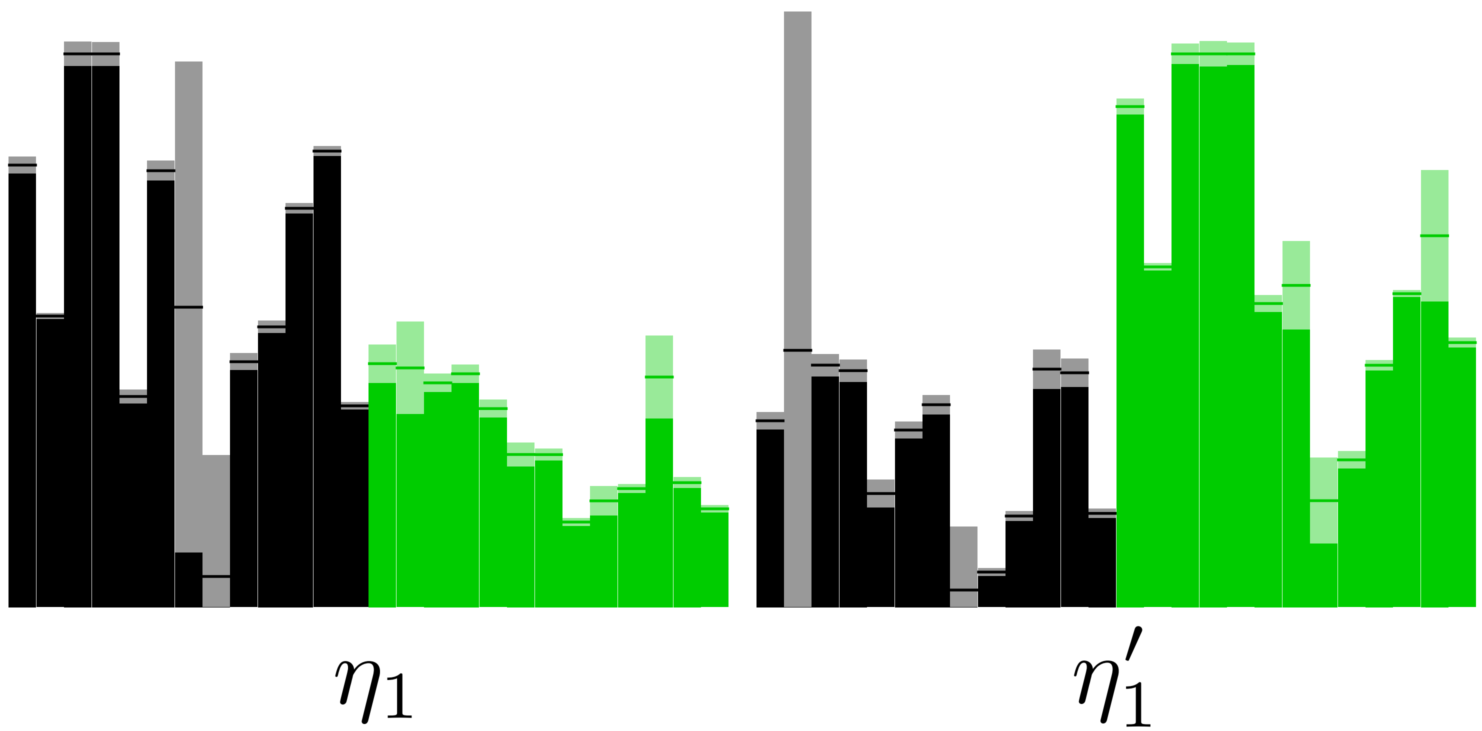
$$m(b_2) = .4386(29)\, a_t^{-1}$$
$$m(h_2) = .4377(60)\, a_t^{-1}$$
$$m(h_2') = .4624(29)\, a_t^{-1}$$

$|\alpha| \sim 2^\circ$

exotics - η1

1^{-+}

light-strange (ℓs) basis



neither flavour-mixed nor SU(3) mixed !

$|\alpha| \sim 24^\circ$

83% , 17%

$$m(\pi_1) = .3380(49)\, a_t^{-1}$$

$$m(\eta_1) = .3777(20)\, a_t^{-1}$$

$$m(\eta_1') = .4042(27)\, a_t^{-1}$$

summary

possible now to compute a spectrum of **isoscalar** mesons

very computationally intensive - use GPUs

computation on a single lattice - 400 MeV pions, 2.0^3 fm^3 volume

pseudoscalars → η ~ **8**, η′ ~ **1** & similar excited states

vectors → ω(light), ϕ(strange) & similar excited states

tensors → f_2(mostly light), f_2'(mostly strange) with small mixing angle

even parity exotics mixed as light, strange

1^{-+} non-ideal admixture of light, strange