scattering & elastic resonances from finite-volume field theory

Jo Dudek

HadronSpectrumCollab. *(JLAB) :* **arXiv:1011.6352** ($\pi\pi \to \pi\pi$ I=2)

EuropeanTwistedMassCollab. : arXiv:1011.5288 ($\pi\pi \to \rho \to \pi\pi$ I=1)

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or
"why don't you guys just calculate decay widths using lattice QCD"

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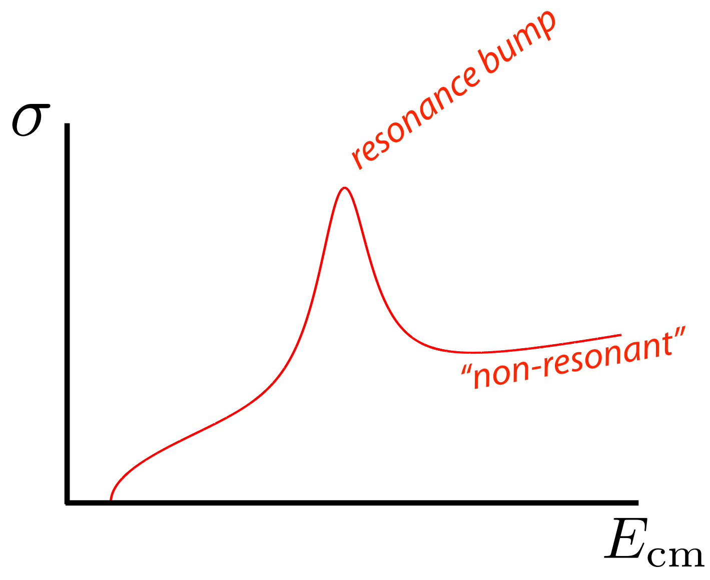
scattering & resonances

e.g. elastic $\pi\pi \to \pi\pi$

experiment

cross-section

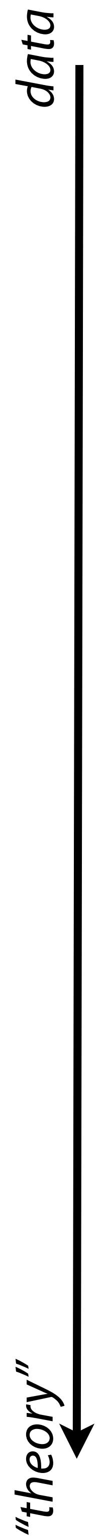


data

"theory"

scattering & resonances

e.g. elastic $\pi\pi \to \pi\pi$



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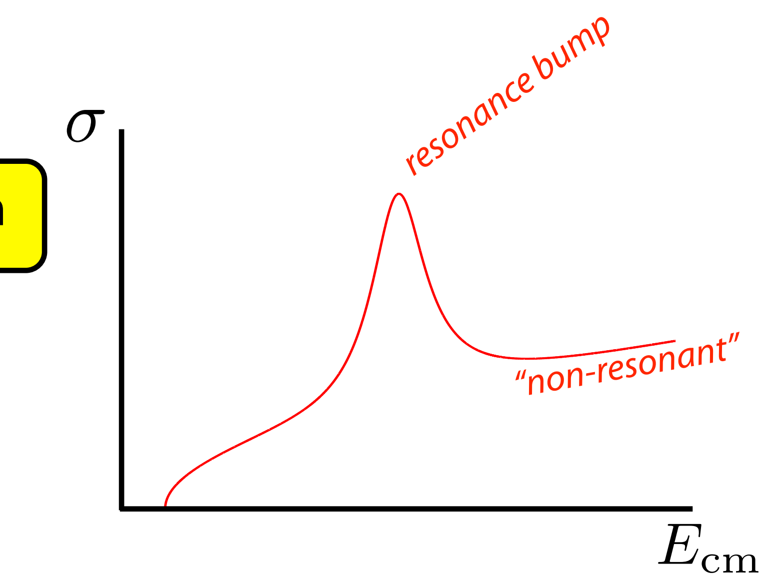
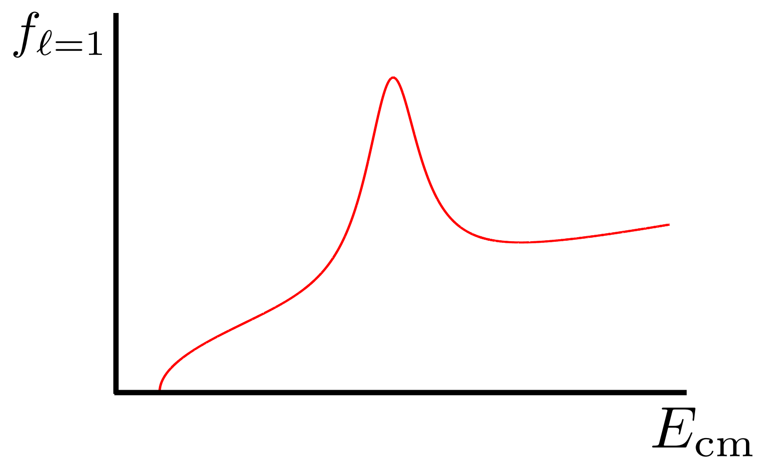


partial-wave amplitude



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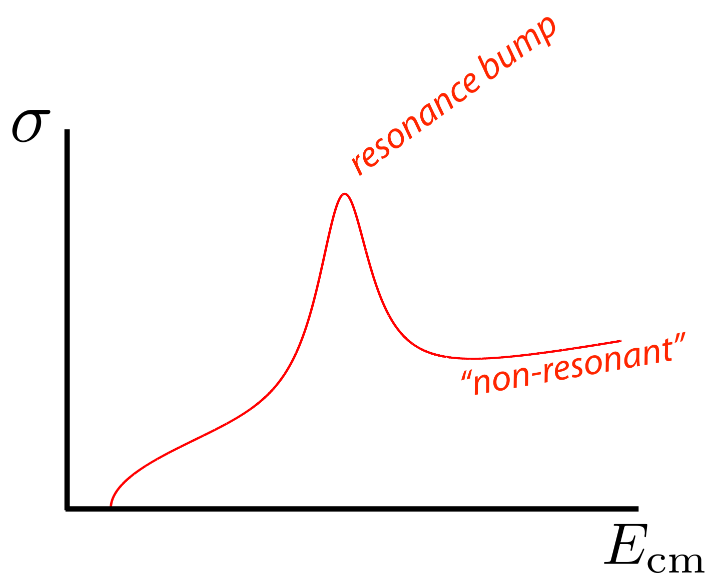

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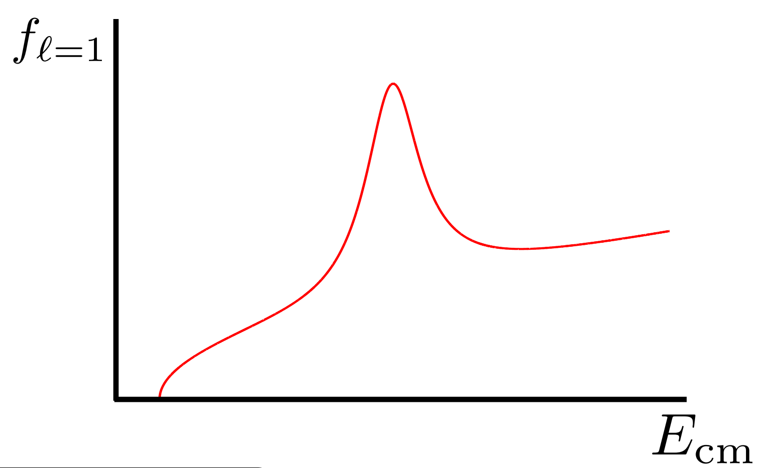
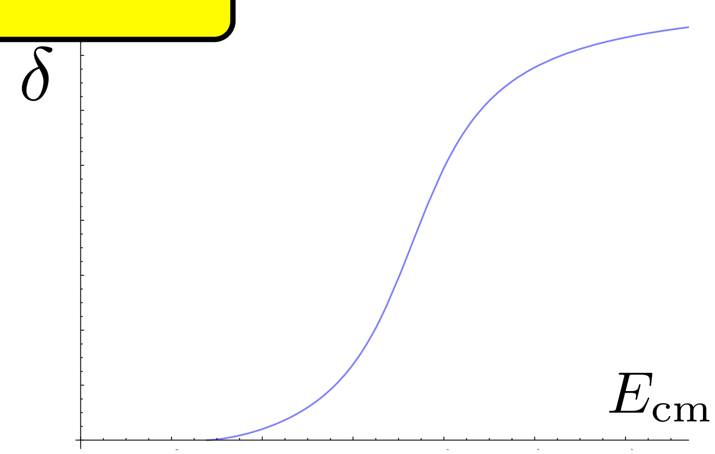


elastic phase-shift



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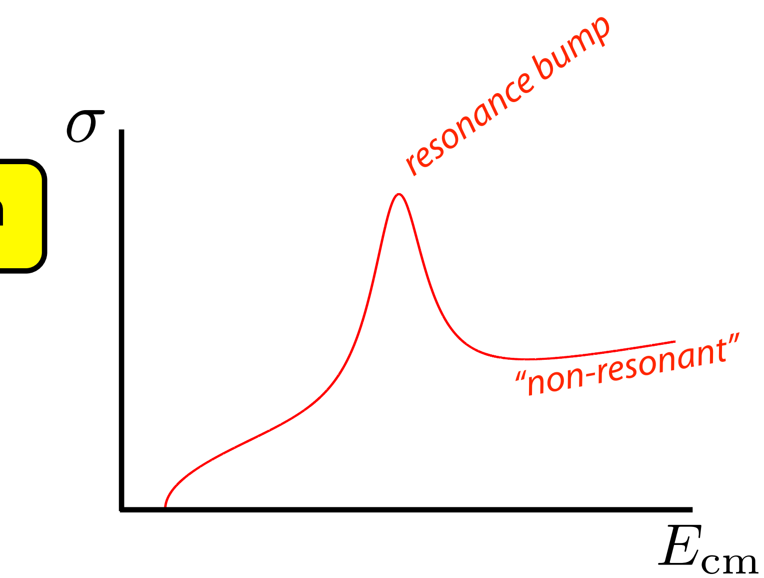

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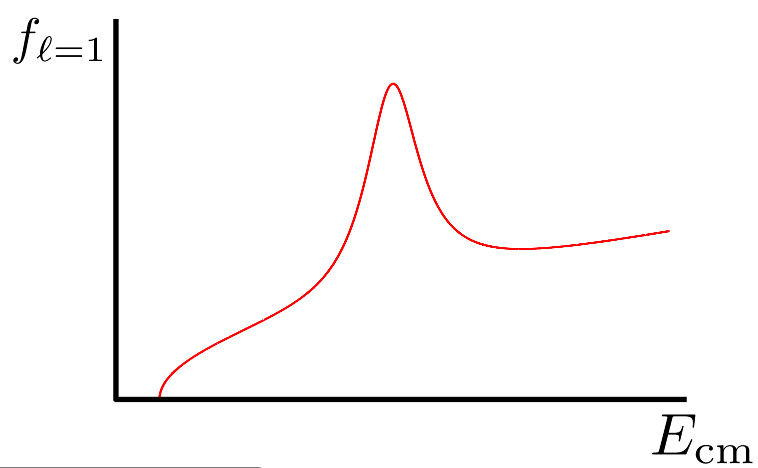

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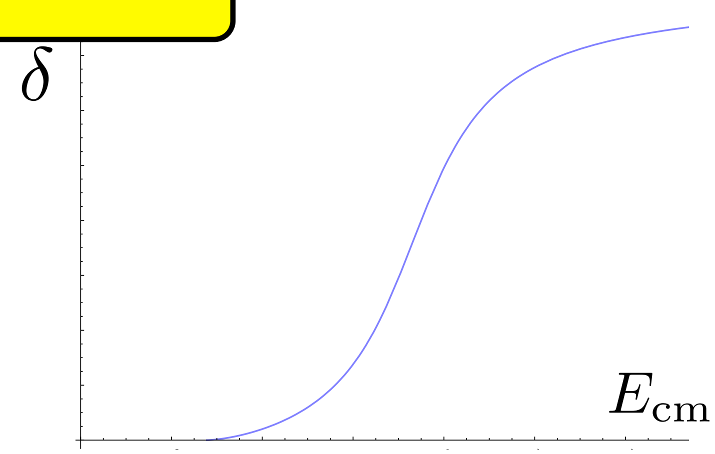


"theory"

Euclidean field theory

correlation function

e.g. $\langle 0 \big| \big(\bar{\psi}\gamma_i\psi\big)_t \big(\bar{\psi}\gamma_i\psi\big)_0 \big| 0 \rangle$

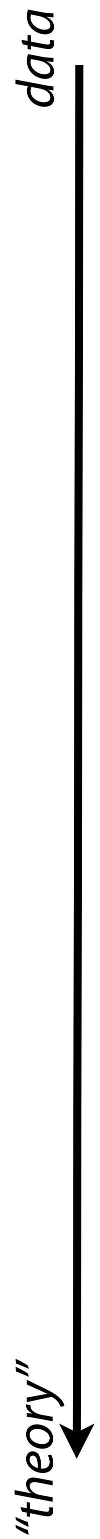
$$= \int dE\, \rho(E)\, e^{-Et}$$

spectral density

scattering & resonances

e.g. elastic $\pi\pi \to \pi\pi$

experiment



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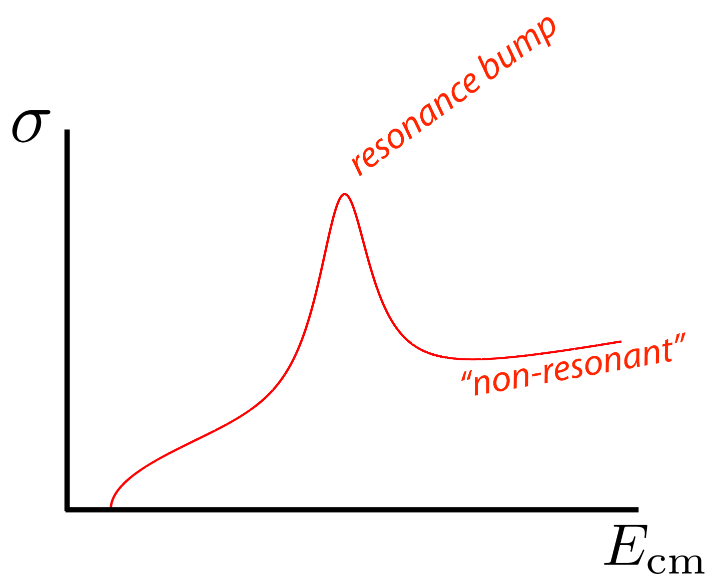

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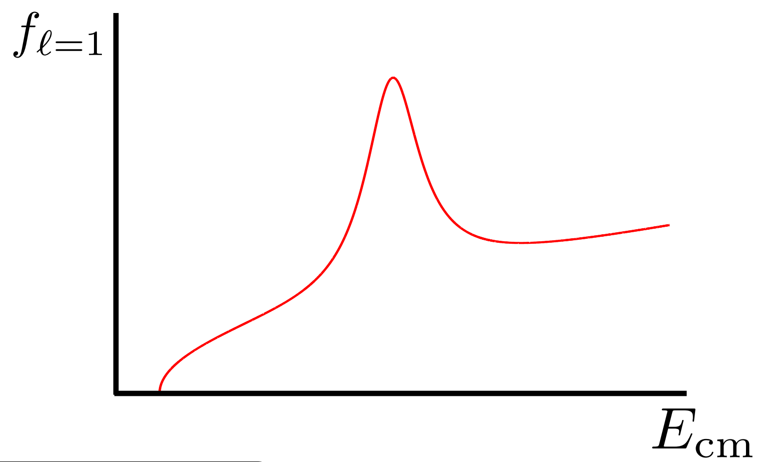

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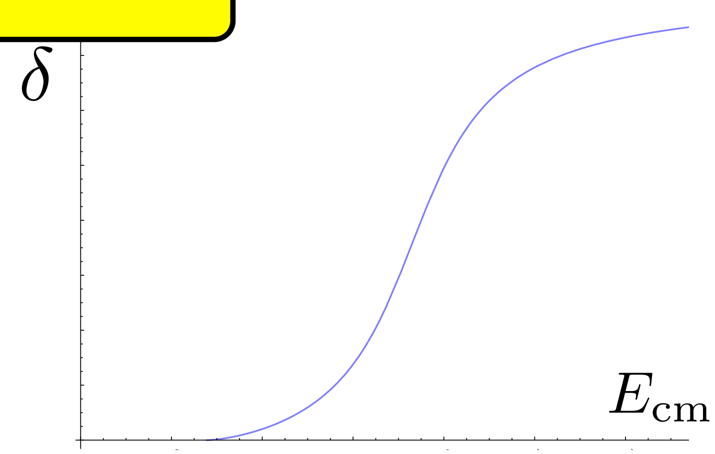

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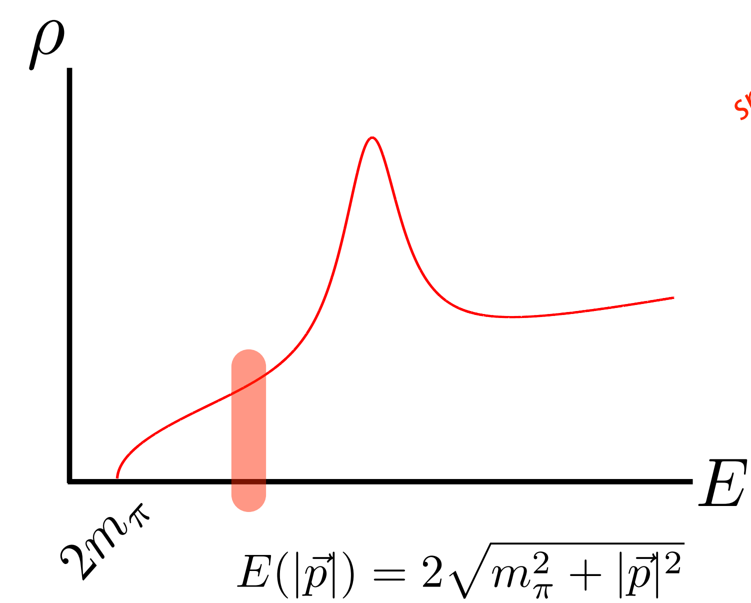

$E(|\vec{p}|) = 2\sqrt{m_\pi^2 + |\vec{p}|^2}$

$\int d\hat{p}\; Y_1(\hat{p})\, \big|\pi(\vec{p})\pi(-\vec{p})\big\rangle$

continuous pion momenta

⇒ continuous energy spectrum

in a finite volume ...

spatially a cube with periodic boundary conditions (torus)

$$e^{ip(x+L)} = e^{ipx}$$

$$\implies e^{ipL} = 1$$

$$\implies p = \frac{2\pi}{L}$$

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

in a finite volume ...

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$$E_n = 2\sqrt{m_\pi^2 + \left(\tfrac{2\pi}{L}\right)^2 n^2}$$

$$n^2 = |\vec{n}|^2$$

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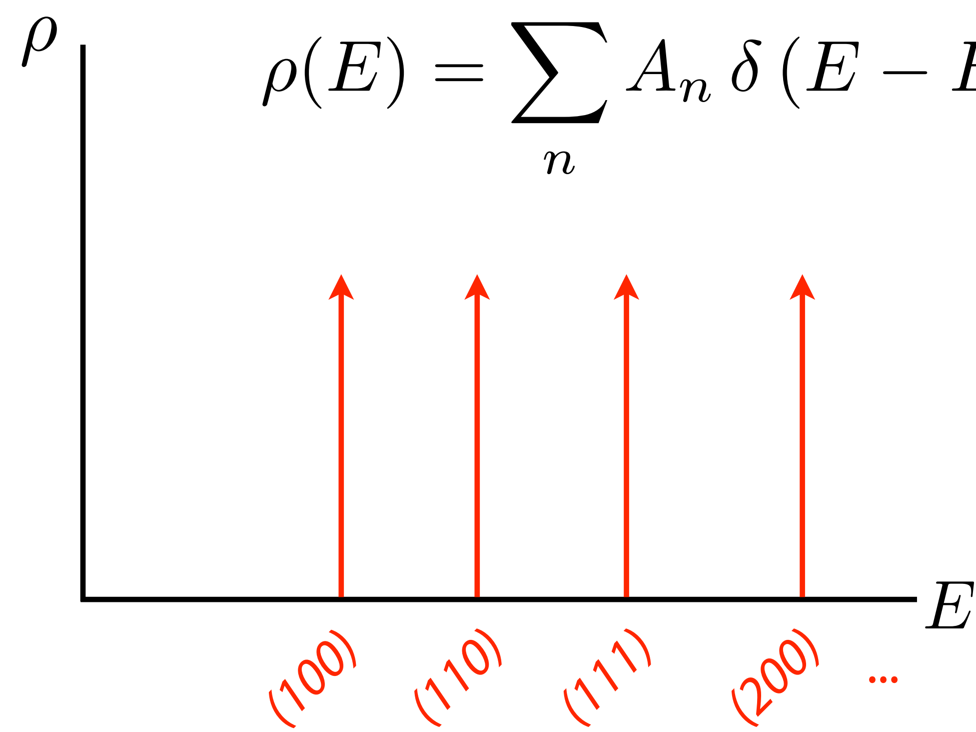
$$\rho(E) = \sum_n A_n \, \delta\left(E - E_n\right)$$



discrete allowed pion momenta

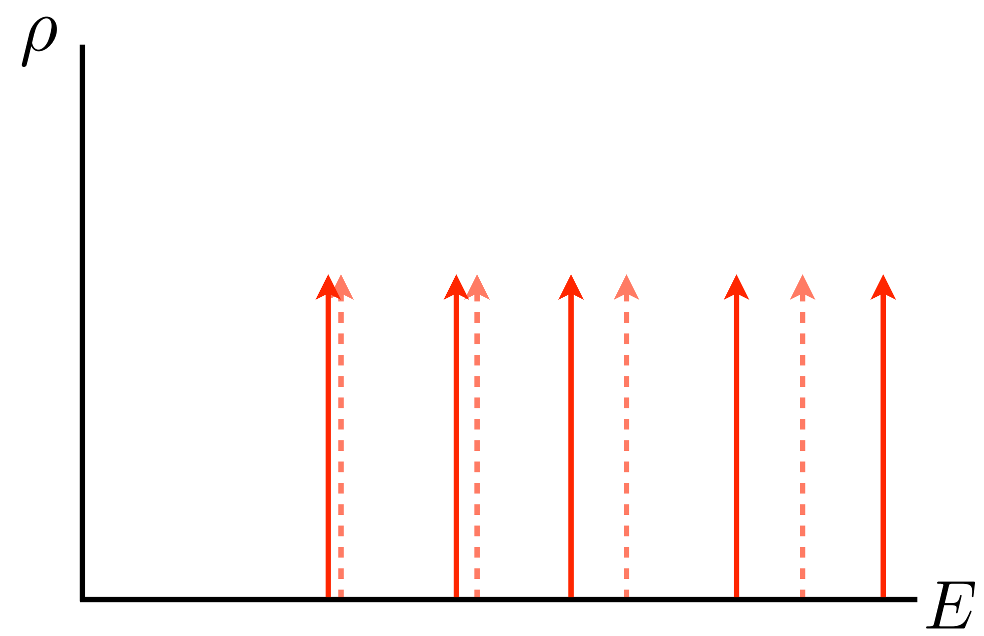
⇒ discrete energy spectrum

in a finite volume ...

in fact this spectrum only present for ***non-interacting*** pions

$\pi\pi$ interaction manifests itself as a shifting of the discrete levels

$$E_n = E_n^{(0)} + \Delta E(\text{“}V_{\pi\pi}\text{”}, L)$$



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Lüscher's finite-volume formalism

NPB 354 p531 (1991)
NPB 364 p237 (1991)

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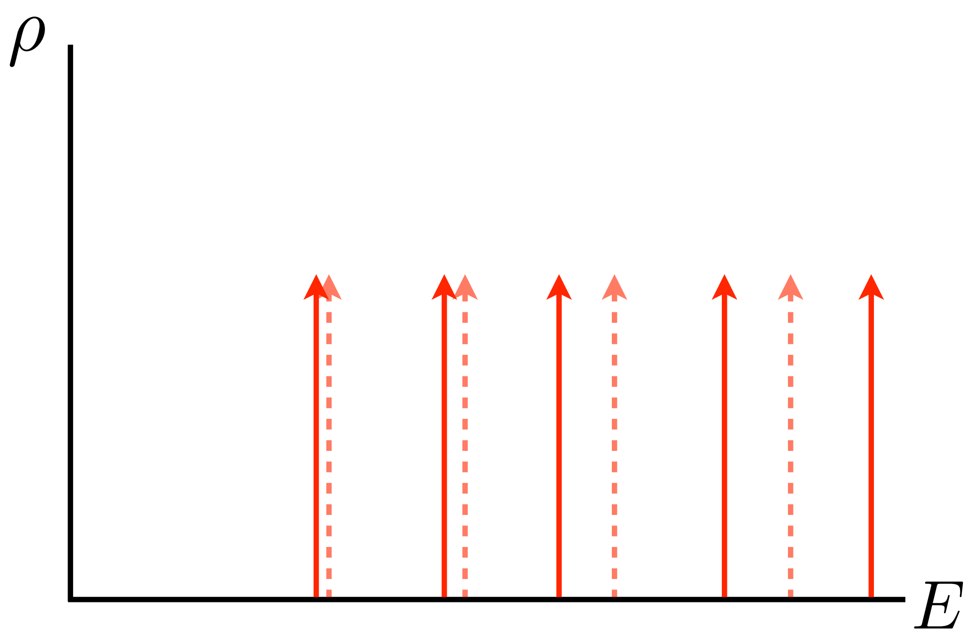

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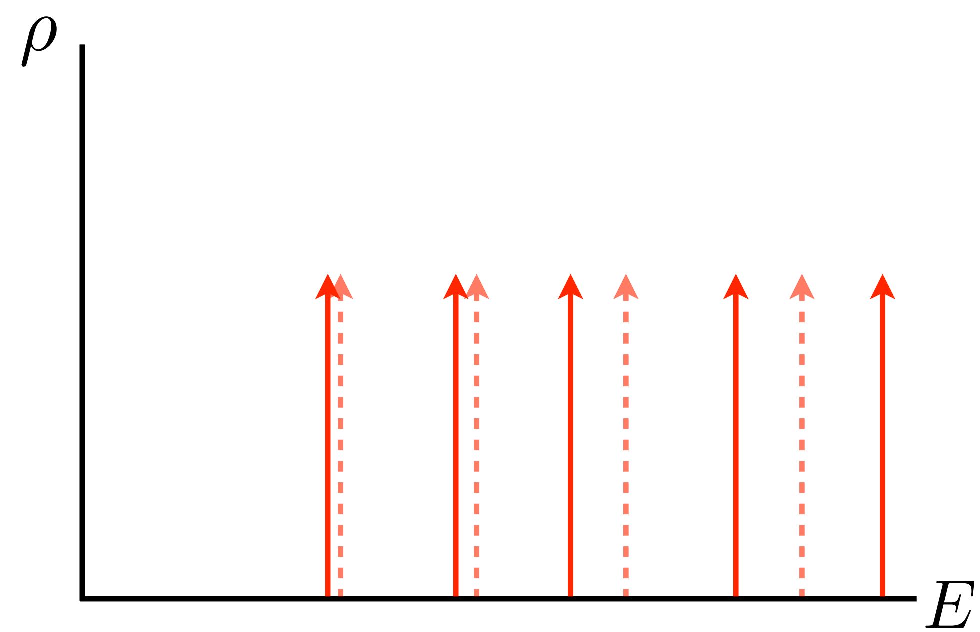

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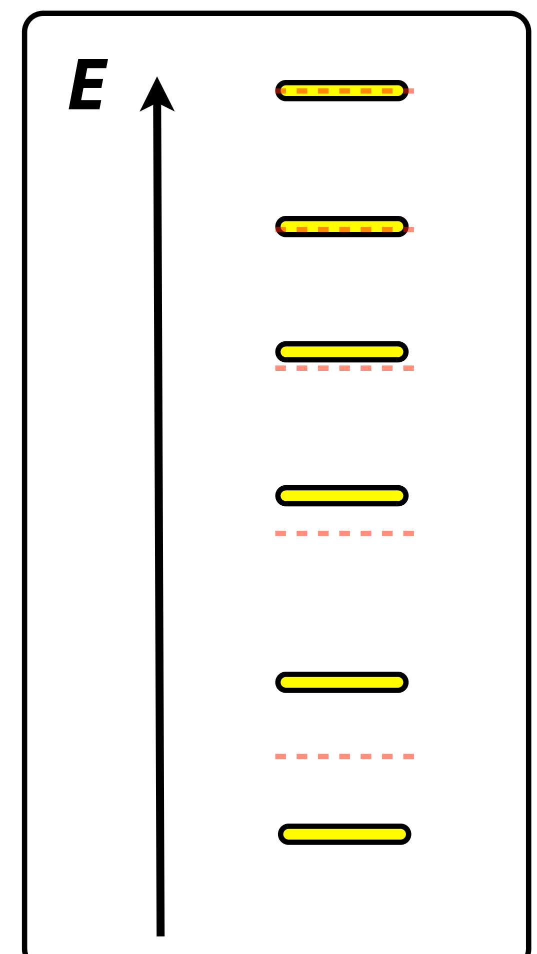
$$E_n = E_n^{(0)} + \Delta E(\delta(E_n), L)$$

“invert” the equation

$$\delta(E_n) = \mathrm{fn}(E_n, L)$$

finite volume energy spectrum maps to the phase-shift

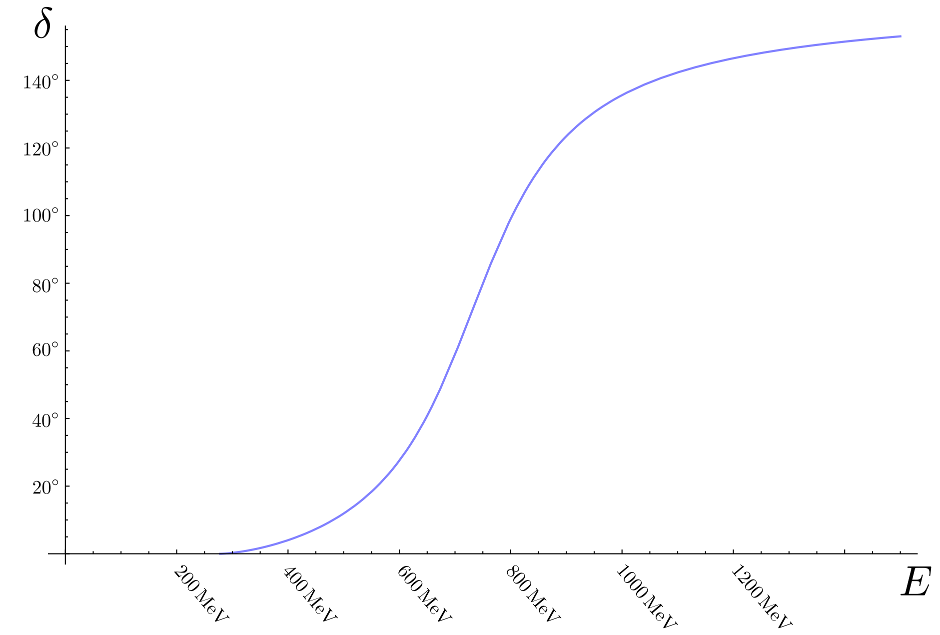


an example

suppose we could do calculations at the physical pion mass

we'd expect to see the ρ appear as a resonance in $\pi\pi$

input the experimental ρ phase-shift in the "uninverted" equations to give the finite volume energy spectrum



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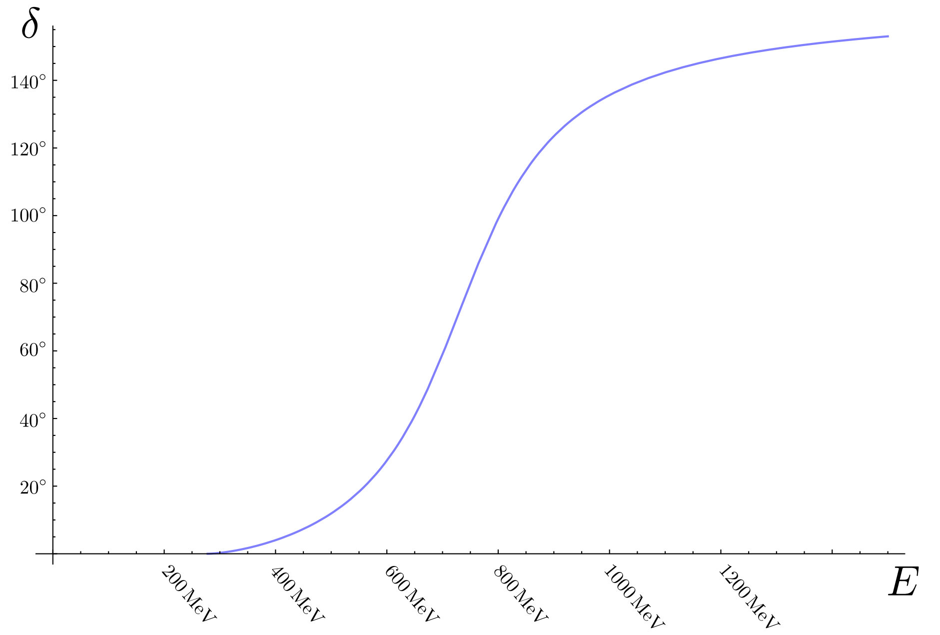

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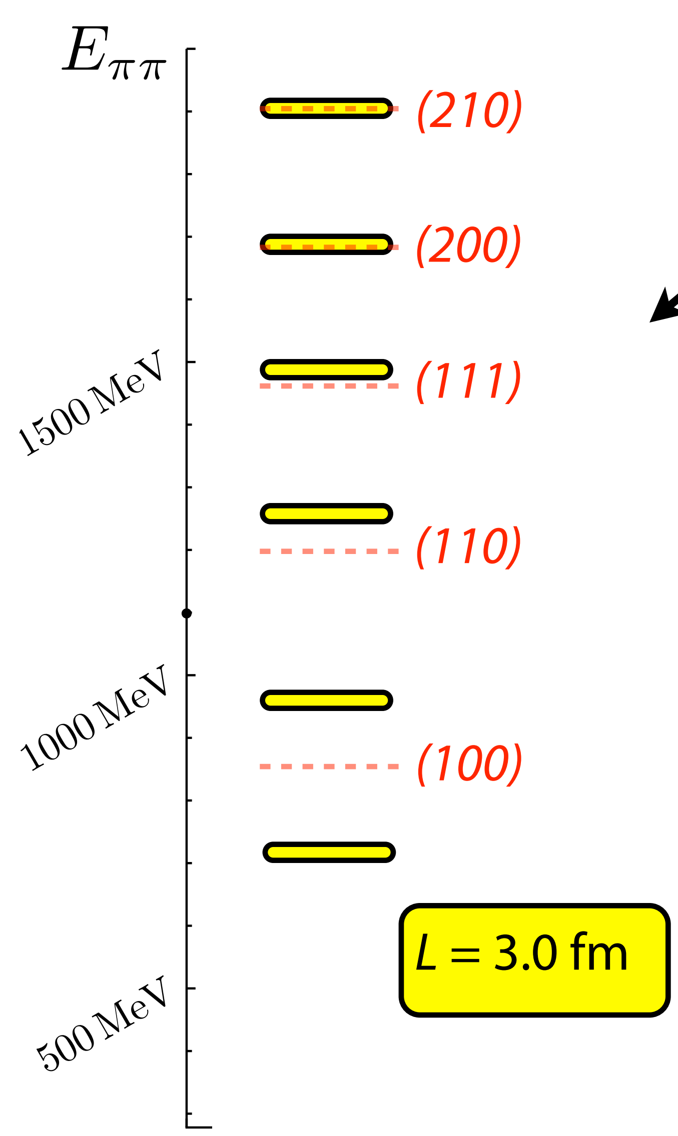

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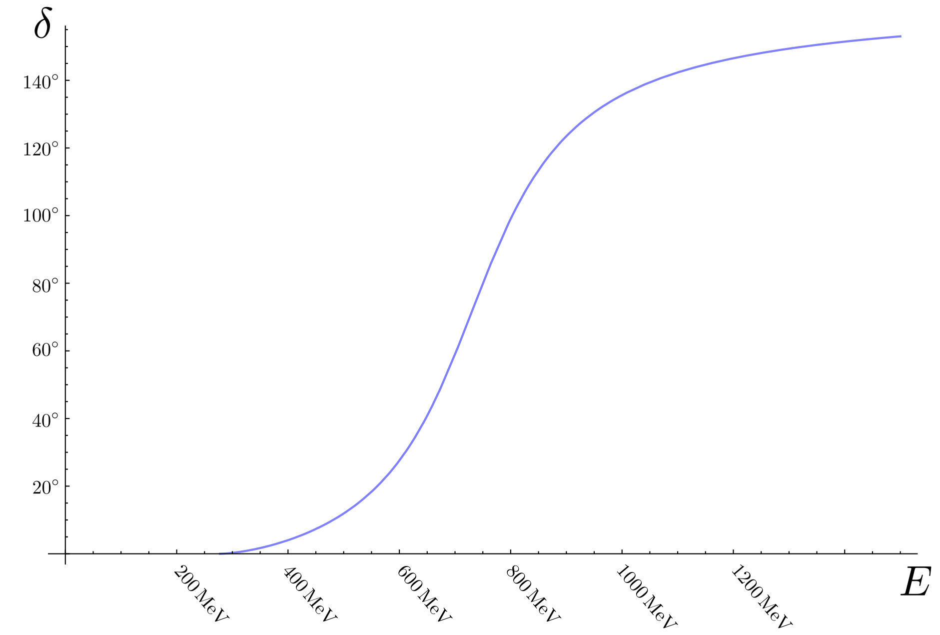

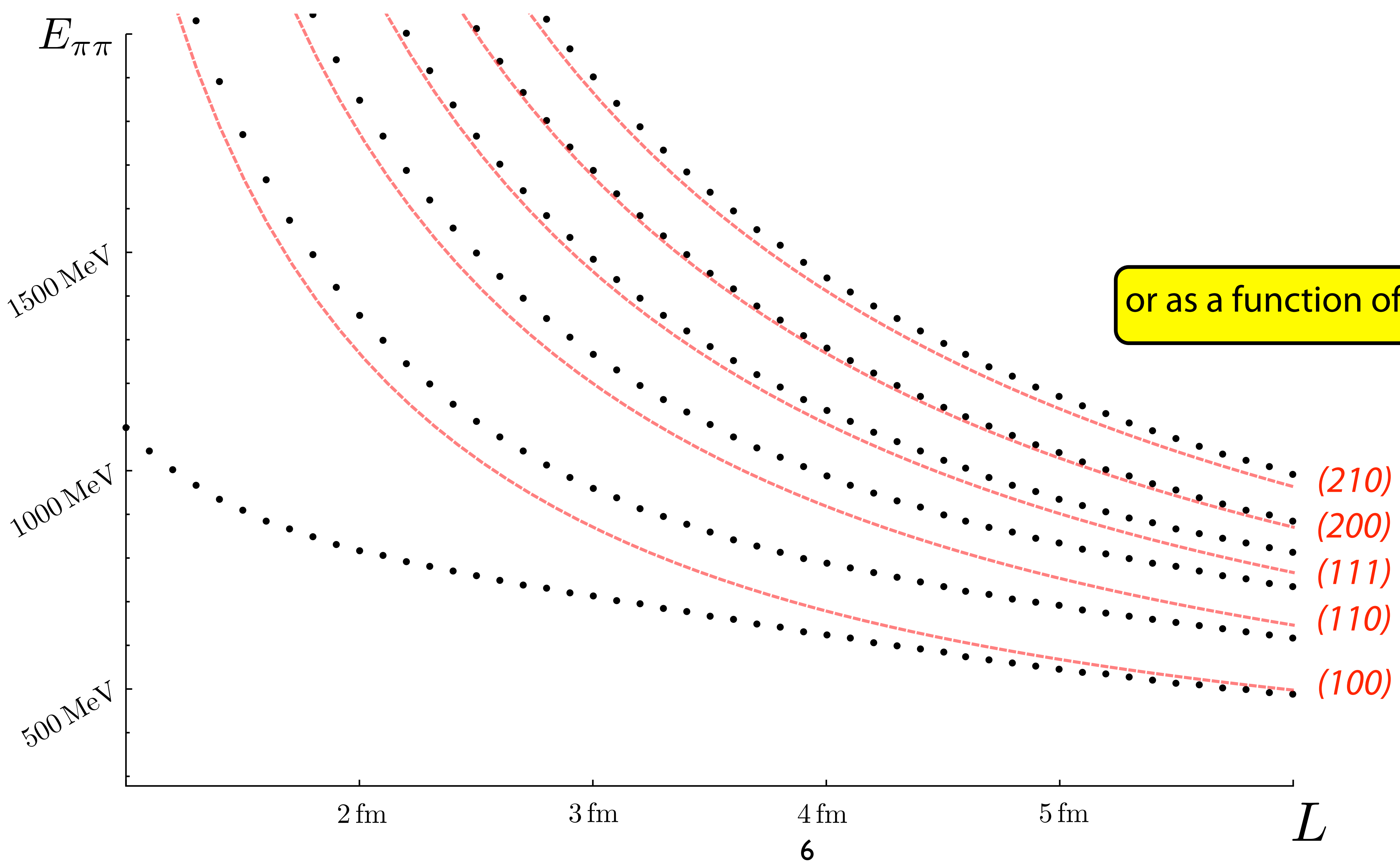

or as a function of the box-size

an example

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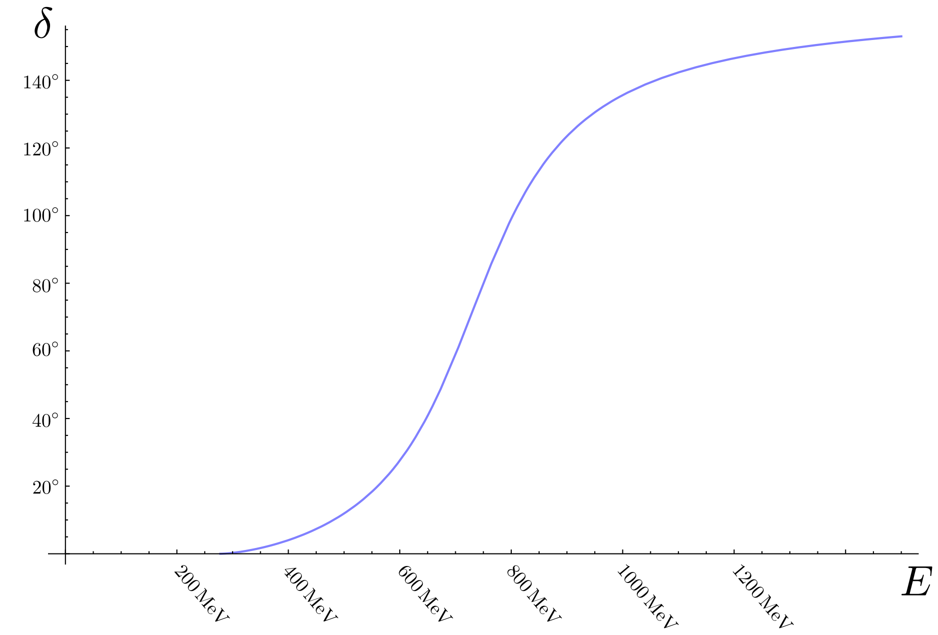
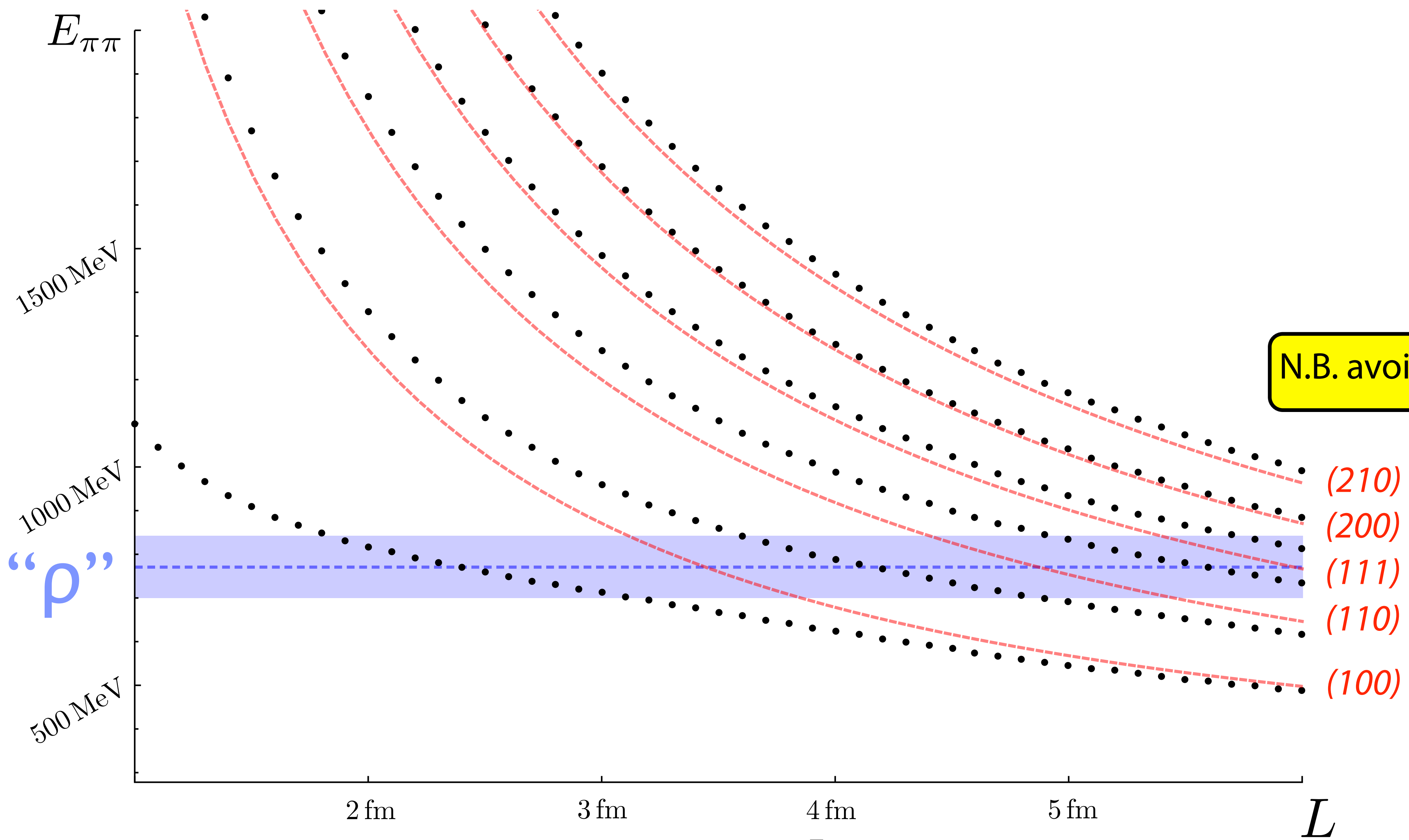
we'd expect to see the ρ appear as a resonance in ππ

input the experimental ρ phase-shift in the "uninverted" equations to give the finite volume energy spectrum

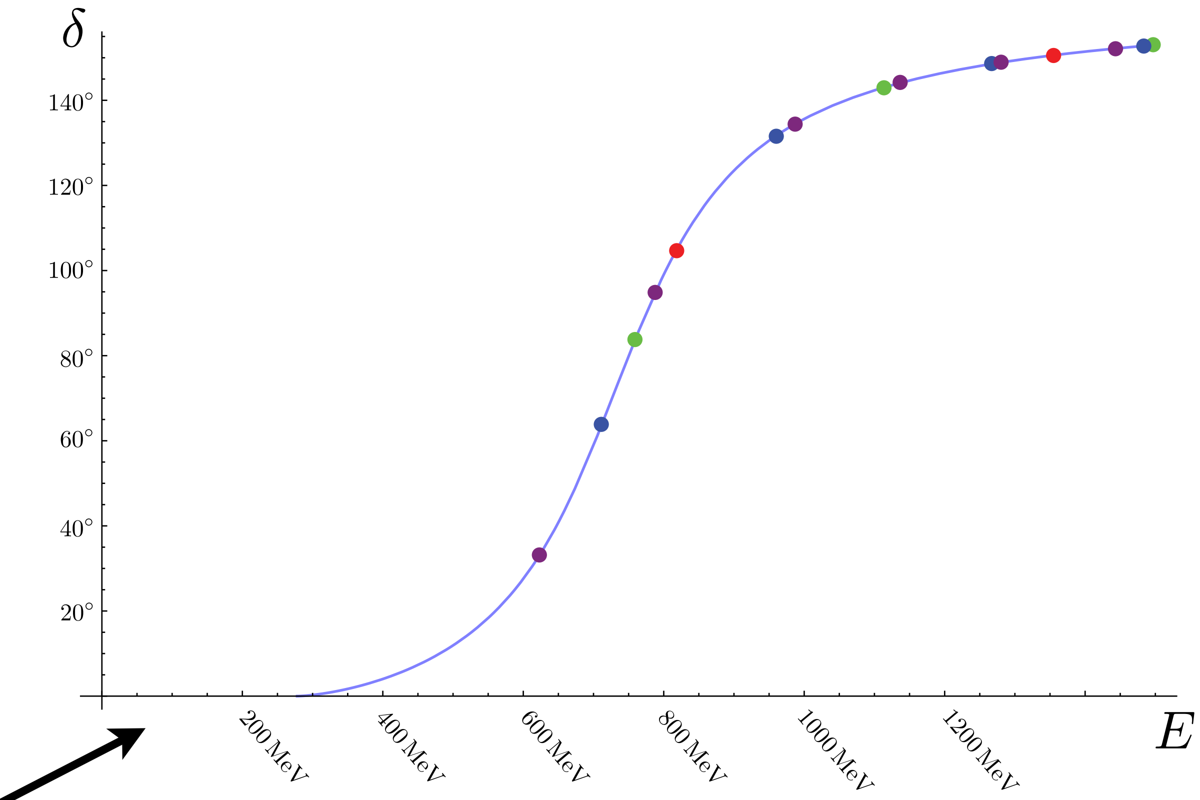


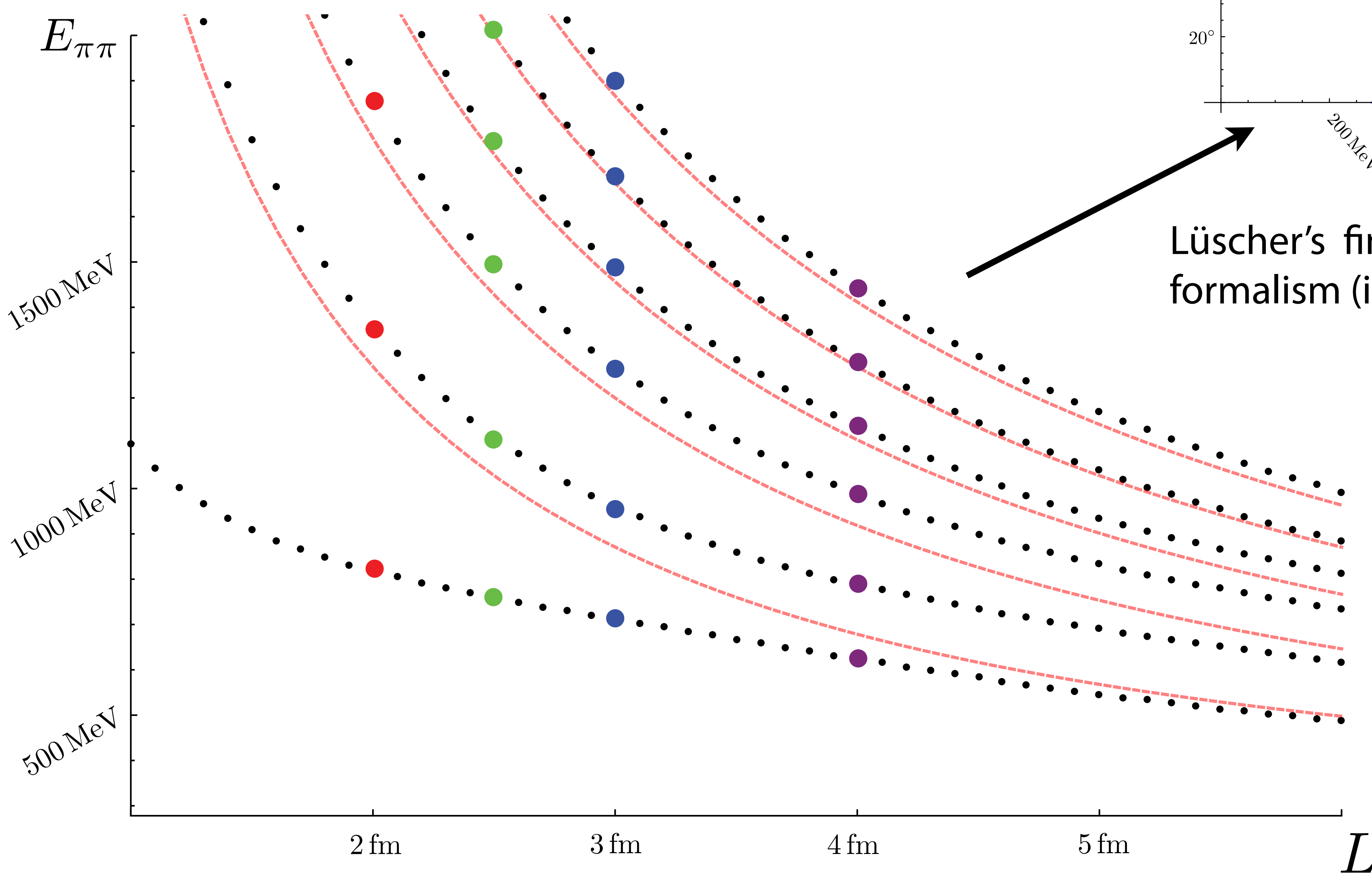


N.B. avoided level crossings

an example

do the lattice calculation at finite volume(s)





Lüscher's finite-volume formalism (inverted)

$$\delta(E_n) = \mathrm{fn}(E_n, L)$$

EuropeanTwistedMassCollab. : arXiv:1011.5288 ($\pi\pi \rightarrow \rho \rightarrow \pi\pi$ I=1)

two flavour dynamical lattices (no strange)

a = 0.079 fm (reasonably fine)

twisted mass formalism introduces some 'oddities'

probably not important here

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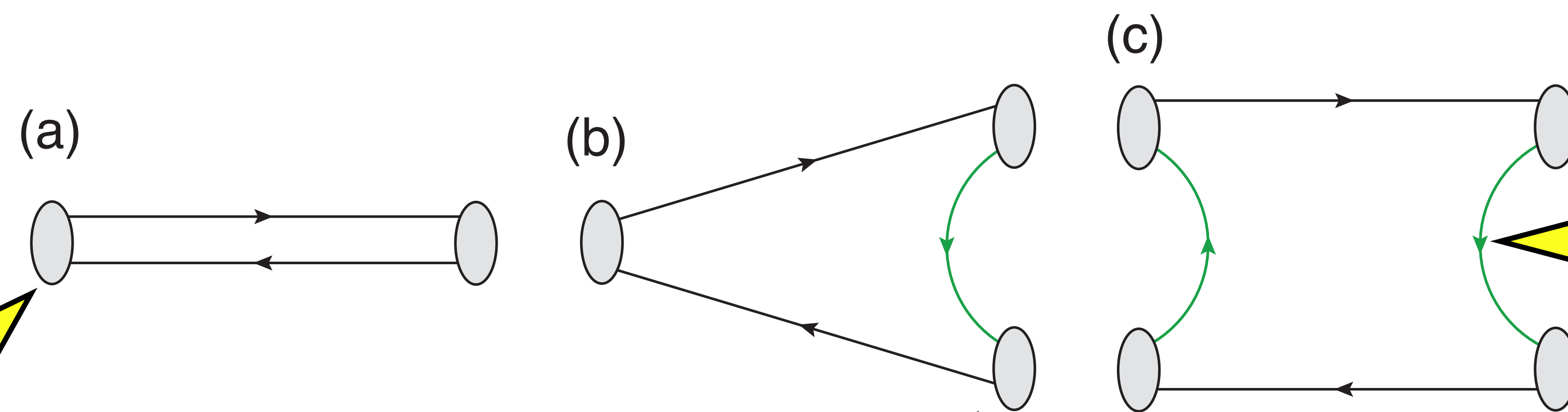

one rho-like operator

one $\pi\pi$-like operator

"noisy" correlator construction

European Twisted Mass Collab. : arXiv:1011.5288 ($\pi\pi \to \rho \to \pi\pi$ I=1)

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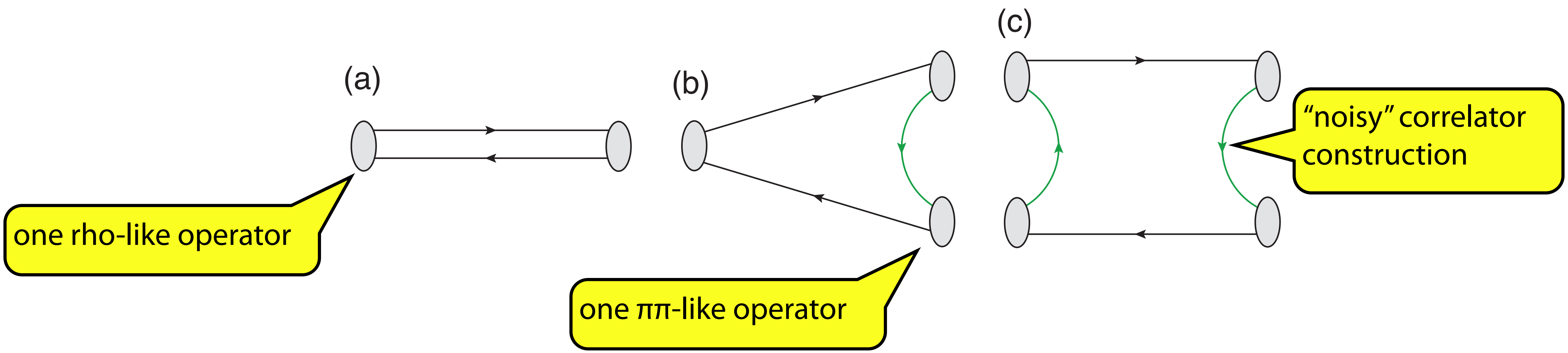

variational analysis in a two-operator basis → "lowest" two finite-volume eigenstates

EuropeanTwistedMassCollab. : arXiv:1011.5288 ($\pi\pi \to \rho \to \pi\pi$ I=1)

rest frame

P=100

P=110

EuropeanTwistedMassCollab. : arXiv:1011.5288 (ππ→ρ→ππ I=1)

rest frame
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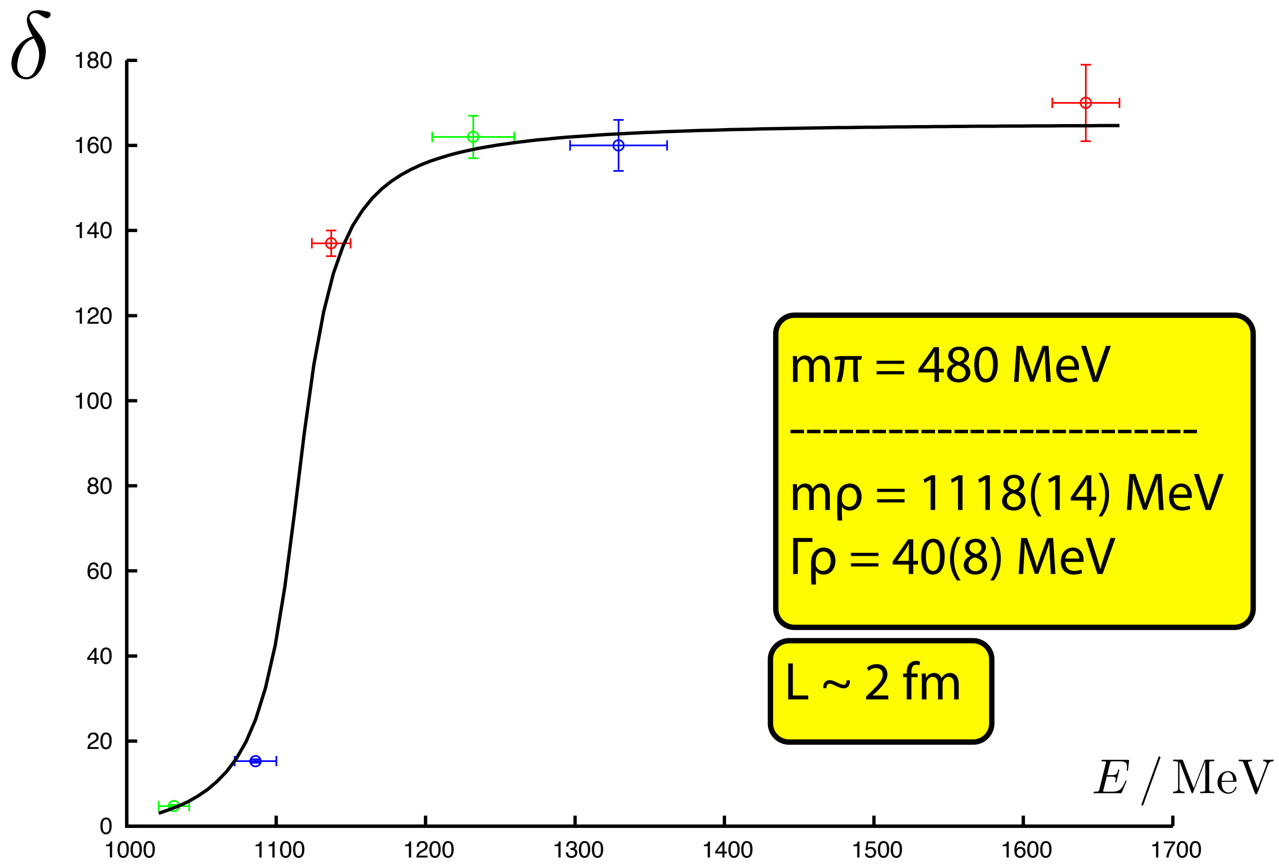
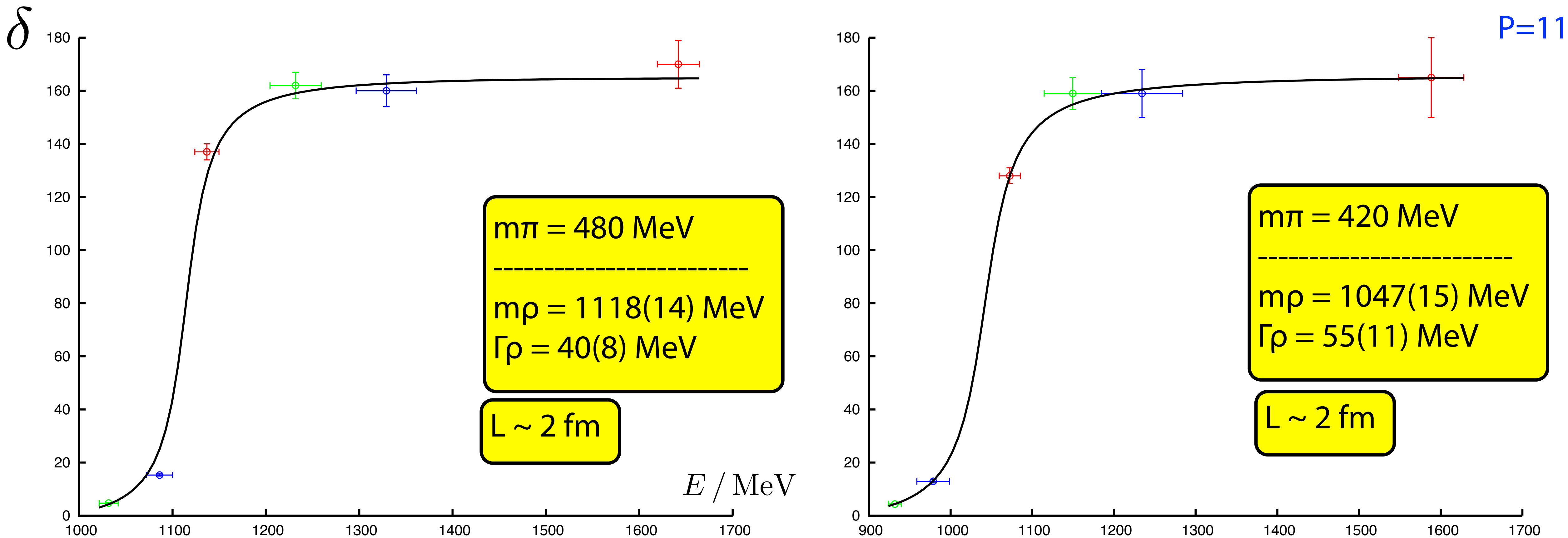


mπ = 480 MeV
----
mρ = 1118(14) MeV
Γρ = 40(8) MeV

L ~ 2 fm

EuropeanTwistedMassCollab. : arXiv:1011.5288 (ππ→ρ→ππ I=1)

rest frame
P=100
P=110



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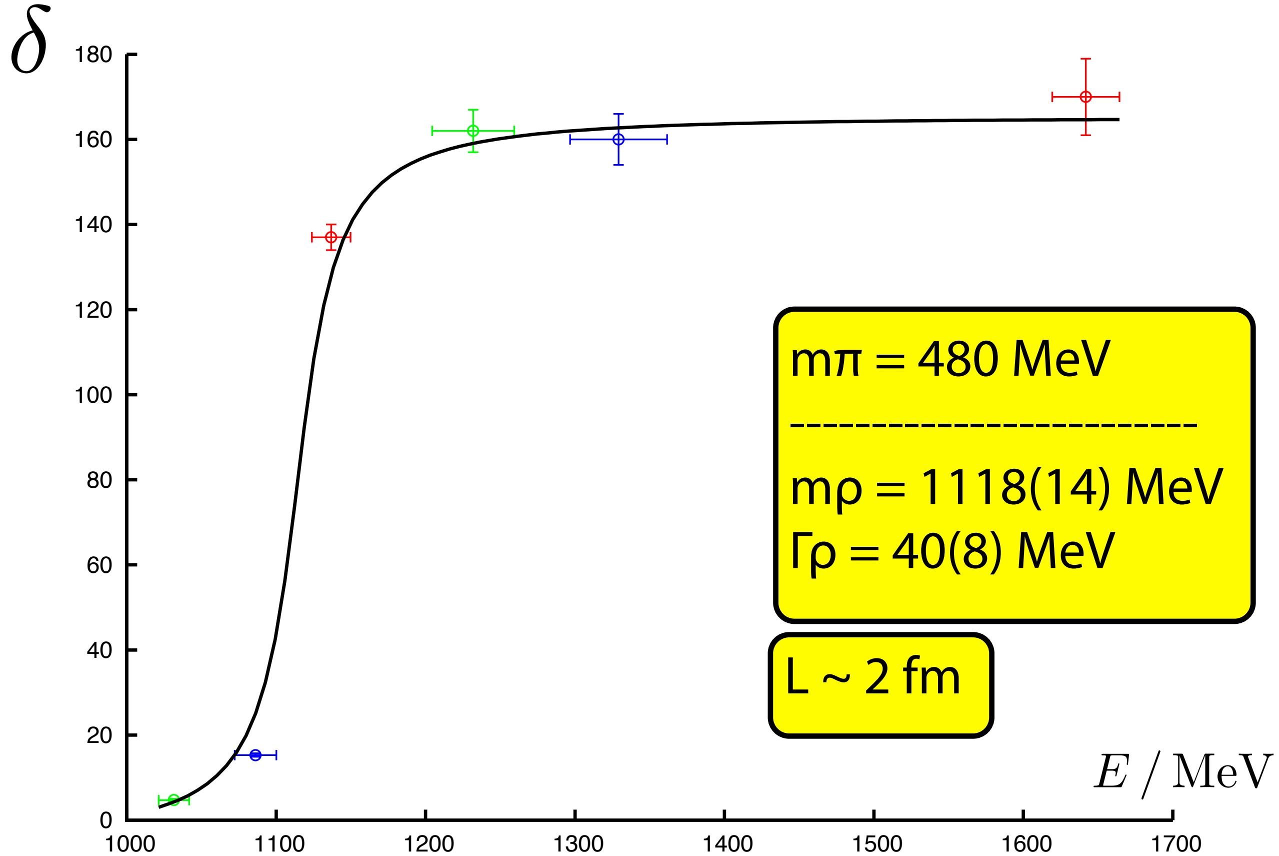

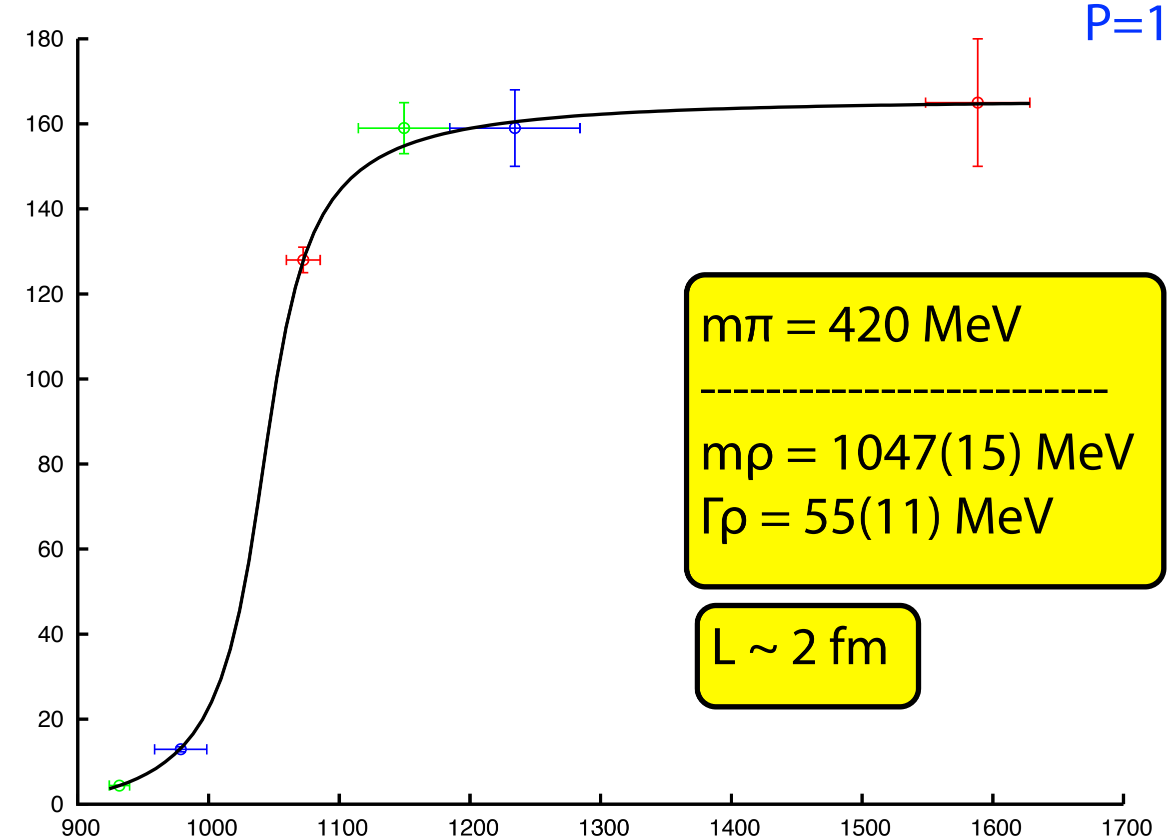

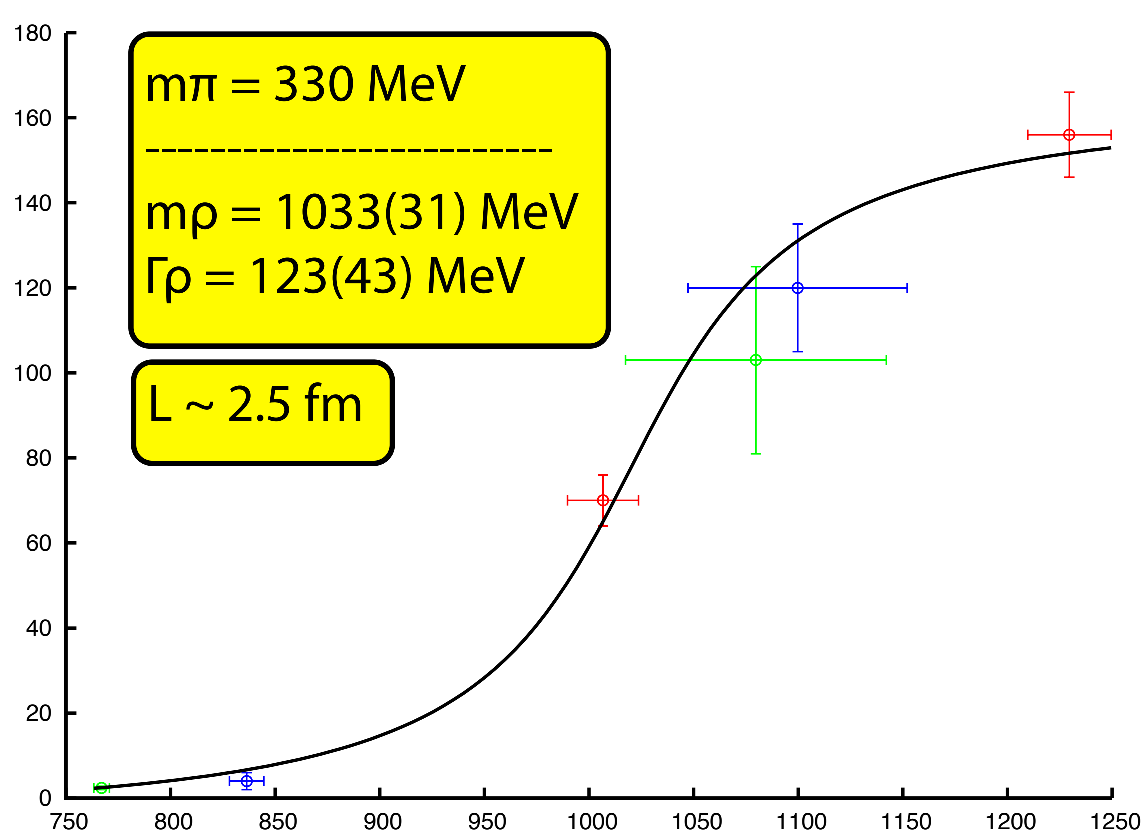

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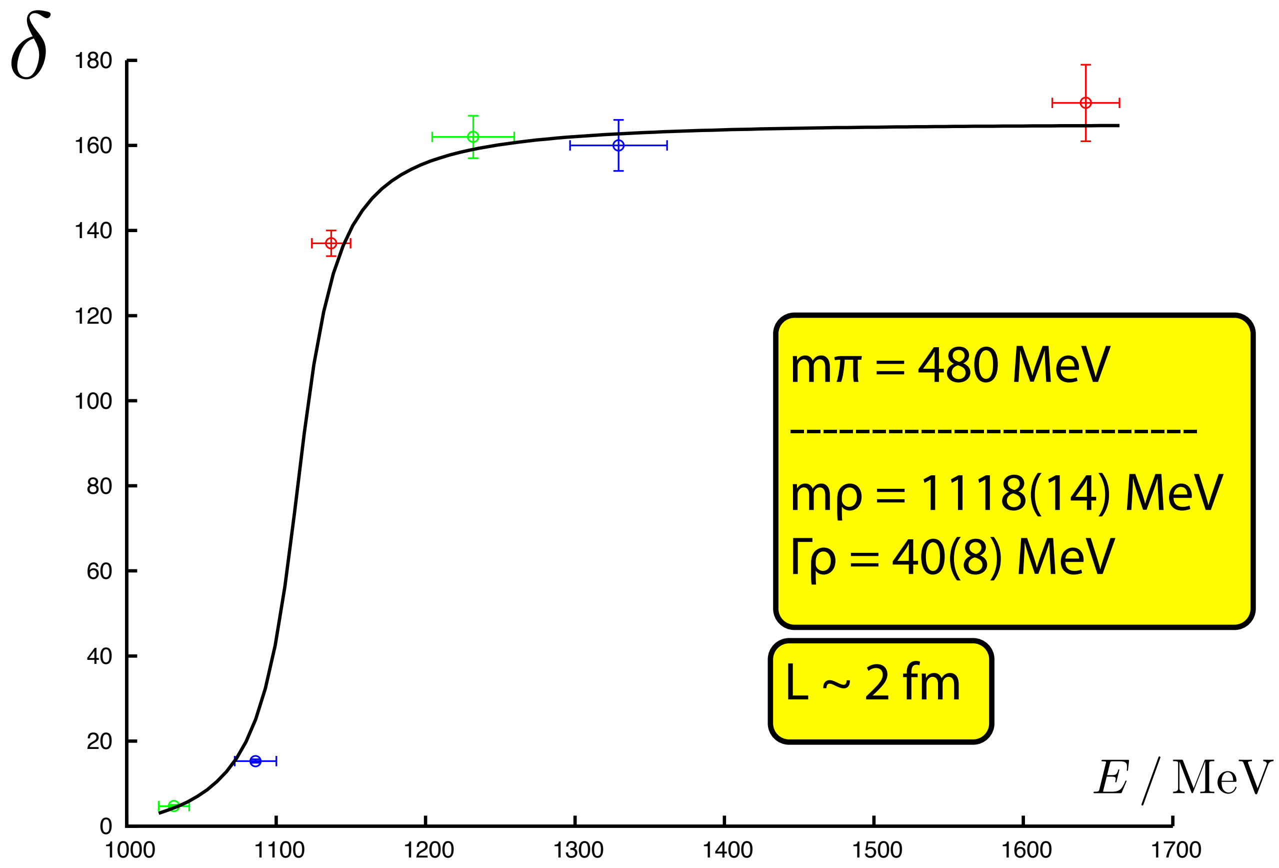

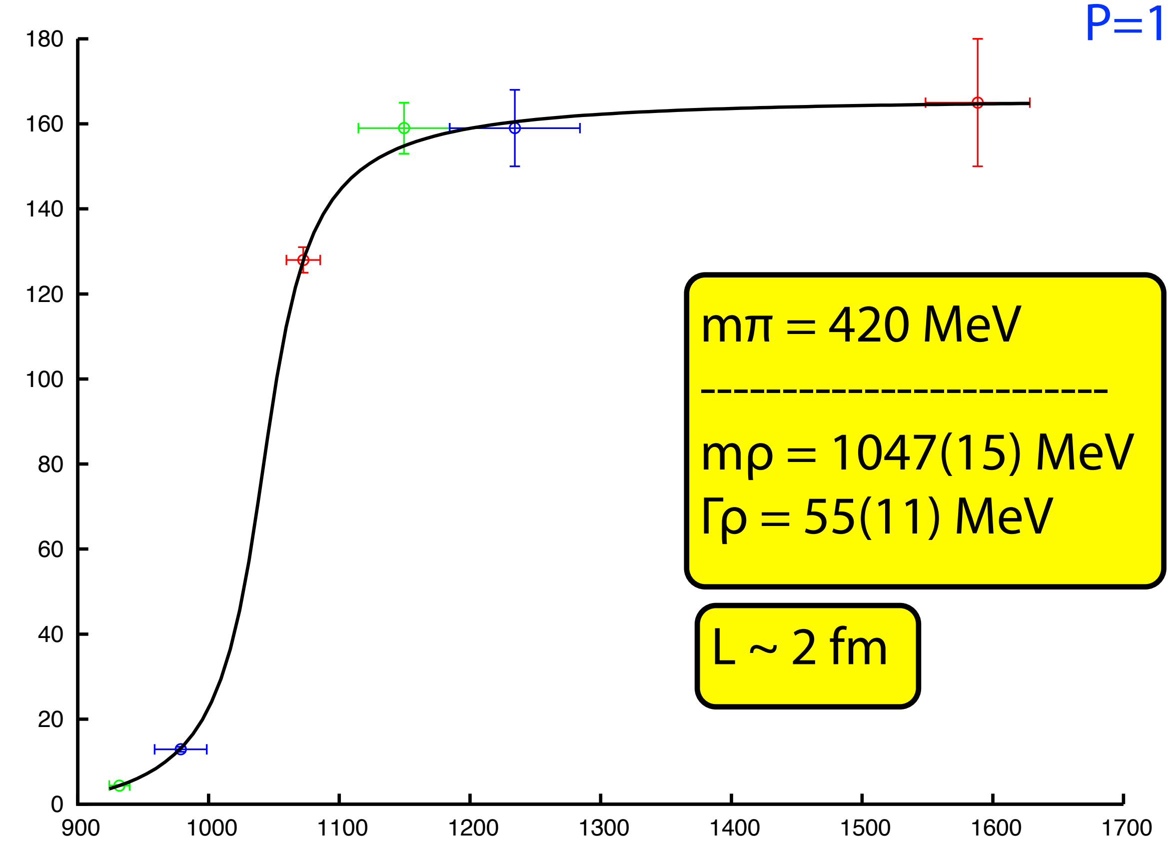

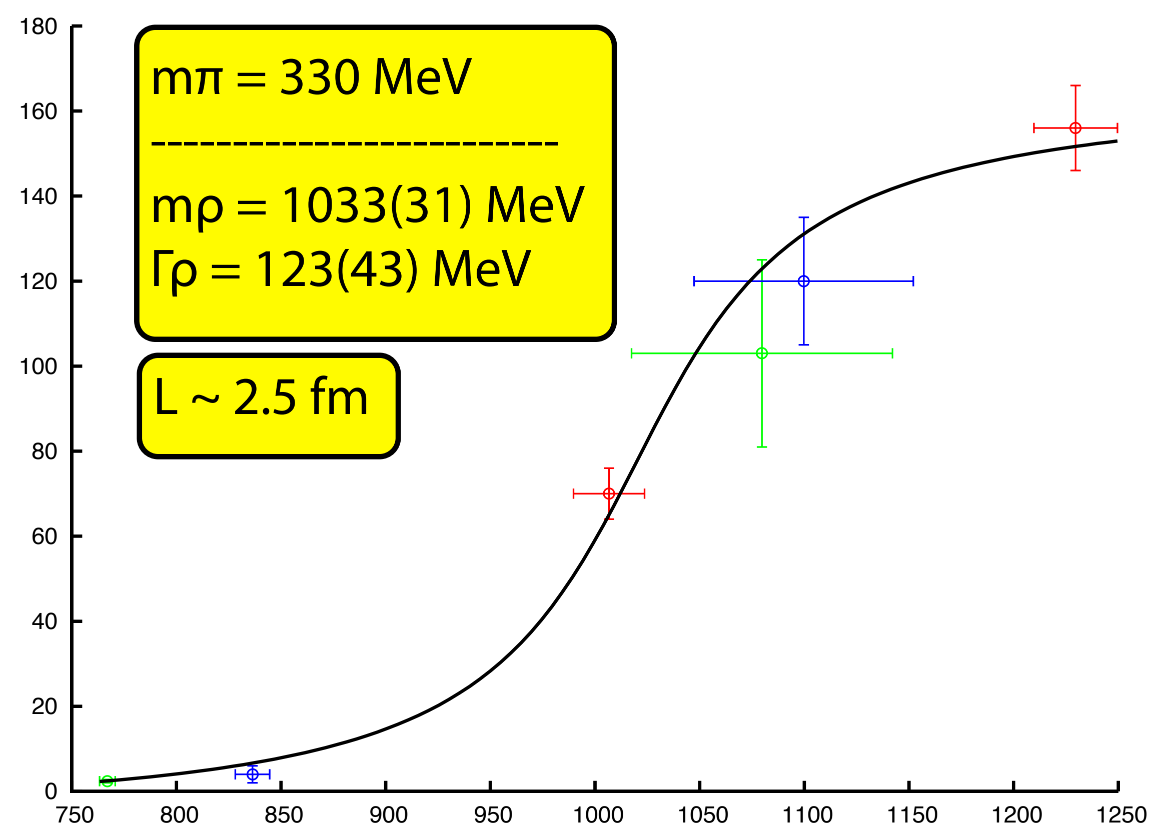

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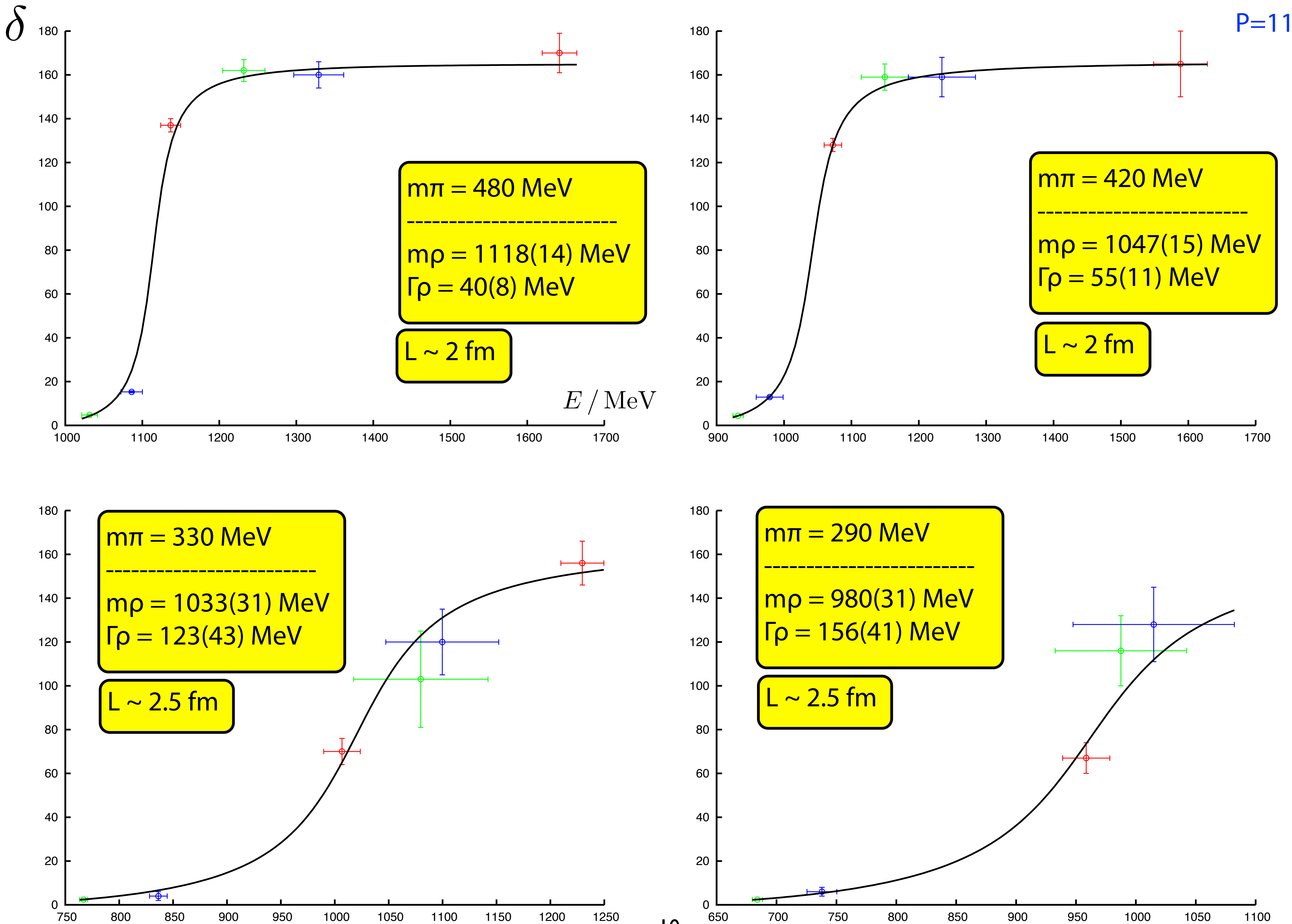

complications - higher spins

cubic symmetry (at best) ⇒ J not a good quantum number → "irreps"

at rest : T_1 (1^-, 3^- ...)

P = 100 : A_2 (1^-, 3^- ...) , E (1^-, 3^- ...)

P = 110 : A_2 (1^-, 3^- ...) , B_1 (1^-, 3^- ...) , B_2 (1^-, 3^- ...)

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multiple spins enter together in Lüscher formula:

$$0 = \det\left[\mathrm{diag}\left(e^{2i\delta_1}, e^{2i\delta_3}, \dots\right) - \mathbf{U}_\Gamma(k, L)\right]$$

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ideally consider multiple irreps to determine effect of higher spins

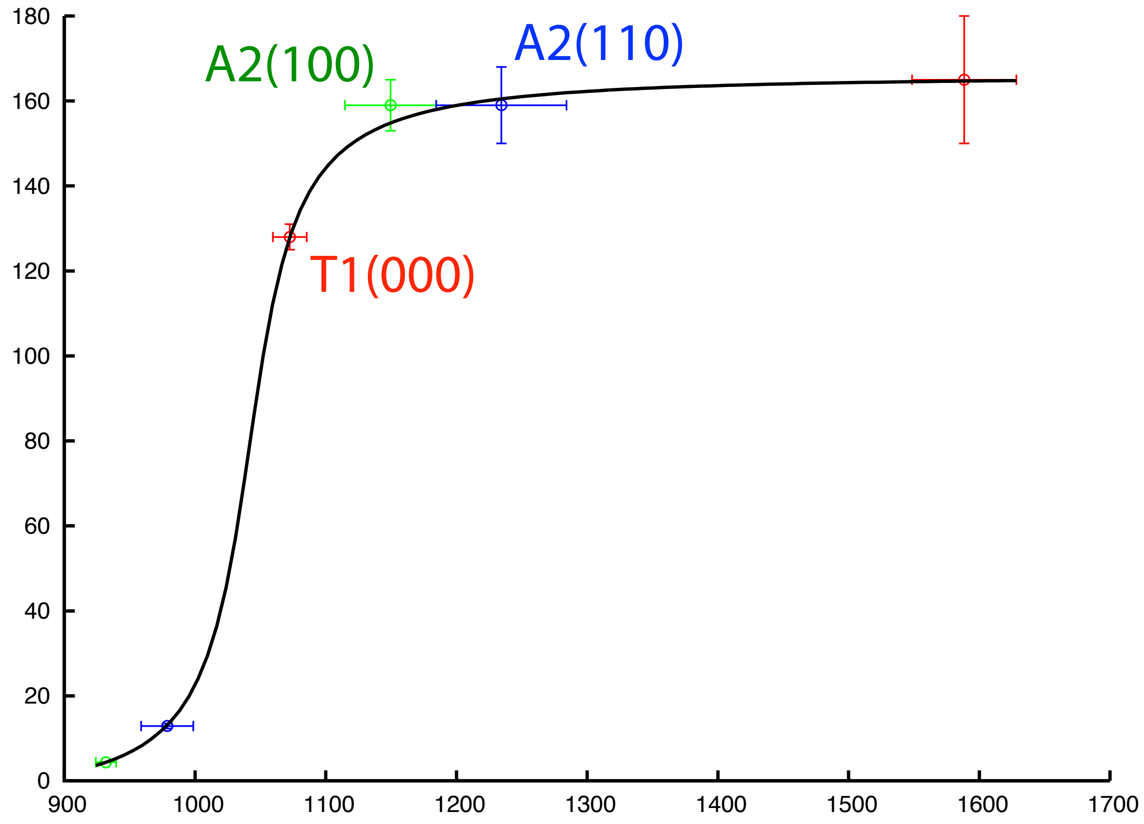
(probably) not a big deal for the ρ
- tail of the ρ_3 very small at ρ energy

will be important for other mesons
(π_1 and π_2 ?)

using the irreps

using all the irreps

still ignoring ρ_3



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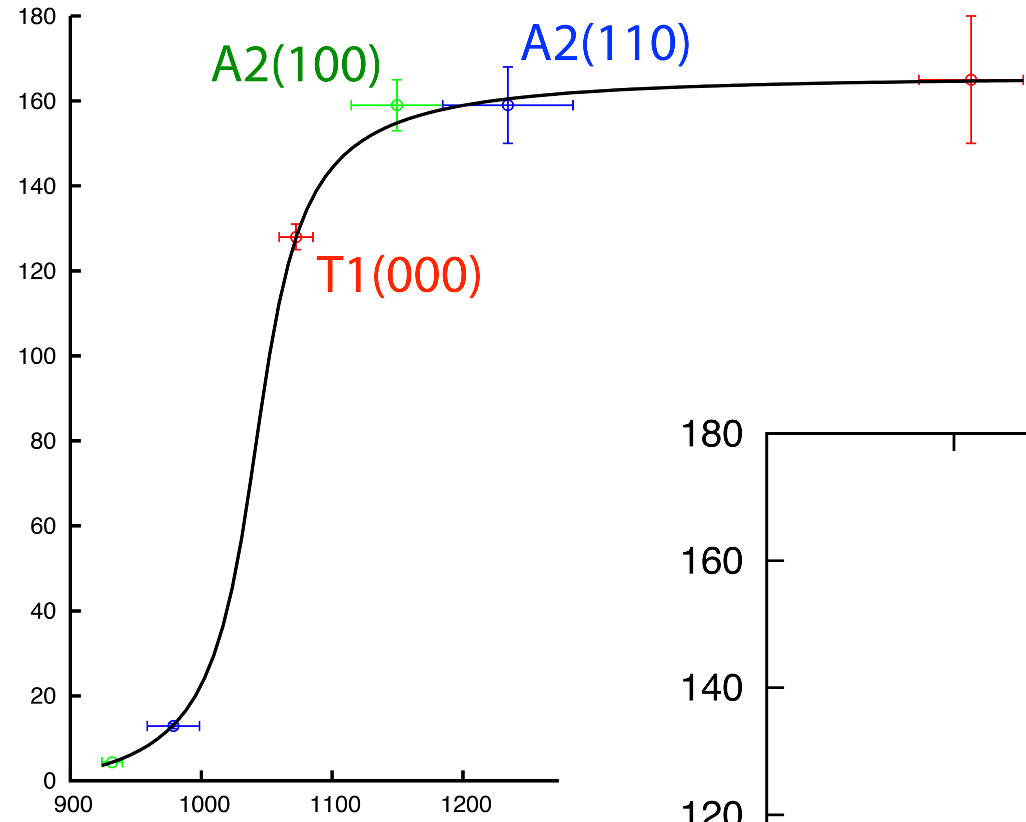
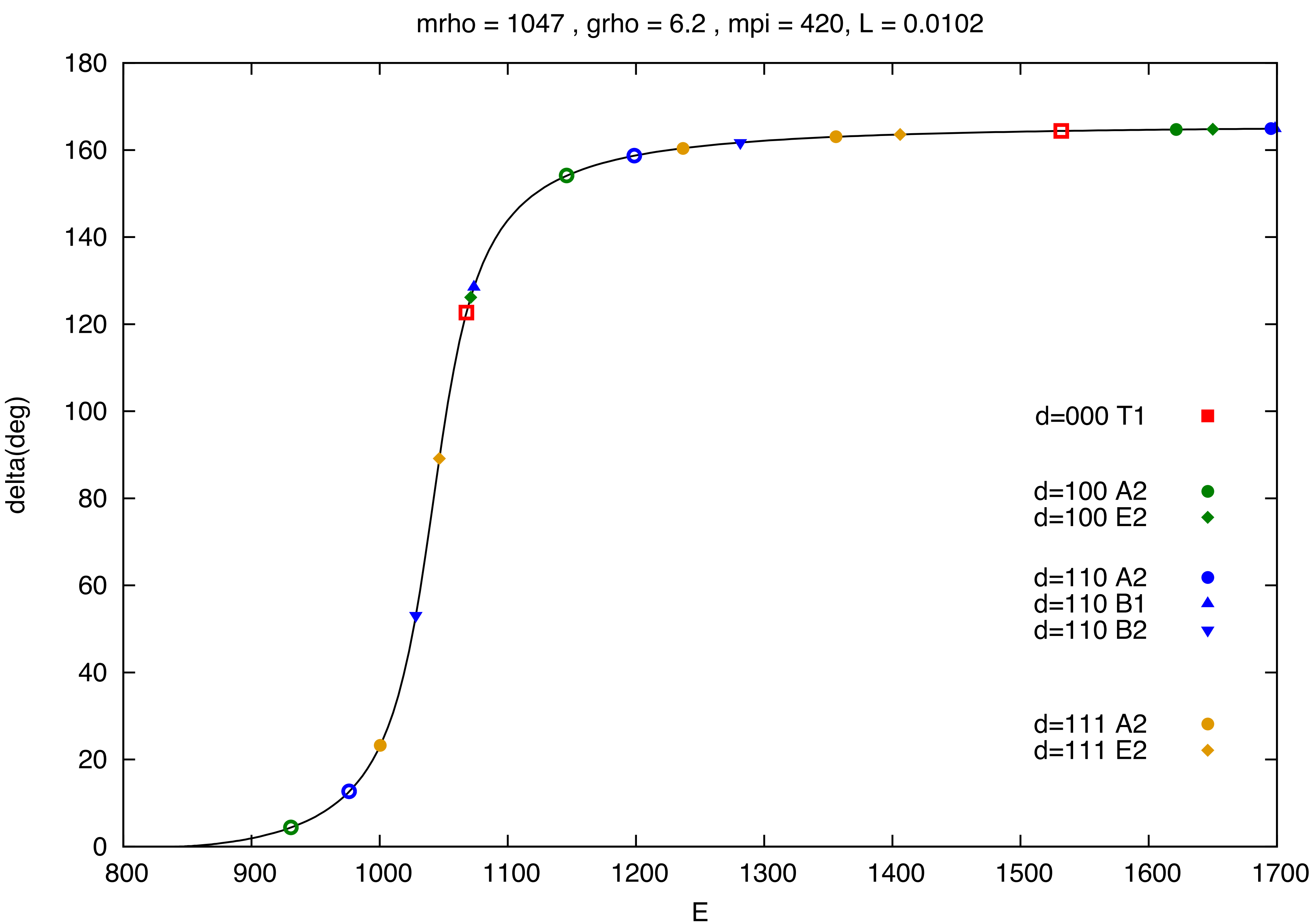
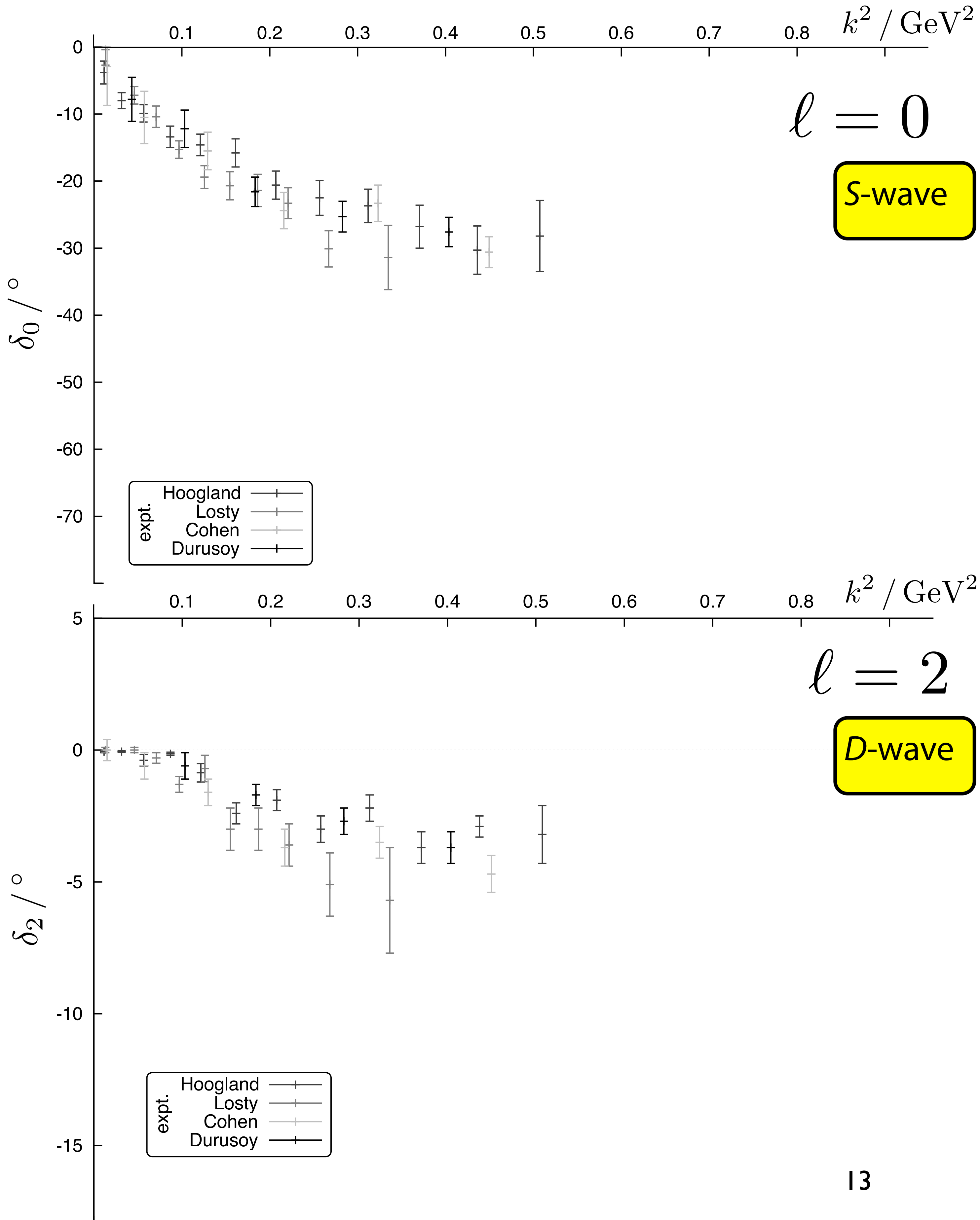


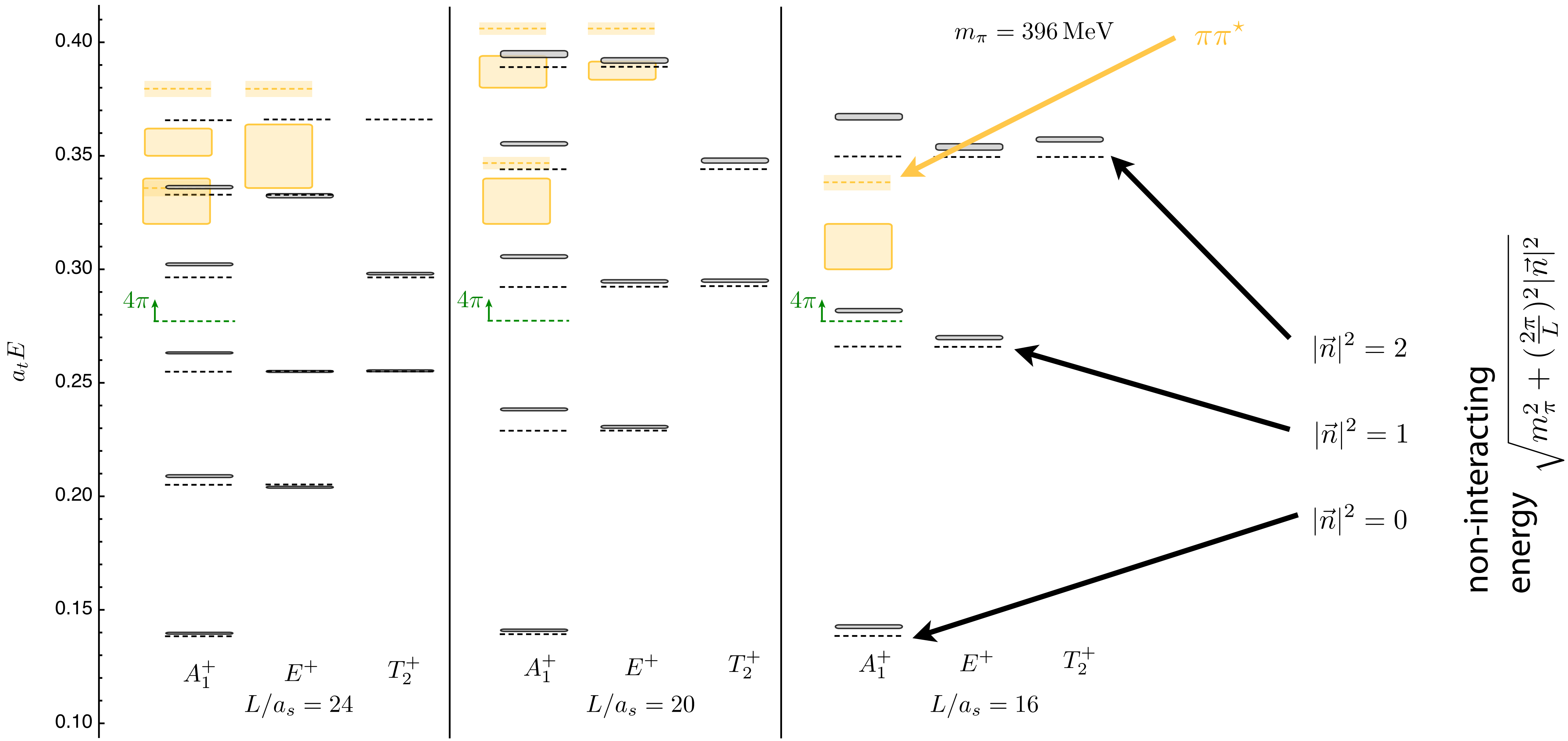


ππ I=2 - a non-resonant example



weak, repulsive interactions

ππ spectrum



$m_\pi = 396\,\mathrm{MeV}$
$\pi\pi^\star$
$a_t E$
0.40
0.35
0.30
0.25
0.20
0.15
0.10
4π
A_1^+
E^+
T_2^+
$L/a_s = 24$
$L/a_s = 20$
$L/a_s = 16$
$|\vec{n}|^2 = 2$
$|\vec{n}|^2 = 1$
$|\vec{n}|^2 = 0$
non-interacting energy
$\sqrt{m_\pi^2 + (\frac{2\pi}{L})^2|\vec{n}|^2}$

applying Lüscher method

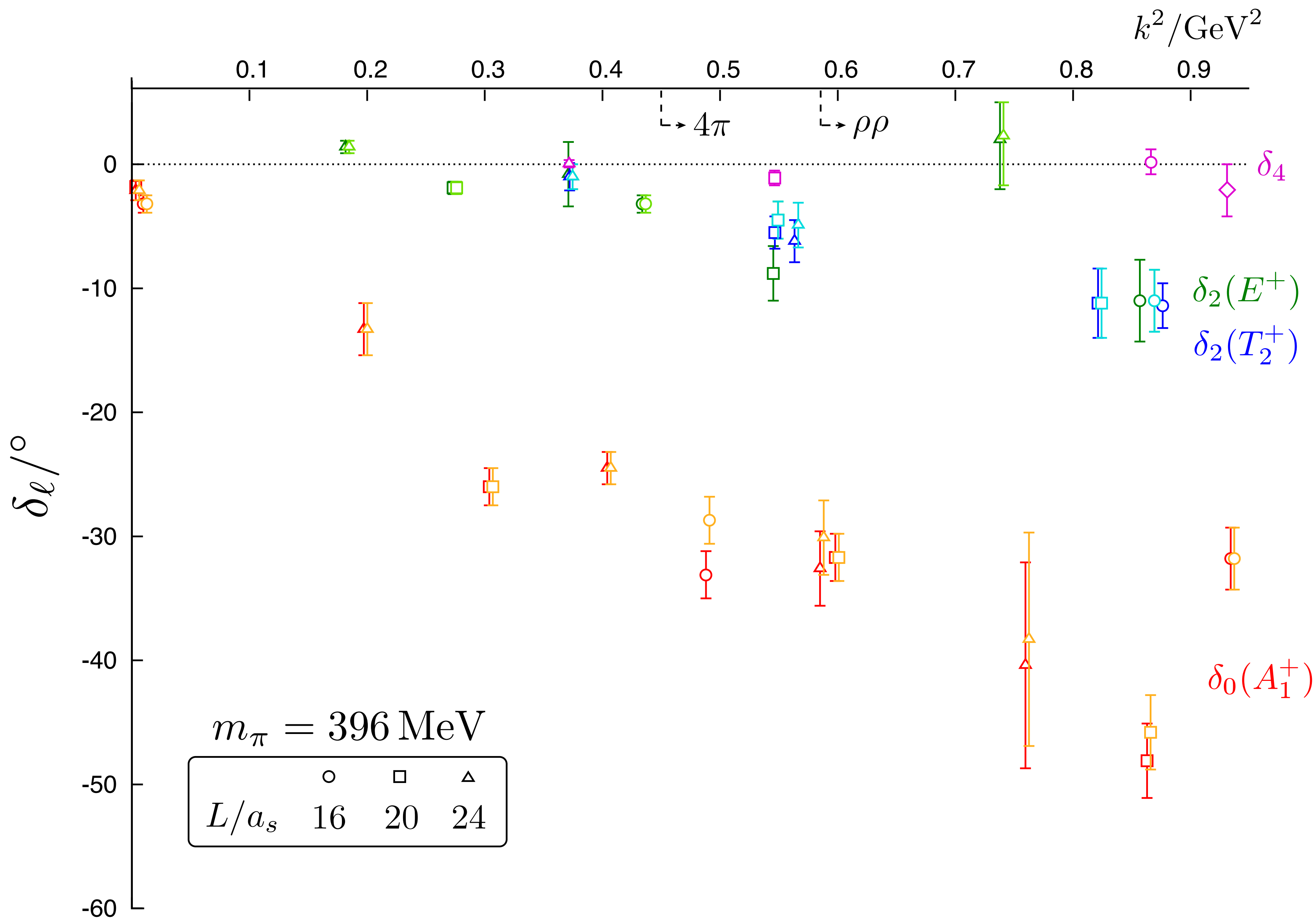
assume $\delta_{l>4} = 0$:

$A_1^+ \rightarrow \delta_0, \delta_4$

$E^+ \rightarrow \delta_2, \delta_4$

$T_2^+ \rightarrow \delta_2, \delta_4$

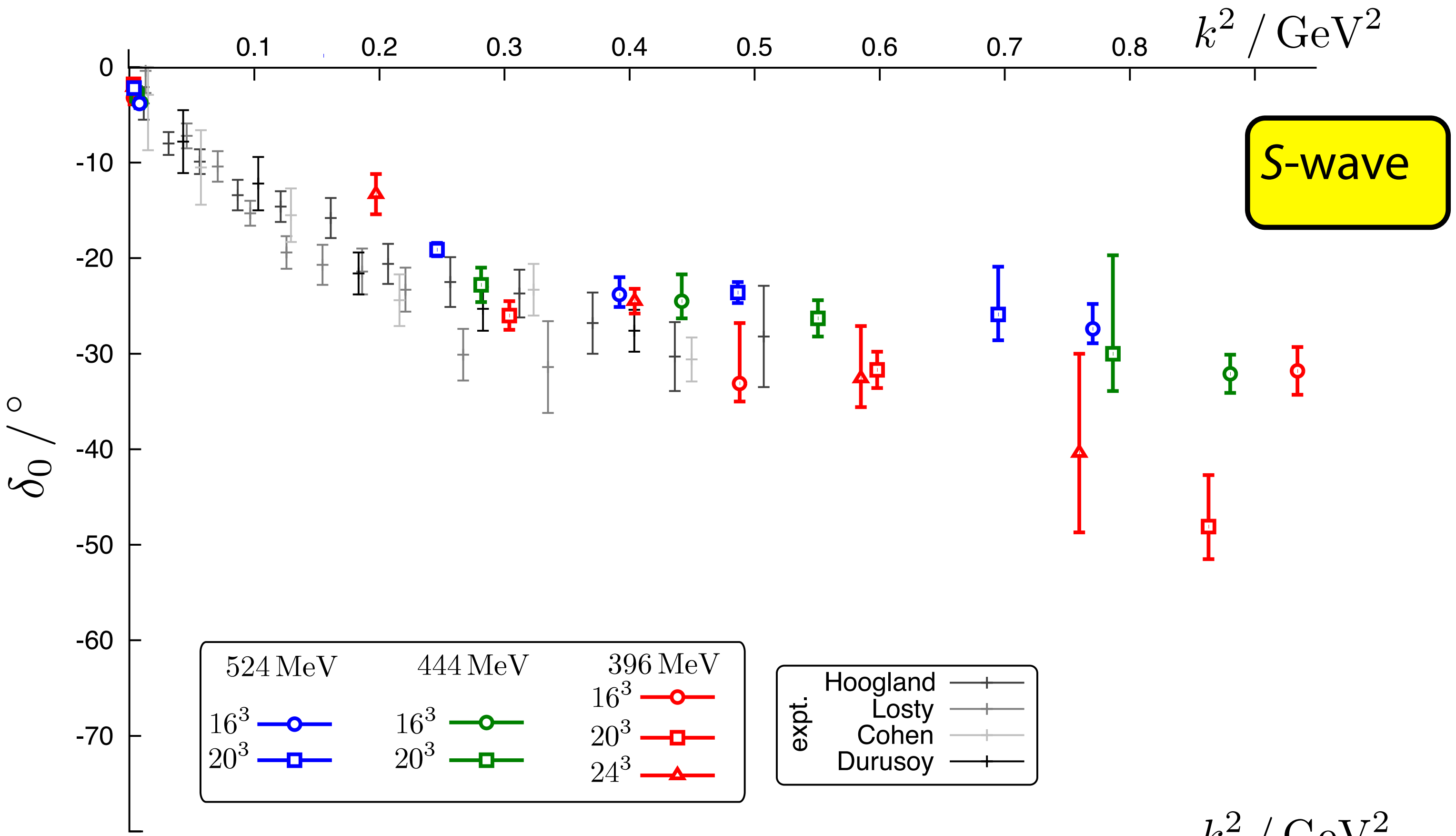
$$\left.\begin{array}{l} \det\left[\begin{pmatrix} e^{2i\delta_2(k)} & 0 \\ 0 & e^{2i\delta_4(k)} \end{pmatrix} - \mathbf{U}_{E^+}\left(\tfrac{kL}{2\pi}\right)\right] = 0 \\ \det\left[\begin{pmatrix} e^{2i\delta_2(k)} & 0 \\ 0 & e^{2i\delta_4(k)} \end{pmatrix} - \mathbf{U}_{T_2^+}\left(\tfrac{kL}{2\pi}\right)\right] = 0 \end{array}\right\} \delta_2(k),\ \delta_4(k)$$

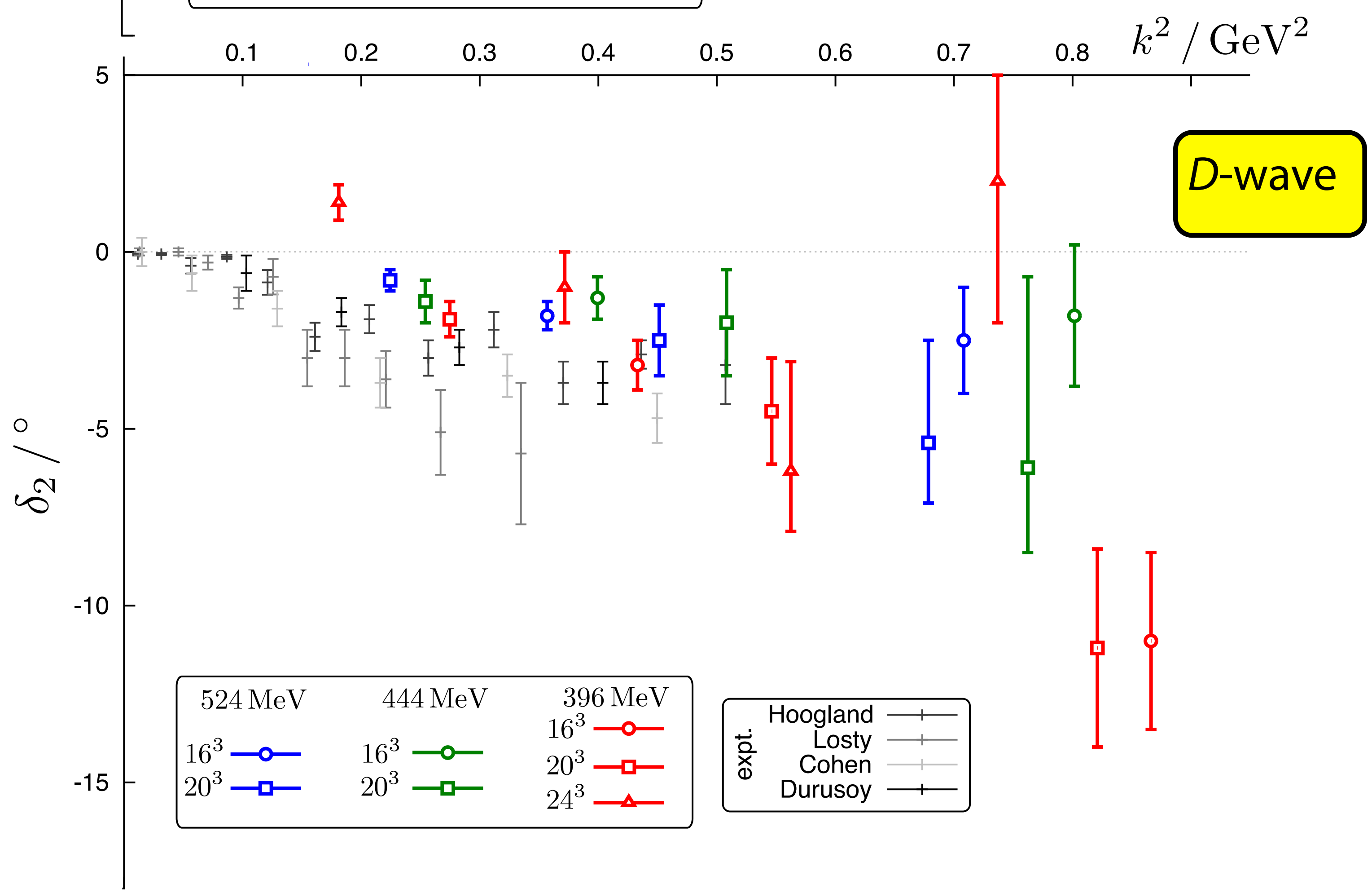


$|\delta_4(k)| < 2^\circ$

direct check from T_1^+ [210](24^3)

mass-dependence





very little mass dependence observed
... but limited range considered

summary

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- scattering of J>0 particles — think we have the formalism
- ... inelastic resonances ? — suggestions for a formalism

start using our m_π~280 MeV dynamical lattices