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Derivation: Polarization Figure-of-Merit

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* Suppose that I have some signal in a region of final-state phase space $d\hat{\Omega} = dt\, d\hat{M}$ where $\hat{M}$ represents mass and angle variables internal to the mesonic final state X in $\vec{\gamma} p \to X b$ (b is any final-state baryon)

* Let the variable ϕ represent the plane of $\hat{E}$ of the photon in the G-J frame, $\phi \in [-\pi, \pi]$.

* Then, events in $d\hat{\Omega}$ have a general ϕ distribution

$$f(\phi) = 1 + P(a \cos 2\phi + b \sin 2\phi)$$

where a and b measure the relative contributions from natural/unnatural exchanges. Given a sample of size N, how does the statistical error on extracted values for a, b depend on P? I use a maximum likelihood estimator:

$$\ln \mathcal{L} = \sum_{i=1}^{N} \ln f(\phi_i)$$

Solution: $$\frac{\partial \ln \mathcal{L}}{\partial a} = \sum_{i=1}^{N} \frac{P \cos 2\phi_i}{f(\phi_i)} = \frac{\partial \ln \mathcal{L}}{\partial b} = \sum_{i=1}^{N} \frac{P \sin 2\phi_i}{f(\phi_i)} = 0$$

These 2 equations can be linearized and solved by iteration in the usual way (see standard refs. on the M.L. method).

* Errors on M.L. estimators: $\hat{a}$, $\hat{b}$

In the large N limit, $\ln \mathcal{L}$ can be expanded as a parabolic form around the maximum, and the curvature gives the error inverse matrix.

$$\frac{\partial^2}{\partial a^2} \ln \mathcal{L} = -\sum_{i=1}^{N} \frac{P^2 \cos^2 2\phi_i}{f^2(\phi_i)} \sim -\frac{1}{\sigma_a^2}$$

There is a P-dependence in the denominator $\frac{1}{f^2}$, but the dominant behavior goes as the canonical form

$$\sigma_a^2 \propto \underbrace{(P^2 N)}^{-1}$$

· figure of merit.

That is, errors on parameter a go like $\frac{1}{P\sqrt{N}}$.

If that is what people mean by "linear in P" then we are in agreement with that. Traditional figure-of-merit is scaled to be linear in beam time, which gives a feeling for how much savings in operations are obtained with a given improvement in beam properties.